Chapter 3
The Plasma Focus—Numerical Experiments, Insights and Applications

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3.1 Introduction

3.1.1 Introduction to the Plasma Focus—Description of the Plasma Focus. How It Works, Dimensions and Lifetimes of the Focus Pinch

The Plasma Focus is a compact powerful-pulsed source of multi-radiation [1–3]. Even a small table top sized 3 kJ plasma focus produces an intense burst of radiation with extremely high powers. For example when operated in neon, the X-ray emission power peaks at $10^9$ W over a period of tens of nanoseconds. When operated in deuterium the fusion neutron burst produces rates of neutron typically $10^{15}$ neutrons per second over burst durations of tens of nanosecond. The emission comes from a point-like source making these devices among the most powerful laboratory pulsed radiation sources in the world. These sources are plasma-based. There are two main types of plasma focus classified according to the aspect ratio of the anode. The Filippov type [4] has an anode with a radius larger than its length. The Mather type [5] has a radius smaller than its length. In this chapter, we discuss the Mather-type plasma focus.

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When matter is heated to a high enough temperature, it ionizes and becomes plasma. It emits electromagnetic radiation. The spectrum depends on the temperature and the material. The higher the temperature and the denser the matter, the more intense is the radiation. Beams of electrons and ions may also be emitted. If the material is deuterium, nuclear fusion may take place if the density and temperature are high enough. In that case, neutrons are also emitted. Typically the temperatures are above several million K and compressed densities above atmospheric density starting with a gas a hundredth of an atmospheric density.

One way of achieving such highly heated material is by means of an electrical discharge through gases. As the gas is heated, it expands, lowering the density and making it difficult to heat further. Thus it is necessary to compress the gas whilst heating it, in order to achieve sufficiently intense conditions. An electrical discharge between two electrodes produces an azimuthal magnetic field which interacts with the column of current, giving rise to a self-compression force which tends to constrict (or pinch) the column. In order to ‘pinch’, or hold together, a column of gas at about atmospheric density at a temperature of 1 million K, a rather large pressure has to be exerted by the pinching magnetic field. Thus an electric current of at least hundreds of kA is required even for a column of small radius of say 1 mm. Moreover, the dynamic process requires that the current rises very rapidly, typically in under 0.1 μs in order to have a sufficiently hot and dense pinch. Such a pinch is known as a super fast super dense pinch, and requires special MA fast-rise (ns) pulsed-lines. These lines may be powered by capacitor banks, and suffer the disadvantage of conversion losses and high cost due to the cost of the high technology pulse-shaping line, in addition to the capacitor banks.

A superior method of producing the super dense and super hot pinch is to use the plasma focus. Not only does this device produce superior densities and temperatures, moreover its method of operation does away with the extra layer of technology required by the expensive and inefficient pulse-shaping line. A simple capacitor discharge is sufficient to power the plasma focus.

The plasma focus

The Mather-type plasma focus is divided into two sections, the axial and the radial sections (see Fig. 3.1). The function of the axial (pre-pinch) section is primarily to delay the pinch until the capacitor discharge (rising in a distorted sinusoidal fashion) approaches its maximum current. This is done by driving a current sheet down an axial (acceleration) section until the capacitor current approaches its peak. Then the current sheet is allowed to undergo transition into a radial compression phase. Thus the pinch starts and occurs at the top of the current pulse. This is equivalent to driving the pinch with a super fast rising current without necessitating the fast line technology. Moreover, the intensity which is achieved is superior to the line driven pinch.

The simplified two-phase mechanism of the plasma focus [6] is shown in Fig. 3.1. The inner electrode (anode) is separated from the outer concentric cathode by an insulating back wall. The electrodes are enclosed in a chamber, evacuated and
typically filled with gas at about 1/100 of atmospheric pressure. When the capacitor voltage is switched onto the focus tube, breakdown occurs axisymmetrically between the anode and cathode across the back wall. The 'sheet' of current lifts off the back wall as the magnetic field ($B_\theta$) and it's inducing a current ($J_r$) rises to a sufficient value.

**Axial phase:** The $J_r \times B_\theta$ force then pushes the current sheet, accelerating it supersonically down the tube. This is very similar to the mechanism of a linear motor. The speed of the current sheet, the length of the tube and the rise time of the capacitor discharge are matched so that the current sheet reaches the end of the axial section just as the discharge reaches its quarter cycle. This phase typically lasts 1–3 $\mu$s for a plasma focus of several kJ.

**Radial Phase:** The part of the current sheet in sliding contact with the anode then 'slips' off the end 'face' of the anode forming a cylinder of current, which is then pinched inwards. The wall of the imploding plasma cylinder has two boundaries (see Fig. 3.1 radial phase). The inner face of the wall, of radius $r_s$ is an imploding shock front. The outer side of the wall, of radius $r_p$ is the imploding current sheet, or magnetic piston. Between the shock front and the magnetic piston is the annular layer of plasma. Imploding inwards at higher and higher speeds, the shock front coalesces on-axis and a super dense, super hot plasma column is pinched onto the axis (see Fig. 3.2). This column stays super hot and super dense for typically tens of ns for a small focus. The column then breaks up and explodes. For a small plasma focus of several kJ, the most intense emission phase lasts for the order of several tens of ns. The radiation source is spot-like (1 mm diameter) when viewed end-on.

Figure 3.3 shows a drawing of a typical plasma focus, powered by a single capacitor, switched by a simple parallel-plate spark gap [7]. The anode may be a hollow copper tube so that during the radial pinching phase the plasma not only elongates away from the anode face but also extends and elongates into the hollow anode (see Fig. 3.2). Figure 3.3 shows the section where the current sheet is
accelerated axially and also the radial section. Also shown in the same figure are shadowgraphs [8] taken from the actual radially imploding current sheet-shock front structure. The shadowgraphs are taken in a sequence, at different times. The times indicated on the shadowgraphs are relative to the moment judged to be the moment of maximum compression. That moment is taken as $t = 0$. The quality of the plasma compression can be seen to be very good, with excellent axisymmetry, and a very well compressed dense phase. In the lower left of Fig. 3.3 are shown the current and voltage signatures of the radial implosion [9], occurring at peak current. The implosion speeds are measured and have a peak value approaching 30 cm/μs. This agrees with modelling, and by considering shock wave theory together with modelling [10] of subsequent reflected shock wave and compressive effects, a temperature of 6 million K (0.5 keV) is estimated for the column at peak compression, with a density of $10^{19}$ ions per cm$^3$. The values quoted here are for the UNU/ICTP PFF 3 kJ device [7]. Dimensions and lifetimes of the pinch are indicated by these images.

An observed property of plasma focus machines is that they operate at similar speeds. A sub-kJ focus [11] has about the same peak axial speed and the same (higher) peak radial speed as a 1 MJ focus [12]. Since the square of speed represents energy density (energy per unit mass) this means the machines achieve similar temperatures in each of their dynamical phases and also similar highest temperatures in the plasma focus pinch. This is a remarkable scaling property of the plasma focus which will be discussed further in this chapter.
3.1.2 Review of Models and Simulation

Observations of the axial acceleration phase of the plasma focus show that typically the current sheath travels at greater than Mach 10 speeds over more than 95% of the axial phase. Thus the axial phase is in the strong-shock electromagnetic regime and may be approximated by a snowplow model essentially considering a thin sheath trapping all encountered mass which is driven by the self-generated electromagnetic force of the current flowing through the ionized sheath. Such a model was used by Rosenbluth [13].

Such one-fluid formulation of a thin (impermeable piston-like ‘snowplow’) current sheath was also presented by various authors [14–16] justifiable on assumptions of small ion–electron collision times, ion–ion collision lengths being short compared to shock layer thickness, and infinite conductivity or at least large magnetic Reynolds number. Shock conservation equations in 2-D were used.
A similar one-fluid time-dependent model was solved by Amsden [17] by the particle-in-cell method with good agreement with experimental results of the shape of the sheath for a coaxial gun with no outer electrode. The one-fluid model has a serious limitation, as it does not consider the effects of ionization. Whilst that may not affect the equation of conservation of momentum it certainly affects the calculation of temperature and of the structure of layers near the interaction zones with the electrodes.

Two-dimensional two-fluid models [18, 19] were developed for MHD modelling of an entire plasma focus discharge (until the final pinch phase) assuming a thin fully ionized plasma layer as an initial condition. Potter’s work [18] was used to discuss many aspects of flow dynamics within the plasma focus pinch. Cylindrical symmetry is assumed so that each dependent variable is a function of \( r, z, t \) radius, axial position and time, respectively. The plasma is described by six dependent variables namely \( \rho, \rho v, B_\theta, \rho e_e \) and \( \rho e_i \) where \( \rho \) is the plasma density, \( v \) is the centre of mass velocity with two components, \( B_\theta \) is the azimuthal magnetic field and \( e_e, e_i \) are the thermal energy densities per unit fluid mass for the electrons and ions, respectively. The MHD equations are the conservation of mass, momentum and magnetic flux appropriately for the dependent variables \( \rho, \rho v, B_\theta \), respectively, with the total pressure tensor in the momentum equation being written as the sum of the scalar pressure with electron and ion contributions and the current density \( j \) in the momentum equation calculated from the curl of \( B_\theta \). The conservation of energy is written in the form of two separate equations for the electron and ion thermal energy densities in which terms due to Joule heating, bremsstrahlung radiation, equipartition of energy between electrons and ions, viscous heating, electron and ion heat conduction are included. The thermal energy densities of the electrons and ions are, respectively, written in terms of the electron temperature and the ion temperature using the specific heat ratio. Quasi-neutrality is assumed which defines the electron velocity in terms of the ion or ambipolar plasma velocity. Maxwell’s equations and the generalized Ohm’s law, defining the electric field, complete the set of MHD equations. In the process of solving the equations variable transport coefficients for the electron and ion heat conductivities, resistivity, viscosity and equipartition times are used. Ion orbiting effects are included incorporating finite Larmor radius effects. Boundary conditions are specified and the domain of dependence is coupled with an external \( L-C \) circuit. The equations are integrated over small time steps, using second-order two-step Lax-Wendroff method. To increase the spatial resolution in the focus stage a fine scale mesh is introduced at the end of the centre electrode. The results of the computation show three main regions in the focus pinch: an anode cold source, a hot pinch region and an axial shock. The observed anomalously long lifetime of the plasma focus is shown to be the result of axial flow with stabilization of MHD modes through the ion stress tensor in the intermediate collisionless, collision-dominated regime.

For explaining the experimental results of D–D neutron yield from the specific plasma focus device, Potter used a thermonuclear mechanism. Potter extracted results based on the computed temperature and density profiles. Although his visualization of the two-dimensional flows within the pinch column had several
points of agreement with experimental observations, his conclusion of a mainly thermonuclear fusion mechanism within pinch column was presumably affected by computed flow velocities of too high a value. The consensus view today from generations of experiments is that the fusion mechanism of the plasma focus has a main component which is beam-plasma target [1, 2, 12, 20–22].

A three-fluid MHD model by Bazdenkov and Vikhrev took into account the initial ionization [23]. In a similar manner, Behler and Bruhns [24] extended the two-fluid Potter code to a three-fluid model. With the three-fluid treatment, neutral gas was added to the plasma components to incorporate the effects of recombination and ionization. Additional mass, momentum and energy conservation equations were written for the neutral component of the fluid. Treating the elastic processes in a similar way as Potter’s work, for the inelastic processes Behler and Bruhns considered electron impact ionization from the ground state, radiative recombination, three-body Auger recombination and charge exchange. Various approximations were also made in adjustments of the coefficients of these processes. The sheet dynamics were studied including residual gas (or plasma) density behind the current sheet in the run-down phase leading to the occurrence of leak currents.

Behler and Bruhns [24] applied the resultant three-fluid model to SPEED 1 with an initial (starting) current of 1 kA assumed to flow along the insulator. Subsequent current was determined by the circuit equation. The results show the dynamics of the lift-off and the structure of the residual neutral density, followed by the dynamics of the current sheath. At 250 ns the sheet has almost reached the end of the centre electrode (the anode) with an axial speed of 20 cm/µs about 8% faster than the two-fluid model. During this time a neutral density of up to $2 \times 10^{16} \text{ cm}^{-3}$ is left behind, the maximum is close to the anode at $z = 4–6 \text{ cm})$. Beyond the tip of the anode, the accelerated sheet continues to move axially. The radial velocity attains 12.5 cm/µs, leading to a total time of 510 ns from breakdown to maximum compression compared to experimentally observed of 470 ns. The calculated maximum current of 780 kA occurs at 380 ns compared to 770 kA appearing at 370 ns. However, the current dip is less pronounced in the computation than observed experimentally and the computed pinch radius remains much larger than inferred from Schlieren pictures. The late pinch phase could not be simulated due to the finite mesh size and the inapplicability of the MHD model. The model code was also applied to SPEED 2, the 250 kJ high-speed Poseidon and the large Frascati PF (1 MJ). Their conclusion is that there is a comparatively good agreement in all cases between the calculated overall features of the discharges and experiments. In particular, the three-fluid 2-D model could provide the leakage currents and had points of agreement in the axial and radial dynamics in both the dynamics and structure of the current sheet. The computation could not be completed to the later part of the pinch.

Magnetohydrodynamic codes face this same problem. In a paper by Nukulin and Polukhin [25] the MHD computed current waveform of PF1000 using the Vikhrev MHD code [26] was also compared with the measured current waveform. The computed waveform agrees well with the measured down to about a third of the current dip. The computed current had a value of 1.6 MA at peak compression
compared to a measured value of 1.1 MA at peak compression, indicating that the MHD treatment was unable to correctly describe the current waveform for a large part of the pinch which occurs at the end of the pinch towards maximum compression and beyond.

The failure of MHD codes to describe this crucial part of the pinching process is due to the breakdown of the assumptions used in calculating the transport coefficients. To avoid such failures one way is to use the kinetic method. The fluid MHD approach considers each dependent variable as a function of space and time (4 independent variables for 3-D simulation and 3 independent variables for 2-D simulation) with the velocity distribution of each species assumed to be Maxwellian everywhere so that it may be uniquely specified by only one number, the temperature $T$ [27]. The kinetic method being a fundamental approach makes no such assumption so that each dependent variable is a function of space, velocity space and time (7 independent variables for 3-D simulation and 5 independent variables for 2-D simulation). The plasma is described by a distribution function which is a function of space, velocity space and time. The basic conservation law is described by Liouville Theorem which in the form of Boltzmann Equation enables to follow the evolution of the distribution function with time. Once the distribution function is obtained at any time at any point of space, any macroscopic quantity may be obtained by integrating the product of that quantity with the velocity distribution function over all velocity space.

A. Schmidt et al. used fully kinetic simulations applied to a kJ plasma focus [28, 29]. They demonstrated that both kinetic ions and kinetic electrons are needed to reproduce experimentally observed features, such as charged-particle beam formation and anomalous resistivity. A. Schmidt et al. extended the method to megajoule-scale plasma focus devices [30] specifically for the Gemina. The cathode (outer electrode) was simulated by a conducting boundary at $r = 10$ cm to represent the set of 24 rectangular rods arranged in a circular pattern with gaps in between where gas can escape. The experimental anode was 57 cm long. To reduce simulation run time the calculation was initialized with 2-D MHD simulation using ALEGRA [31]. The fluid simulation began with the plasma sheath at the insulator and proceeded into the sheath run-in radial phase at $6.6 \mu$s, when the plasma profiles were transferred to the particle-in-cell (PIC) code large scale plasmas ($LSP$) [32] for kinetic simulation with time step dynamically varied from $2.5 \times 10^{-4}$ to $8 \times 10^{-6}$ ns to resolve the electron cyclotron frequency. Densities at $z < 9.5$ cm were set to 0 to reduce the total number of particles in the simulation. The kinetic simulation was then run for a total of 26 ns; 11 ns prior to the formation of the pinch and for an additional 15 ns of the pinch. The voltage drive was modelled with a prescribed incoming voltage wave travelling the length of the anode, with a reflected wave travelling back. The voltage was ramped up during the first 10 ns of the simulation and then kept constant for the remainder of the simulation, resulting in a steady state current of 1.94 MA before the pinch formation. At the end of the 26 ns of kinetic simulation, the pinch had not completely stopped producing neutrons. The simulation was stopped due to computing resource limitations and the remaining yield was extrapolated from the simulated yield curve. Extrapolated
estimates of the neutron yield are $1.5 - 2 \times 10^{11}$ at 3.6 Torr which is consistent with an experimentally measured yield of $1.5 \times 10^{11}$ at 3.5 Torr. The 26 ns of kinetic simulation also predicted ion and neutron spectra, neutron anisotropy, neutron spot size of 0.7 mm for Gemini DPF and time history of neutron production. In the forward direction 5 ns after the start of the pinch, the bulk of ions had energies below 100 eV with a high-energy tail extending to 1 meV. Preliminary measurements of ion beam energies on the Gemini DPF using a Faraday cup time of flight (TOF) measurement observed deuterons with energies up to 310 keV. The main neutron production appeared to be produced from the pinch region ($r = 0-2.5$ mm) and was apparently dominated by beam-target fusion, producing a wide spread in energies around a central peak at 2.45 meV. Outside this region, neutrons were predominantly produced at 2.45 meV with an 11 keV width, characteristic of thermonuclear fusion or low-energy beam-target fusion. The results also indicated anomalously high plasma resistivity during the pinch with plasma resistance rising to 0.7 $\Omega$ at peak neutron production.

The fully kinetic simulation of Schmidt et al. [28–30] may be the most advanced simulation so far carried out for the plasma focus pinch. This work has demonstrated the capability of such fundamental methods to show details of the pinch plasmas. However considerable theoretical sophistication and computing resources are required as can be seen in the work of Schmidt et al. [30] for just 26 ns of simulation. Hence the technique is not available for general use on any machine.

On the other hand, simple methods with varying degrees of utility may be used in attempts to look at experimental neutron yields. For example, Moreno et al. [33] and Gonzalez et al. [34] applied modelling codes based on thermonuclear fusion mechanism by adjusting axial and radial mass sweeping factors in their particular plasma focus devices until acceptable matching between computed neutron yield $Y_n$ and measured $Y_n$. In addition, their method of calculating shock speeds was based upon an old version of Lee Model Code (pre-1995) [6, 35] which did not include the important properties of ‘communication delay’ between the shock front and driving magnetic piston in the radial plasma slug [36]. That pre-1995 version over-estimated the shock speed by factor 2, shock temperature by factor 4 and D-D fusion cross section by factors exceeding 1000. After 1995, the Lee Model Code [10, 37] has included this ‘communication delay’ and its results in terms of dynamics and radiation yields have an acceptable correlation with experimental results.

In 2009, Gonzalez et al. [38] used Von Karman approximations of radial velocity and density profiles to fit the experimental $Y_n$ versus gas pressure curve of the seven plasma focus devices using thermonuclear mechanism. Four parameters namely axial and radial shape parameters, velocity profile exponent and density profile exponent were used to describe this model and these values were adjusted until the $Y_n$ versus pressure curve for each machine fitted the measured $Y_n$ versus pressure curve. It appears that that is the sole purpose of this modelling. There is no discussion that this model has any predictive value, especially in the case of neutron emission yield.

From the above review, it is seen that much of the numerical work that has been carried out is motivated by a desire to simulate the plasma focus to obtain a computed picture of flow dynamics and the density and temperature structure of the
plasma focus pinch. There has been a particular motivation to simulate the $Y_n$ to determine the mechanism of neutron production and to have a method to predict the $Y_n$ of a machine. Additionally, several studies towards other uses and applications may also be noted here.

Trunk carried out a numerical study of the parameters of the plasma focus machines at Stuttgart [39] using MHD equations coupled to the electrical circuit. The influence of varying circuit parameters, focus apparatus dimensions, and filling pressure on the discharge characteristics, especially the maximum current, and the plasma variables in the pinch phase were examined and compared with experimentally determined neutron outputs. An experimentally derived scaling law for the dependence of maximum neutron output on bank energy, filling pressure and length of the inner electrode was confirmed by the results of the MHD computations. In the process, Trunk observed that the optimum conditions for the focus experiment “NESSI” occurred at an external inductance $L_0$ of 20 nH.

The harnessing of nuclear fusion energy has been described as indispensable to the salvation of our planet [40], indeed the next giant step of Mankind [41]. In this context, early work on the plasma focus generated great excitement in demonstrating that fusion yield was proportional to stored energy squared in the plasma focus. Conditions for plasma focus to achieve energy breakeven had been a topic of discussion. The currents required for breakeven fusion was a highly optimistic 10 MA as predicted by Imshennick et al. [42] using a similarity calculation. Vikhrev and Korolev [43] predicted a more realistic though still highly optimistic value of 30 MA. In the context of very large machines, an attempt had been made to discuss neutron saturation in terms of a proportional relationship between tube inductance and storage capacitance [21, 25, 44]. Using a completely different approach Lerner et al. [45] proposed to achieve controlled fusion with hydrogen-boron (p-B$^{11}$) fuel in the dense plasma focus. The proposal is based on a theory of plasmoids within the plasma focus pinch. The theory envisages the trapping of magnetic energy within the plasmoid and plasma instability conditions relating to electron gyro-frequency, and ion and electron plasma frequencies. Estimates of dimension, magnetic field, ion density and lifetime of these plasmoids are made resulting in a favourable picture to support aneutronic fusion. The concept is being tested in the Focus-Fusion-1 [46].

An enterprising ongoing project to obtain so-called ‘global optimization’ has been discussed recently by Auluck [47] regarding the use of the Gratton-Vagras 2-D electromechanical model for the construction of an appropriate design tool to search for a globally optimized “best possible” design to maximize quantitative performance criteria per unit of stored energy. According to Auluck, the Gratton-Vargas model [48] can currently calculate optimality parameters of designs at the rate of 150,00 designs a day with considerable scope for further improvements in speed. This model maps the 10 parameters of a DPF installation—capacitance, inductance, resistance, voltage, anode radius, anode length, insulator radius, insulator length and gas pressure—on to 7 dimensionless model parameters, out of which 5 are relevant for optimization. This circumstance enables generation of a database of a limited set of optimality properties (such as average power transfer efficiency, electromagnetic work, energy per particle, a fraction of energy dissipated in circuit resistance, etc.) of
Mather-type DPF devices in a practically relevant range of parameters. The cases presented by Auluck do not include variation in thermodynamics or the effect of radiation. Therefore the ‘global optimisation’ could not have an application in most of plasma focus operation which spans not just plasma focus of varied geometry and electrical parameters but also many gases in which the variations of thermodynamics and the effects of radiation on the dynamics and compression are significant, even dominant. Without including these effects even the dynamics is not accurately computed and electromagnetic work, energy and power transfer efficiencies are incorrectly scaled for most of the gases in which plasma focus machines operate.

3.1.3 A Universal Code for Numerical Experiments of the Mather-Type Plasma Focus

The Lee model code [10, 37] uses the snowplow model [13] in the axial phase, the slug model [36] with ‘communication delay’ and thermodynamics in the early radial phase and a radiation-coupled compression in the pinch phase; all the phases being rigorously circuit-coupled so as to be energy and charge consistent.

The code has on the one hand been successful in developing a number of deep insights and on the other hand has been successful in applications ranging from correlation with experiments in dynamics, neutron and soft X-ray (SXR) yields, in fast ion beams (FIB) and fast plasma streams (FPS) properties and in designing plasma focus and variants. Its success on so many fronts appears to be due to its use of 4 parameters (fitted to a measured current waveform) which in one sweep incorporates all the mechanisms and effects occurring in the plasma focus including mechanisms and effects difficult to compute or even as yet unknown.

The simple treatment of the axial and early radial phases has nevertheless produced several important insights into plasma focus such as an optimum static inductance [49] and current and neutron scaling and yield deterioration (for which a misnomer would be the word ‘saturation’) [21, 44]. The radiation-coupled equation of motion used in the pinch phase has enabled pioneering work on radiative collapse in the plasma focus [50–54].

On a more practical level, SXR experiments using Pin Diode system in INTI PF operated in Ne to correlate typically measured profile with dynamics computed from the code have been demonstrated [55, 56] and optimization of UNU/ICTP PFF plasma focus guided by the code for Ne soft X-ray operation has been presented [57]. A recent extension of the code to compute ion beam has pioneered the establishment of reference units and numbers for fast ion beams (FIB) and fast plasma streams (FPS) from the dense plasma focus device [58, 59]. These numerical computations of FIB and FPS properties have been incorporated into studies for damage assessment of prospective materials for plasma facing walls of fusion reactors [60] using conventional fast focus mode (FFM), and additionally generated a concept of running the plasma focus in a slow focus mode (SFM) to produce less damaging FIB and FPS with bigger and more uniform interaction with targets for materials fabrication [61–63].
Such a wide range of insights and applications has not been demonstrated by any other single model or code. Its features include the following [10, 37]:

- Numerical Experimental Facility
- Simulate any Mathers-type plasma focus, computes axial and radial dynamics
- Design new plasma focus machines
- Thermodynamics included; H, D, Ne, Ar, Xe, He, N, Kr and D-T
- Model parameters to fit experimental axial, radial phase times implicit in measured current traces
- Radiative phase computes bremsstrahlung, line radiation, recombination and total radiation power yield. Computes neutron yield for D and D-T operation; based on a combined thermonuclear and beam-target model. Computes FIB properties and FPS properties. Plasma self-absorption incorporated in the code
- Code computes radiative cooling and collapse
- Code includes choice of tapered anode and has been approximated to curved electrodes and also SPF (spherical plasma focus).

3.2 Lee Model Code

3.2.1 The Physics Foundation and Wide-Ranging Applications of the Code

In the early 1960s, Filippov [4] and Mather [5] independently invented the plasma focus and carried out groundbreaking research establishing the Filippov- and Mather-type devices. In 1971, D. Potter published his *Numerical Studies of the Plasma Focus*, a two-dimensional fluid model which estimated neutron yield concurring with experimental yields, and concluded that these neutrons were the result of thermally reacting deuterons in the hot pinch region [18]. Since then some five decades of research have been conducted, computing and measuring all aspects of the plasma focus [1, 2]: imaging for dynamics, interferometry for densities, spectroscopy for temperatures, measurements on neutrons and radiation yields, and MeV particles. The result is the commonly accepted picture today that mechanisms within the focus pinch, such as micro- and MHD instabilities, acceleration by turbulence and ‘anomalous’ plasma resistance are important to plasma focus behaviour and that the bulk of the emitted neutrons do not originate from thermonuclear reactions.

In conjunction with the development of the plasma focus known as the UNU/ICTP PFF [7, 9] during the UNU Training Programme in Plasma and Laser Technology in 1985 [7, 9, 64] a 2-phase code had been developed [6, 7, 10, 65] to describe and optimize the design of the plasma focus. The code [10] couples the electrical circuit with PF dynamics, thermodynamics and radiation. It is energy-, charge- and mass-consistent. It was used in the design and interpretation of experiments [7, 20, 66–69]. An improved 5-phase code [10, 37] incorporating finite small
disturbance speed [36] and radiation-coupled dynamics evolved and this version was used [49–63, 70–81], and was first web-published [82] in 2000. Plasma self-absorption was included [10, 37] in 2007. It has been used extensively as a complementary facility in several machines, for example: UNU/ICTP PFF [7, 9, 20, 62, 70–76], NX2 [74, 78–80, 83, 84], NX1 [74, 78], and modified for Filippov configuration for DENA [85]. It has also been used in several machines for design, optimization [7, 10, 55–57, 62, 83, 84, 86–89] and interpretation including sub-kJ PF machines [90], FNII [91], the UBA hard X-ray source [92], KSU PF [89] and a cascading plasma focus [93]. Information computed includes axial and radial dynamics [7, 62, 68–81, 94, 95], SXR emission characteristics and yield [55–57, 73–79, 83, 87, 88, 96–99] for various gases and applications including as a source for microelectronics lithography [74, 78], adaptation in the form of ML (Modified Lee) to Filippov-type plasma focus devices [85]. Speed-enhanced PF [70–72] was demonstrated. Plasma focus neutron yield calculations [21, 37, 44, 100–102], current and yield limitations [49, 102–104], deterioration of neutron scaling (neutron saturation) [21, 44], radiative collapse [50–54], current-stepped PF [105], extraction of diagnostic data [106–109] and anomalous resistance data [110–112] from current signals have been studied using the code [10, 37] or variants. Yield enhancement effects of high operational pressure and voltage have been studied [44]. It has recently been used to produce reference numbers for deuteron beam number and energy fluence and flux and scaling trends for these with PF storage energy; and subsequently extended for beam ion calculations for all gases [58, 59]. Fast ion beams FIB and fast plasma streams FPS for damage studies have been simulated [60] and calculations for the production of short-lived radioisotopes have been made [113]. Radiation and particle yields scaling laws [21, 22, 44, 77, 98, 100, 114–123] have been deduced. Arwinder Singh [77] has used the code as a tool to tabulate the characteristics and properties of 44 machines collated from all over the world, including sealed, small and big plasma focus devices. Considerable effort has also been made to collect neutron and SXR data for comparison with computed results using the code [124–129]. The code opens up so many fronts in plasma focus numerical experiments that it has been proposed as an advanced training system for the fusion energy age [130] to further advance the successful international training programmes established by the Asian African Association for Plasma Training (AAAPT) [9, 64]. The range and scope of this Model code is shown in the following Fig. 3.4.

The code has been continuously developed over the past 3 decades and in recent years many details, as they evolve, are described in the website of the Institute for Plasma Focus Studies [10]. This section presents the complete description of the Lee Model code in its basic 5-phase version. The chapter also briefly describes the development into the 6-phase version for Type-2 (high inductance plasma focus) machines which have been found to be incompletely fitted with the 5-phase model due to a dominant anomalous resistance phase [110].

This model has been developed for Mather-type [5] plasma focus machines. It was developed for the 3 kJ machine known as the UNU/ICTP PFF [7, 9] (United Nations University/International Centre for Theoretical Physics Plasma Focus
facility), which now forms an international network. However, it has since been generalized to all machines. In principle there is no limit to energy storage and electrode configuration, though house-keeping may need to be carried out in extreme cases, in order to keep within efficient ranges, e.g. of graph plotting.

### 3.2.2 The Five Phases of the Plasma Focus

A brief description of the five phases is summarized as follows:

1. Axial Phase (see Fig. 3.1): This is described by a snowplow model with an equation of motion which is coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors $f_m$ and $f_c$ [55, 131–133]. The mass swept-up factor $f_m$ accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheet-shock front structure, boundary layer effects and current shunting and fragmenting and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor, $f_c$ accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back wall, current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.

2. Radial Inward Shock Phase (see Figs. 3.1 and 3.6): Described by 4 coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath (CS), also called the magnetic piston, allowing...
the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations (as well as for all radial phases), these effects being especially important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock front and current sheath due to the finite small disturbance speed is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors, $f_{mr}$ and $f_{cr}$, are incorporated in all three radial phases. The mass swept-up factor $f_{mr}$ accounts for all mechanisms including current sheet curvatures and necking leading to axial acceleration and ejection of mass, and plasma/current disruptions. These effects may give rise to localized regions of high density and temperatures. The detailed profile of the discharge current is influenced by these effects and during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase, the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely, all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor $f_{cr}$ accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

3. Radial Reflected Shock (RS) Phase: When the shock front hits the axis (Fig. 3.6), because the focus plasma is collisional, a reflected shock RS develops which moves radially outwards, whilst the radial current sheath (CS) piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the RS moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters, $f_{mr}$ and $f_{cr}$, are used as in the previous radial phase. The plasma temperature behind the RS undergoes a jump by a factor nearly 2.

4. Slow Compression (Quiescent) or Pinch Phase (Fig. 3.6): When the outgoing RS hits the incoming piston the compression enters a radiative phase in which for gases such as Ne, Ar, Kr and Xe, radiation emission may strongly enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects [134] are incorporated into this phase. Radiation yields are computed incorporating the effects of plasma self-absorption. Thermonuclear and beam-gas target
components of neutron yields are computed as are properties of fast ion beams (FIB) and fast plasma streams (FPS) exiting the focus pinch [56–58, 135]. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.

5. Expanded Column Phase: To simulate the current trace beyond this point, we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase, the snowplow model is used, and two coupled equations are used; similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

We note that the transition from Phase 4 to 5 is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These localized regions are not modelled in the code, which consequently computes only an average uniform density and an average uniform temperature which is considerably lower than measured peak density and temperature. We have investigated profiling techniques to estimate these peaks [136]. However, because the 4 model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modelled. Hence the computed gross features such as speeds and trajectories and integrated soft X-ray yields have been extensively tested in numerical experiments for many machines across the range of machines and are found to be comparable with measured values. The statements in this paragraph apply to both Type-1 (low inductance) and Type-2 (high inductance) plasma focus machines [110]. However it has been found that whilst Type-1 current waveforms can be fitted adequately with the 5-phase code, the current waveform of a Type-2 machine typically contains current dip with a first portion that is well fitted by the 5-phase code. Beyond the first portion of the dip, there is an extended dip which cannot be fitted by the 5-phase model however much the model parameters are stretched. Therefore for Type-2 machines, an additional sixth phase (termed Phase 4a) has been coded occurring between Phase 4 and 5 above which is fitted by assuming anomalous resistance terms [110]. Despite the need for this additional phase for Type-2 machines it is found that the dynamics up to the slow compression (pinch) phase and neutron and soft X-ray yields for the same Type-2 machines are correctly described by the 5-phase code which already incorporates a compensatory feature for the neutron yield, basically a multiplier to the beam deuteron energy deduced from inductive voltage and fitted with global experimental data (see section on neutron calculation below). The conclusion is that the anomalous resistance phase which dominates an additional phase after the pinch phase is needed to fit the current trace but otherwise is likely not needed for the description of the dynamics up to the pinch phase or for the estimation of the neutron, SXR and other yields.

We proceed to a detailed description of the basic 5-phase model code.
3.2.3 The Equations of the Five Phases

3.2.3.1 Axial Phase (Snowplow Model)

We refer to the left image of Fig. 3.1:

Rate of change of momentum at current sheath, position $z$, is

$$\frac{d}{dt} \left[ \rho_0 \pi (b^2 - a^2) z f_m \frac{dz}{dt} \right] = \rho_0 \pi (c^2 - 1) a^2 f_m \frac{d}{dt} \left( z \frac{dz}{dt} \right)$$

Magnetic force on current sheath is

$$\int_a^b \left[ \left( \frac{\mu I f_c}{2\pi r} \right)^2 / (2\mu) \right] 2\pi rdr = \frac{\mu f_c^2 \ln(c) I^2}{4\pi}$$

where $f_m = \text{fraction of mass swept down the tube in the axial direction}; f_c = \text{fraction of current flowing in piston (or current sheet CS)}; c = b/a = \text{cathode radius/anode radius}$, $\rho_0 = \text{ambient density}, I = \text{time-varying circuit current}, \mu = \text{permeability}.$

Equation of Motion

From the above equating rate of change of momentum to the magnetic force, we derive:

$$\frac{d^2 z}{dt^2} = \frac{f_c^2}{f_m 4\pi^2 \rho_0 (c^2 - 1)} \left[ \frac{I}{a} \right]^2 - \left( \frac{dz}{dt} \right)^2$$  \hspace{1cm} (3.1)

Circuit (Current) Equation

We ignore $r_p(t)$, plasma resistance, hence not shown in the circuit diagram of Fig. 3.5. This is the approximation which is generally used for electromagnetic drive. Using the $C_0-L_0-L(t)-r_0$ mesh of Fig. 3.5 we derive the circuit equation as follows:

$$\frac{d}{dt} \left[ (L_0 + L f_c) I \right] + r_0 I = V_0 - \int \frac{Idt}{C_0} \frac{dI}{dt} + \left[ \frac{V_0}{C_0} - r_0 I - \frac{\mu f_c}{2\pi} (\ln(c) \frac{dz}{dt}) \right] / \left[ L_0 + \frac{f_c \mu}{2\pi} (\ln(c) z) \right]$$  \hspace{1cm} (3.2)
Equations (3.1) and (3.2) are the Generating Equations of the axial model. They contain the physics built into the model. They are coupled equations. The equation of motion is affected by the electric current $I$. The circuit equation is affected by the current sheath motion $\frac{dz}{dt}$ and position $z$.

Normalizing the Generating Equations to Obtain Characteristic Axial Transit Time, Characteristic Axial Speed and Speed Factor $S$; and Scaling Parameters of Times, $\tau$ and Inductances $\beta$

Replace variables $t, z,$ and $I$ by non-dimensionalised quantities as follows:

$$\tau = t/t_0, \zeta = z/z_0 \text{ and } i = I/I_0$$

where the normalizing quantities $z_0 = \text{the length of the anode, } t_0 = (L_0 C_0)^{0.5}$ (note that $2\pi t_0$ is the periodic time of $L_0-C_0$ discharge circuit) and $I_0 = V_0/Z_0$ where $Z_0 = (L_0/C_0)^{0.5}$ is the surge impedance.

Normalizing, we have equation of motion:

$$\frac{d^2\zeta}{d\tau^2} = \left[\frac{f_c^2}{I_m 4\pi^2 \rho_0 (c^2 - 1)} \left(\frac{I_0}{a}\right)^2 \frac{t_0^2}{z_0^2} \tau^2 - \left(\frac{d\zeta}{d\tau}\right)^2\right]/\zeta$$

which we write in the following form

$$\frac{d^2\zeta}{d\tau^2} = \left[\frac{\tau^2 - \left(\frac{d\zeta}{d\tau}\right)^2}{\zeta}\right]$$

with

$$\tau^2 = \frac{t_0^2}{t_a^2}$$

(3.3)

(3.4)
and

\[ t_a = \left[ \frac{4\pi^2(c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{\sqrt{f_m}}{f_c} \frac{z_0}{(I_0/a) / \sqrt{\rho}} \]  

(3.5)

which is identified as the characteristic axial transit time of the CS for the anode axial phase.

\[ \alpha = (I_0/t_a) \]  

(3.6)

is identified as the first scaling parameter being the ratio of characteristic electrical discharge time to the characteristic axial transit time. This scaling parameter is seen as an indicator of the matching of electrical drive time to the axial transit time for efficient energy transfer.

We further identify a characteristic axial transit speed \( v_a = z_0/t_a \) where

\[ v_a = \left[ \frac{\mu \ln c}{4\pi^2(c^2 - 1)} \right]^{1/2} \frac{f_c}{\sqrt{f_m}} \frac{(I_0/a)}{\sqrt{\rho}} \]  

(3.7)

The quantity \( (I_0/a) / \rho^{0.5} \) is the all-important \( S \) (speed or drive) factor [137] of the plasma focus axial phase and as we shall see also the radial phase; and indeed for all electromagnetically driven devices.

Normalizing the circuit (current) Equation, we have:

\[ \frac{dt}{d\tau} = \left( 1 - \int t d\tau - \beta t \frac{d\zeta}{d\tau} - \delta t \right) / (1 + \beta \zeta) \]  

(3.8)

where

\[ \beta = (L_a/L_0) \]  

(3.9)

and \( L_a = f_c (\mu / 2\pi)(\ln c)z_0 \) is the inductance of the axial phase when CS reaches anode end at \( z = z_0 \).

Thus this second scaling parameter has a great effect on the electrodynamics of the system.

The third scaling parameter \( \delta = r_0/Z_0 \) is the ratio of circuit stray resistance to surge impedance. This has a damping effect on the current.

Equations (3.3) and (3.8) are the Generating Equations (in normalized form) that are integrated step-by-step for the time variation of current \( t \) and axial position \( \zeta \).
Calculate Voltage Across Input Terminals of Focus Tube

\[ V = \frac{d}{dt} (Lj) = f_c I \frac{dL}{dt} + f_c L \frac{dI}{dt} \quad \text{where} \quad L = \frac{\mu}{2\pi} (\ln c) z \] (3.10)

Normalised to capacitor voltage \( V_0 \):

\[ v = \frac{V}{V_0} = \beta I \frac{d\zeta}{d\tau} + \beta \frac{d\zeta}{d\tau} \] (3.11)

Integration Scheme for Normalized Generating Equations

Define initial conditions:

\[ \tau = 0, \ \frac{d\zeta}{d\tau} = 0, \ \zeta = 0, \ \tau = 0, \ \int id\tau = 0, \ \frac{dI}{d\tau} = 1, \ \frac{d^2\zeta}{d\tau^2} = (1/2)^{0.5} z \]

Set time increment: \( D = 0.001 \); Increment time: \( \tau = \tau + D \)

Next step values (LHS) are computed from current-step values (RHS) using the following linear approximations:

\[ \frac{d\zeta}{d\tau} = \frac{d\zeta}{d\tau} + \frac{d^2\zeta}{d\tau^2} D \]

\[ \zeta = \zeta + \frac{d\zeta}{d\tau} D \]

\[ \tau = \tau + \frac{dI}{d\tau} D \]

\[ \int id\tau = \int id\tau + iD \]

Use new values of \( \frac{d\zeta}{d\tau}, \zeta, \tau \) and \( \int id\tau \) to calculate new generating values of \( dI/d\tau \) and \( d^2\zeta/d\tau^2 \) using generating Eqs. (3.3) and (3.8). Increment time again and repeat calculations of next step values and new generating values. Continue procedure until \( \zeta = 1 \). Then go on to radial phase inward shock.

### 3.2.3.2 Radial Inward Shock Phase (Slug Model)

The snowplow model is used for axial phase just to obtain axial trajectory and speed (from which temperature may be deduced) and to obtain a reasonable current profile. As the plasma structure is assumed to be infinitesimally thin, no information of density is contained in the physics of the equation of motion, although an estimate of density may be obtained by invoking additional mechanisms, e.g. using shock wave theory [138, 139].
In the radial phase, however, a snowplow model (with infinitesimally thin structure) would lead to all current flowing at $r = 0$, with infinite inductance and density. This is obviously unrealistic.

We thus replace the snowplow model by a slug model [10, 36]. In this model, the magnetic pressure drives a shock wave ahead of it, creating a space for the magnetic piston (also called current sheet CS) to move into. The speed of the inward radial shock front (see the left image of Fig. 3.1) is determined by the magnetic pressure (which depends on the drive current value and CS position $r_p$). A radius-time representation of the slug model is shown in Fig. 3.6.

The speed of the magnetic piston (CS) is determined by the first law of thermodynamics applied to the effective increase in volume between shock front (SF) and CS, created by the incremental motion of the SF. The compression is treated as an elongating pinch.

Four generating equations are needed to describe the motion of (a) radial SF (see right image of Fig. 3.1); (b) radial CS; (c) pinch elongation and (d) the electric current; in order to be integrated for the four variables $r_s$, $r_p$, $z_f$ and $I$.

**Motion of Shock Front**

From shock theory [138, 139], shock pressure

$$P = 2\rho_0 v_s^2/(\gamma + 1)$$

(3.13)

where shock speed $v_s$ into ambient gas $\rho_0$ causes the pressure of the shocked gas (just behind the shock front) to rise to value $P$ (see Fig. 3.7); $\gamma$ is the specific heat ratio of the gas.

![Fig. 3.6 Schematic of the radial phase—in radius versus time format](image)
If we assume that this pressure is uniform from the SF to the piston or CS (infinite acoustic, or small disturbance speed approximation) then across the piston (Fig. 3.7), we may apply \( P = P_m \) where
\[
P_m = \left( \frac{\mu I_f c}{2\pi r_p} \right)^2 / 2 \mu v_s^2 = \frac{\mu (I_f c)^2}{8\pi^2 r_p^2} \times \frac{\gamma + 1}{2 \rho_{mr}}
\]
(3.13a)

where \( I \) is the circuit current and \( I_f c \) is the current flowing in the cylindrical CS, taken as the same \( f_c \) as in the axial phase, and \( \rho_{mr} \) is the effective mass density swept into the radial slug; where \( f_{mr} \) is a different (generally larger) factor than \( f_m \) of the axial phase.

Thus
\[
\frac{dr_s}{dt} = - \left( \frac{\mu (\gamma + 1)}{\rho_0} \right)^{1/2} \frac{f_c}{\sqrt{f_{mr}}} \frac{I}{4\pi r_p}
\]
(3.14)

Elongation Speed of CS (Open-Ended at Both Ends)

The radial compression is open-ended. Hence an axial shock is propagated in the \( z \)-direction, towards the downstream anode axis. We take \( z_f \) as the position of the axial CS. The pressure driving the axial shock is the same as the pressure driving the inward radial shock. Thus the axial shock speed is the same as the radial shock speed. The CS speed is slower, from shock wave theory, by an approximate factor of \( 2/(\gamma + 1) \). Thus axial elongation speed of CS is:
\[
\frac{dz_f}{dr} = - \left( \frac{2}{\gamma + 1} \right) \frac{dr_s}{dt}
\]
(3.15)

Radial Piston Motion

We inquire: for an incremental motion, \( dr_s \), of the shock front, at a driving current \( I \), what is the relationship between plasma slug pressure \( P \) and plasma slug volume \( V_{ol} \)?
We assume an adiabatic relationship [10, 36, 37] (assuming infinite small disturbance speed for which we will apply a correction subsequently) to a fixed mass of gas in the slug during the incremental motion \(dr_s\). We have \(P \text{Vol} = \text{constant or } c d(Vol)/\text{Vol} + dP/P = 0\) (3.16)

where slug pressure \(P \sim v_s^2\) (see Eq. 3.13); so \(dP/P = 2d\ln v_s\) but \(v_s \sim r_p\) (see Eq. 3.13a); so

\[
\frac{dP}{P} = 2 \left( \frac{dI}{I} - \frac{dr_p}{r_p} \right) \tag{3.16a}
\]

Now slug volume \(\text{Vol} = \pi(r_p^2 - r_s^2)z_f\).

Here we note that although the motion of the piston \(dr_p\) does not change the mass of gas in the slug, the motion of the shock front, \(dr_s\), does sweep in an amount of ambient gas. This amount swept in is equal to the ambient gas swept through by the shock front in its motion \(dr_s\). This swept-up gas is compressed by a ratio \((\gamma + 1)/(\gamma - 1)\) and will occupy part of the increase in volume \(d\text{Vol}\).

The actual increase in volume available to the original mass of gas in volume \(\text{Vol}\) does not correspond to increment \(dr_s\) but to an effective (reduced) increment \(dr_s(2/(\gamma + 1))\). (Note \(\gamma\) is specific heat ratio of the plasma, e.g. \(\gamma = 5/3\) for atomic gas, \(\gamma = 7/5\) for molecular gas; for strongly ionizing argon \(\gamma\) has value closer to 1, e.g. 1.15.) The specific heat ratio and effective charge \(Z_{\text{eff}}\), where needed are computed from a corona model and placed in the code in the form of a series of polynomials. This is described in Sect. 3.2.3.4 slow compression phase below. Thus:

\[
d\text{Vol} = 2\pi \left( r_p dr_p - \frac{2}{\gamma + 1} r_s dr_s \right) z_f + \pi \left( r_p^2 - r_s^2 \right) dz_f
\]

and we have:

\[
\frac{\gamma d\text{Vol}}{\text{Vol}} = 2\gamma \left( r_p dr_p - \frac{2}{\gamma + 1} r_s dr_s \right) z_f + \gamma \left( r_p^2 - r_s^2 \right) dz_f \tag{3.16b}
\]

From Eqs. (3.16), (3.16a) and (3.16b); (and taking effective increment of \(dz_f\) as \(dz_f(2/(\gamma + 1))\) for the same reason as explained above for effective increment of \(dr_s\)) we have

\[
\frac{dr_p}{dr} = \frac{2}{\gamma + 1} \frac{r_s}{r_p} \frac{dr_s}{dr} - \frac{r_s}{r_p} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dr} - \frac{r_p}{(\gamma + 1)z_f} \left( 1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dr} \tag{3.17}
\]
Circuit Equation During Radial Phase

The inductance of the focus tube now consists of the full inductance of the axial phase and the inductance of the radially imploding and elongating plasma pinch. Thus

$$L = \frac{\mu}{2\pi} (\ln c)z_0 + \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) z_f$$

(3.18)

where both $z_f$ and $r_p$ vary with time.

Thus the circuit (current) equation is obtained as:

$$\frac{dI}{dt} = \frac{V_0 - \int \frac{f_{dr}}{c_0} - r_0 I - f_{cr} \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) I \frac{dz_f}{dr} + f_{cr} \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dr}}{L_0 + f_{cr} \frac{\mu}{2\pi} (\ln c)z_0 + f_{cr} \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) z_f}$$

(3.19)

The four Generating Eqs. (3.14), (3.15), (3.17) and (3.19) form a closed set of equations which are integrated for $r_s$, $r_p$, $z_f$ and $I$.

Normalizing the Generating Equations to Obtain Characteristic Radial Transit Time, Characteristic Radial Transit Speed and Speed Factor $S$; and Scaling Parameters for Times $z_1$ and Inductances $\beta_1$; also Compare Axial to Radial Length Scale, Time Scale and Speed Scale

For this phase, the following normalization is adopted.

$$\tau = t/t_0, \ i = I/I_0$$ as in axial phase but with $\kappa_s = r_s/a$, $\kappa_p = r_p/a$, and $\zeta_f = z_f/a$, i.e. distances are normalized to anode radius, instead of anode length.

After normalization we have:

**Radial shock speed**

$$\frac{d\kappa_s}{d\tau} = -\alpha z_1 \frac{i}{\kappa_p}$$

(3.20)

**Axial column elongation speed (both ends of column defined by axial piston)**

$$\frac{d\zeta_f}{d\tau} = -\frac{2}{\gamma + 1} \frac{d\kappa_s}{d\tau}$$

(3.21)

**Radial piston speed:**

$$\frac{d\kappa_p}{d\tau} = \frac{2}{\gamma + 1} \frac{\kappa_s}{\kappa_p} \frac{d\kappa_s}{d\tau} - \frac{\kappa_p}{\gamma + 1} \left( 1 - \frac{\kappa_s^2}{\kappa_p^2} \right) \frac{d\kappa_s}{d\tau} - \frac{1}{\gamma + 1} \frac{\kappa_p}{\kappa_s} \left( 1 - \frac{\kappa_s^2}{\kappa_p^2} \right) \frac{d\zeta_f}{d\tau}$$

(3.22)
Current:
\[
\frac{dt}{d\tau} = 1 - \int \frac{d\tau}{dt} + \beta_1 \left[ \ln(\kappa_p/c) \right] \frac{d\kappa_p}{d\tau} + \beta_1 \frac{d\kappa_p}{d\tau} - \delta_t \left\{ 1 + \beta - (\beta_1) \left[ \ln(\kappa_p/c) \right] \frac{1}{\tau} \right\} 
\]  
(3.23)

where scaling parameters are
\[
\beta_1 = \beta/(F \ln c) 
\]  
(3.24)

and
\[
\alpha_1 = \left[ (\gamma + 1)(c^2 - 1)/(4 \ln c) \right]^{1/2} F\left[ f_m/f_mr \right]^{1/2}[f_{cr}/f_c]. 
\]  
(3.24a)

We note that \( F = z_0/a \) (the length/radius ratio of the anode) may be considered to be the controlling parameter of \( \beta_1 \) and \( \alpha_1 \). In other words, \( \beta_1 \) and \( \alpha_1 \) may not be independently assigned, but should be assigned as a pair with the value of each fixed by the value of \( F \).

Note that whereas we interpret \( \alpha = t_0/t_a \), (Eq. 3.6) we may interpret
\[
\alpha_1 = t_a/t_r 
\]  
(3.25)

where \( t_r \) is the characteristic radial transit time.

\[
t_r = \frac{4\pi}{[\mu(\gamma + 1)]^{1/2}} f_{mr} a \left\{ \frac{f_{cr}}{(I_0/a)/\sqrt{\rho}} \right\} 
\]  
(3.26)

The product \( \alpha \alpha_1 \) may then be interpreted as \( \alpha \alpha_1 = \frac{t_a}{t_a} = t_0/t_r \)

The characteristic speed of the radial inward shock to reach focus axis is:
\[
v_r = a/t_r = \frac{[\mu(\gamma + 1)]^{1/2}}{4\pi} \frac{f_{cr}}{\sqrt{f_{mr}}} \left\{ \frac{(I_0/a)}{\sqrt{\rho}} \right\} 
\]  
(3.27)

The ratio of characteristic radial and axial speeds is also essentially a geometrical one, modified by thermodynamics. It is
\[
v_r/v_a = \left[ \frac{(c^2 - 1)(\gamma + 1)}{4 \ln c} \right]^{1/2} \left\{ \frac{f_m/f_{mr}}{1/2}[f_{cr}/f_c] \right\}. 
\]  
(3.28)

with a value typically 2.5 for a small deuterium plasma focus with \( c \sim 3.4 \), and \( \gamma = 5/3 \). We note [137] that the radial characteristic speed has same dependence as axial transit speed on the all-important drive factor (see Eq. 3.7). \( S = (I_0/a)/\sqrt{\rho} \).
Calculate Voltage V Across PF Input Terminals

As in the axial phase, the tube voltage is taken to be inductive: $V = \frac{d}{dt}(LI)$

$L = \frac{\mu}{2\pi}(\ln c)z_0 + \frac{\mu}{2\pi}\left(\ln \frac{b}{r_p}\right)z_f$

where

$V = \frac{\mu}{2\pi}\left[(\ln c)z_0 + \left(\ln \frac{b}{r_p}\right)z_f\right]f_{cr}\frac{df}{dt} + \frac{\mu}{2\pi}\left[(\ln \frac{b}{r_p})\frac{dz_f}{dt} - \frac{z_f}{r_p} \frac{dr_p}{dt}\right]f_{cr}I$ (3.29)

We normalize to the capacitor charging voltage $V_0$; so that $v = V/V_0$

$v = \left[\beta - \beta_1\left(\ln \frac{\kappa_p}{c}\right)\zeta_f\right]\frac{di}{dt} - \beta_1 I\left[\left(\frac{\zeta_f}{\kappa_p}\right)\frac{d\kappa_p}{dt} + \left(\ln \frac{\kappa_p}{c}\right)\frac{d\zeta_f}{dt}\right]$ (3.30)

Integrating for the Radial Inward Shock Phase

The 4 normalized generating Eqs. (3.20)–(3.23) may now be integrated using the following initial conditions: $\tau =$ the time at which the axial phase ended, $\kappa_s = 1$, $\kappa_p = 1$; $\zeta_f = 0$ (taken as a small number such as 0.00001 to avoid numerical difficulties for Eq. (3.21); $i =$ value of current at the end of the axial phase; $\int id\tau =$ value of ‘flowed charge’ at the end of the axial phase.

Smaller time increments of $D = (0.001/100)$ are taken.

$d\kappa_s/d\tau, d\zeta_f/d\tau, d\kappa_p/d\tau$ and $di/d\tau$ are sequentially calculated from generating Eqs. (3.20)–(3.23).

Then using linear approximations we obtain next step values (LHS) from current (RHS) values as follows:

$\kappa_s = \kappa_s + Dd\kappa_s/d\tau; \zeta_f = \zeta_f + Dd\zeta_f/d\tau$

$\kappa_p = \kappa_p + Dd\kappa_p/d\tau; i = i + Ddi/d\tau$ and $\int i d\tau = \int i d\tau + iD$

Time is then incremented by $D$, and the next step value of $d\kappa_s/d\tau, d\zeta_f/d\tau, d\kappa_p/d\tau$ and $di/d\tau$ are computed from Eqs. (3.20)–(3.23) followed by linear approximation for $\kappa_s$, $\zeta_f$, $\kappa_p$, $i$ and $\int i d\tau$.

The sequence is repeated step-by-step until $\kappa_s = 0$. 

S. Lee and S.H. Saw
Correction for Finite Acoustic (Small Disturbance) Speed

In the slug model above we assume that the pressure exerted by the magnetic piston (current $I$, position $r_p$) is instantaneously felt by the shock front (position $r_s$). Likewise, the shock speed $dr_s/dt$ is instantaneously felt by the piston (CS). This assumption of infinite small disturbance speed (SDS) is implicit in Eqs. (3.14) and (3.17) [or in normalized form Eqs. (3.20) and (3.22)].

Since the SDS is finite, there is actually a time lapse $\Delta t$ communicating between the SF and CS. This communication delay has to be incorporated into the model. Otherwise, for the PF, the computation will yield too high values of CS and SF speed.

Consider the instant $t$, SF at $r_s$, CS at $r_p$, the value of current is $I$. SF actually feels the effect of the current not of value $I$ but of a value $I_{\text{delay}}$ which flowed at time $(t - \Delta t)$, with the CS at $r_p$-delay. Similarly, the piston ‘feels’ the SF speed is not $dr_s/dt$ but $(dr_s/dt)_{\text{delay}}$ at time $(t - \Delta t)$.

To implement this finite SDS correction we adopt the following procedure:

Calculate the SDS, taken as the acoustic speed.

$$SDS = \left(\frac{\gamma P}{\rho}\right)^{1/2} \text{ or } \left(\frac{\gamma R_0 D_c T}{M}\right)^{1/2} \text{ or } \left(\frac{\gamma D_c k T}{M m_i}\right)^{1/2}$$  \hspace{1cm} (3.31)

$M =$ Molecule Weight, $R_0 =$ Universal Gas constant $= 8 \times 10^3$ (SI units); $m_i =$ mass of proton, $k =$ Boltzmanns constant. $D_c =$ departure coefficient $= DN \ (1 + Z)$; where $Z$, here, is the effective charge of the plasma $Z = \sum_i^J r \alpha_r$, summed over all ionization levels $r = 1, \ldots, J$. This is computed using a corona model [140]. The procedure is described in more details in the description of the pinch phase.

$DN =$ dissociation number, e.g. for deuterium $DN = 2$, whereas for argon $DN = 1$.

From shock theory, the shocked plasma temperature $T$ is:

$$T = \frac{M}{R_0 D} \frac{2(\gamma - 1)}{(\gamma + 1)^2} \left(\frac{dr_s}{dt}\right)^2$$  \hspace{1cm} (3.32)

The communication delay time is then:

$$\Delta T = (r_p - r_s)/SDS$$  \hspace{1cm} (3.33)

In our programme using the Microsoft EXCEL VISUAL BASIC, data of the step-by-step integration is stored row-by-row, each step corresponding to one row. Thus the $\Delta T$ may be converted to $\Delta$(row number) by using $\Delta$(row number) = $\Delta T$/ (timestep increment); this $\Delta$(row number) being, of course, rounded off to an integer.

The correction then involves ‘looking back’ to the relevant row number to extract the corrected values of $I_{\text{delay}}$, $r_p$-delay and $(dr_s/dt)_{\text{delay}}$. Thus in the actual
numerical integration, in Eq. (3.20) $i$ and $k_p$ are replaced by $I_{\text{delay}}$ and $k_p\text{-delay}$ and in Eq. (3.22) $d\kappa_p/d\tau$ is replaced by $(d\kappa_p/d\tau)_{\text{delay}}$.

### 3.2.3.3 Radial Reflected Shock (RS) Phase

When the inward radial shock hits the axis, $\kappa_s = 0$. Thus in the computation, when $\kappa_s \leq 1$ we exit from radial inward shock phase. We start computing the RS phase. The RS is given a constant speed of 0.3 of on-axis inward radial shock speed [37]. In this phase, computation is carried out in real (SI) units.

**Reflected Shock Speed**

\[
\frac{dr_e}{dt} = -0.3 \left( \frac{dr_s}{dt} \right)_{\text{on-axis}} \tag{3.34}
\]

**Piston Speed**

\[
\frac{dr_p}{dt} = -\frac{r_p}{\beta} \left( 1 - \frac{r^2}{r_p^2} \right) \frac{dr_s}{dt} - \frac{r_p}{(\gamma + 1)z_0} \left( 1 - \frac{r^2}{r_p^2} \right) \frac{dz_f}{dt} \left( \frac{\gamma - 1}{\gamma} + \frac{1}{\gamma} \frac{r^2}{r_p^2} \right) \tag{3.35}
\]

**Elongation Speed**

\[
\frac{dz_f}{dt} = -\left( \frac{2}{\gamma + 1} \right) \left( \frac{dr_s}{dt} \right)_{\text{on-axis}} \tag{3.36}
\]

**Circuit Equation**

\[
\frac{dI}{dt} = V_0 - \frac{\int \mu dl}{c_0} - r_0 I - f_{cr} \frac{\mu}{2\pi} \ln \left( \frac{b}{r_p} \right) I \frac{dz_f}{dt} + f_{cr} \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dt} - f_{cr} \frac{\mu}{2\pi} \ln \left( c \right) z_0 - f_{cr} \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) z_f \tag{3.37}
\]

The integration of these 4 coupled generating Eqs. (3.34)–(3.37) is carried out step-by-step as in the radial inward shock phase.

**Tube Voltage**

The tube voltage uses Eq. (3.29) above as in the radial inward shock phase.
In this phase as the RS (position \( r_r \)) moves outwards, the piston (position \( r_p \)) continues moving inwards. When the RS position reaches that of the piston the RS phase ends and the slow compression (pinch) phase begins.

### 3.2.3.4 Slow Compression (Pinch) Phase

Radiation-Coupled Dynamics (Piston) Equation

In this phase the piston speed is:

\[
\frac{dr_p}{dt} = \frac{-r_p \frac{dl}{dt} - \frac{1}{\gamma+1} \frac{r_p}{\zeta_l} \frac{d\zeta_l}{dr} + \frac{4\pi (\gamma-1) - r_p}{f_c r^2} \frac{dQ}{dt}}{\frac{\gamma-1}{\gamma}}
\] (3.38)

Here we have included energy loss/gain terms into the equation of motion. The plasma gains energy from Joule heating; and loses energy through bremsstrahlung and line radiation. A positive power term \( dQ/dt \) will tend to push the piston outwards whilst a power loss term will have the opposing effect. The specific heat ratio \( \gamma \) is taken as 5/3 for H, D, T and He gases. For other gases such as Ne, N, O, Ar, Kr, Xe, a sub-routine [140] based on a corona model is used to compute \( \gamma \) as a function of temperature; and for faster computing the values of \( \gamma \) for each gas are represented by a series of polynomials incorporated into the code. At the same time the charge number \( Z \) is also computed and included as another series of polynomials and incorporated into the code.

**Joule Heating Component of \( dQ/dt \)**

The Joule term is calculated from the following:

\[
\frac{dQ_J}{dt} = R I^2 f_{cr}^2
\] (3.39)

where plasma resistance \( R \) is calculated using the Spitzer form [141]:

\[
R = \frac{1290 Z z_f}{\pi \rho_p^2 T^{3/2}}
\] (3.40)

And using Bennett [142] formula:

\[
T = \frac{\mu}{8\pi^2 k} I^2 f_{cr}^2 / (DN_0 a^2 f_{m_F})
\] (3.41)
Radiation Components of $\frac{dQ}{dt}$

The bremsstrahlung loss term may be written as:

$$\frac{dQ_B}{dt} = -1.6 \times 10^{-40} N_i^2 \left( \frac{\pi r_p^2}{a} \right) z_t T^{1/2} Z^3$$ (3.42)

$$N_0 = 6 \times 10^{26} \frac{P_0}{M} ; \quad N_i = N_0 f_{mr} \left( \frac{a}{r_p} \right)^2$$ (3.43)

$Z_n$ = atomic number, $N_0$ = ambient number density, $N_i$ = ion number density.

The line loss term may be written as:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z Z_n^4 \left( \frac{\pi r_p^2}{a} \right) z_t / T$$ (3.44)

And

$$\frac{dQ}{dt} = \frac{dQ_B}{dt} + \frac{dQ_L}{dt} + \frac{dQ_J}{dt}$$ (3.45)

where $\frac{dQ}{dt}$ is the total power gain/loss of the plasma column. In the standard code, recombination radiation is similarly incorporated into $\frac{dQ}{dt}$.

By this coupling, if, for example, the radiation loss $\frac{dQ}{dt}$ is severe, this would lead to a large value of $dr_p/dt$ inwards. In the extreme case, this leads to radiation collapse [50], with $r_p$ going rapidly to such small values that the plasma becomes opaque to the outgoing radiation, thus stopping the radiation loss.

This radiation collapse occurs at a critical current of 1.6 MA (the Pease-Braginski current) for deuterium [143, 144]. For gases such as Ne or Ar, because of intense line radiation, the critical current is reduced to even below 100 kA, depending on the plasma temperature [50, 145].

Plasma Self-absorption and Transition from Volumetric Emission to Surface Emission

Plasma self-absorption [10, 146, 147] and volumetric (emission described above) to surface emission of the pinch column are implemented in the following manner.

The photonic excitation number is written as follows:

$$M = 1.66 \times 10^{-15} r_p Z_n^0.5 n_i / (ZT^{1.5})$$ (3.46)

with $T$ in eV, rest in SI units.
The volumetric plasma self-absorption correction factor $A$:

$$
A_1 = \left(1 + 10^{-14} n_i Z\right) / \left(T^{3.5}\right) \quad A_2 = 1 / 1 \quad A = A_2^{(1+M)}
$$  \hfill (3.47)

Transition from volumetric to surface emission occurs when the absorption correction factor goes from 1 (no absorption) down to $1/e$ ($e = 2.718$) when the emission becomes surface-like given by:

$$
dQ_L = \text{const} Z^{0.5} Z_n^{3.5} r_p z_t T^4
$$  \hfill (3.48)

where the constant ‘$\text{const}$’ is taken as $4.62 \times 10^{-16}$ to conform with numerical experimental observations that this value enables the smoothest transition, in general, in terms of power values from volumetric to surface emission.

Neutron Yield

Neutron yield is calculated with two components, thermonuclear term and beam-target term. The thermonuclear term is taken as:

$$
dY_{th} = 0.5 n_i^2 \pi r_p^2 z_t \sigma v (\text{time interval})
$$  \hfill (3.49)

where $\sigma v$ is the thermalized fusion cross section-velocity product corresponding to the plasma temperature [148], for the time interval under consideration. The yield $Y_{th}$ is obtained by summing up over all intervals during the focus pinch.

The beam-target term is derived using the following phenomenological beam-target neutron generating mechanism [12], incorporated in the code version RADPFV5.13 and later. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modelling, each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with a proportionality constant. The yield is then calibrated against a known experimental point.

The beam-target yield is written as: $Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$

where $n_b$ is the number of beam ions per unit plasma volume, $n_i$ is the ion density, $r_p$ is the radius of the plasma pinch with length $z_p$, $\sigma$ the cross section of the D–D fusion reaction, $n$-branch [148], $v_b$ the beam ion speed and $\tau$ is the beam-target interaction time assumed proportional to the confinement time of the plasma column. Total beam energy is estimated [12] as proportional to $L_p I_{\text{pinch}}^2$, a measure of the pinch inductance energy, with $L_p$ being the focus pinch inductance. Thus the number of beam ions is $N_b \sim L_p^2 I_{\text{pinch}}^2 / v_b^2$ and $n_b$ is $N_b$ divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that [137] $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$
where $U$ is the disruption-caused diode voltage. Here $b$ is the cathode radius. We also assume reasonably that $U$ is proportional to $V_{\text{max}}$, the maximum voltage induced by the current sheet collapsing radially towards the axis.

Hence:

$$Y_{b-t} = C_n n I_{\text{pinch}}^2 \frac{b^2}{r_p} (\ln (b/r_p)) \sigma / V_{\text{max}}^{1/2} \quad (3.50)$$

where $I_{\text{pinch}}$ is the current flowing through the pinch at start of the slow compression phase; $r_p$ and $z_p$ are the pinch dimensions at end of that phase. Here $C_n$ is a constant which in practice we will calibrate with an experimental point. The D–D cross section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute $\sigma$. The code computes $V_{\text{max}}$ of the order of 20–50 kV. However, it is known [12], from experiments that the ion energy responsible for the beam-target neutrons is in the range 50–150 keV, and for smaller lower-voltage machines the relevant energy [149] could be lower at 30–60 keV. Thus to align with experimental observations the D–D cross section $\sigma$ is reasonably obtained by using beam energy fitted to 3 times $V_{\text{max}}$.

A plot of experimentally measured neutron yield $Y_n$ versus $I_{\text{pinch}}$ was made combining all available experimental data [7–10, 21, 44, 100, 150, 151]. This gave a fit of $Y_n = 9 \times 10^{13} I_{\text{pinch}}^{1.8}$ for $I_{\text{pinch}}$ in the range 0.1–1 MA [21, 37, 44, 100]. From this plot, a calibration point was chosen at 0.5 MA, $Y_n = 7 \times 10^9$ neutrons. The model code [10] from version RADPFV5.13 onwards was thus calibrated to compute $Y_{b-t}$ which turns out to be typically the same as $Y_n$ since the thermonuclear component is typically negligible.

Column Elongation

Whereas in the radial RS phase we have adopted a ‘frozen’ elongation speed model, we now allow the elongation to be driven fully by the plasma pressure.

$$\frac{dz_f}{dr} = \left[ \frac{\mu}{4\pi^2(\gamma + 1)\rho_0} \right]^{1/2} \frac{I_{\text{cr}}}{r_p} \quad (3.51)$$

Circuit Current Equation

$$\frac{df}{dr} = \frac{V_0 - \int \frac{dr}{C_0} - \frac{\mu}{2\pi} \left( \ln \frac{b}{r_p} \right) \frac{dz_f}{dr} I_{\text{cr}} + \frac{\mu}{2\pi} \frac{z_f}{r_p} \frac{dz_f}{dr} I_{\text{cr}} - I (R f_{\text{cr}} + r_0)}{L_0 + \frac{\mu}{2\pi} f_{\text{cr}} \left( \ln c \right) z_0 + \left( \ln \frac{b}{r_p} \right) z_f} \quad (3.52)$$
Voltage Across Plasma Focus Terminals

\[
V = \frac{\mu f_{cr}}{2\pi} I \left[ \left( \ln \frac{b}{r_p} \right) \frac{dz_f}{dt} - \frac{z_f}{r_p} \frac{d\rho}{dt} \right] + \frac{\mu f_{cr}}{2\pi} \left[ \left( \ln \frac{b}{r_p} \right) z_f + (\ln C) z_0 \right] \frac{dI}{dt} + R I f_{cr} \tag{3.53}
\]

Pinch Phase Dynamics and Yields of Neutrons, Soft X-rays, Ion Beams and Fast Plasma Stream

Equations (3.38), (3.51) and (3.52) are the coupled Generating Equations integrated for \( r_p, z_f \) and \( I \). At each step the value of \( dQ/dt \) is also evaluated as above using Eqs. (3.39)–(3.45) with the effect of plasma self-absorption implemented using Eqs. (3.46)–(3.48). Soft X-rays of various gases are computed using Eq. (3.44) modified by the effect of plasma self-absorption. Neutron yield is computed using Eqs. (3.50) and (3.49). In the latest version RADPFV5.15FIB fast ion beam (FIB) fluence and flux, energy fluence and flux, power flow and damage factors as well as fast plasma streams (FPS) exiting the pinch are also computed [58, 59].

The step-by-step integration is terminated at the end of a period related to the transit time of small disturbance speed across the plasma pinch column.

3.2.3.5 Expanded Column Axial Phase

We model the expanded column axial phase [10] in the following manner. In the expanded column phase we assume that the current flows uniformly from anode to cathode in a uniform column having the same radius as the anode and a length of \( z \).

The normalized equations (same normalization as in axial phase):

Circuit current:

\[
\frac{dI}{dt} = \frac{1 - \int i d\tau - \beta I \frac{dc}{d\tau} e - \delta I}{1 + \beta + \beta(\zeta - 1)e} \tag{3.54}
\]

where

\[
e = \left( \ln c + \frac{1}{z} \right) / \ln c
\]

Motion:

\[
\frac{d^2 \zeta}{d\tau^2} = \frac{\chi^2 r^2 e_1 - h^2 \left( \frac{dc}{d\tau} \right)^2}{1 + h^2 (\zeta - 1)} \tag{3.55}
\]
with

\[ h = \left[ \frac{c^2}{(c^2 - 1)} \right]^{1/2} \]
\[ e_1 = \left( \frac{\ln c + \frac{1}{4}}{\ln c} \right) \]

The initial conditions for \( i \) and \( \int id\tau \) are the last values of \( i \) and \( \int id\tau \) from the last phase. The initial value of \( \zeta \) is \( \zeta = 1 + \zeta_f \) where \( \zeta_f \) is the last length of the focus column, but normalized to \( z_0 \), rather than \( a \). This phase is terminated when the discharge has proceeded to a half cycle. The purpose of computing this phase is to allow the fitting of the computed current to the measured to the point when the current has dropped to low levels beyond interest. This completes the integration of all five phases.

### 3.2.4 Procedure for Using the Code

The Lee model code is configured [10, 37, 152] to work as any plasma focus by inputting into the configuration panel of the code (see Fig. 3.8a for the main Sheet of the code and guide diagram Fig. 3.8b-block 1 is the configuration panel):

- Bank parameters \( L_0, C_0 \) and stray circuit resistance \( r_0 \);
- Tube parameters \( b, a \) and \( z_0 \);
- Model parameters \( f_m, f_c, f_{mr} \) and \( f_{cr} \) (either fitted or trial values) and
- Operational parameters \( V_0 \) and \( P_0 \) and the fill gas.

The computed total current waveform is fitted to a measured waveform by varying model parameters \( f_m, f_c, f_{mr} \) and \( f_{cr} \) sequentially, until the computed waveform agrees with the measured waveform [10, 37].

First, the axial model factors \( f_m, f_c \) are adjusted (fitted) until the features in Fig. 3.9: ‘1’ computed rising slope of the total current trace; ‘2’ the rounding off of the peak current, as well as ‘3’ the peak current itself, is in reasonable (typically very good) fit with the measured total current trace (see Fig. 3.9, measured trace fitted with computed trace).

Then we proceed to adjust (fit) the radial phase model factors \( f_{mr} \) and \( f_{cr} \) until features ‘4’ the computed slope and ‘5’ the depth of the dip agree with the measured values. Note that the fitting of the computed trace with the measured current trace is done up to the end of the radial phase which is typically at the bottom of the current dip. Fitting of the computed and measured current traces beyond this point is not done. If there is significant divergence of the computed with the measured trace beyond the end of the radial phase, this divergence is not considered important.
Fig. 3.8 a Showing the blocks of sheet 1 of RADPFV5.15—showing display after run—refer 3.8b for guide dividing the data display in blocks (details are deliberately blurred out). b Guide to sheet 1: Block 1 (see above labelled 1 in gold) = main configuration panel; Block 2 (see above labelled 2 in green) = taper configuration sub-panel; Block 3 (see above labelled 3 in blue) = neutron yields; (cell R4 in pink) = selection of 1 of three reference machines; Block 4 (labelled 4 not coloured) = data of applicable gases and suggested model parameters; Block 5 (labelled 5, not coloured) = some computed useful parameters; Block 6 (labelled 6, not coloured) = start and end of phases; Block 7 (labelled 7, in gold) = row (17) of computed results (dataline); Block 8 (labelled 8, in brown, shown partially above) = from row 20 downwards for typically several thousand rows: computed point by point data for various quantities described in the headings of rows 18 and 19.
In this example (Poseidon plasma focus), after fitting the five features ‘1’ to ‘5’ above, the following fitted model parameters are obtained: $f_m = 0.277$, $f_c = 0.6$, $f_{mr} = 0.47$ and $f_{cr} = 0.44$.

Typically refinements to the fitting process require adjustments to $L_0$ and $r_0$, as well as the discharge start time and measured peak value. Because the code is charge consistent, once the measured waveform is accurately fitted, the correct peak current is the computed value which is then be used to calibrate the measured peak current. For Poseidon, we fitted the value of $L_0 = 17.7 \, \text{nH}$ and $r_0 = 1.7 \, \text{m\Omega}$.

Once fitted the code outputs in tabular and graphical forms [10, 37] realistic data of the following: axial and radial dynamics (positions and speeds), pinch length and minimum pinch radius, temperatures and densities, bremsstrahlung and line yields, thermonuclear and beam-target neutron yields. Also, energy distributions and thermodynamics properties are found to be realistic representations of the actual machines.

An extended code also gives fast ion beam flux and fluence, energy flux and fluence, power flow and damage factors, fast plasma stream energies and speeds [58, 59]. This concludes the description of the standard 5-phase Lee Model code. This 5-phase code has been used to fit all low $L_0$ plasma focus with adequate accuracy (an example is given in Fig. 3.10 [12]). However, it was found necessary for high $L_0$ plasma focus devices to include a 6-phase (phase 4a) in order to achieve a complete fit. The development of the code variant version (RADPFV6.1b) [10] for phase 4a is described in the following section.

**Fig. 3.9** The 5-point fitting of computed current trace to measured (reference) current trace. **Point 1** is the current rise slope. **Point 2** is the topping profile. **Point 3** is the peak value of the current. **Point 4** is the slope of the current dip. **Point 5** is the bottom of the current dip. Fitting is done up to **point 5** only. Further agreement or divergence of the computed trace with/from the measured trace is only incidental and not considered to be important.
3.2.5 Adding a 6th Phase: From Pinch (Slow Compression) Phase to Large Volume Plasma Phase-Transition Phase 4a

From experiments, it is well known that after a brief period (few ns for a small plasma focus), the quiescent column is rapidly broken up by instabilities. One effect is a huge spike of voltage, partially observed at focus tube terminals. This voltage spike is responsible for driving ion beams (forward direction) and relativistic electron beam, REB, (negative direction, towards the anode) with energies typically 200 keV. The final result of this instability mechanism is the breaking up of the focus pinch into a large expanded current column.

3.2.5.1 The 5-Phase Model Is Adequate for Low Inductance $L_0$ Plasma Focus Devices

The 5-phase Lee code does not model the transition from Phase 4 to Phase 5. Nevertheless it has been found to be adequate for modelling all the well-known plasma focus with low static inductance $L_0$ [10, 12, 21, 74] which we have fitted; in the sense that the computed current traces can be fitted to the measured current trace by adjustment of the model parameters $f_m$, $f_c$, $f_{mu}$ and $f_{cr}$. This has been the case for the PF1000, PF400J, NX1, NX2, DPF78, Poseidon [106], FMPF1 [153–155], and FN-II [91]. The case of a low inductance machine (Poseidon) is already shown in Fig. 3.9. Another example of a typical 5-phase fit is shown in Fig. 3.10.
Amongst the well-published plasma focus devices only the UNU/ICTP PFF [7–9] which has relatively higher $L_0$ of 110 nH presented a problem in the fitting. This was due to a very small computed current dip and a measured current dip that has always been masked by very large oscillations taken to be noise; although when operated in unusually low-pressure regime, a clear discrepancy was noted between the computed and measured current trace [156, 157].

In 2012 a current trace from the then newly commissioned KSU DPF (Kansas State University Dense Plasma Focus) [89, 110] which had an even higher $L_0$, was obtained by numerically integrating the output of a $\text{d}I/\text{d}t$ coil [108]. An analysis of the frequency response of the coil system and the digital storage oscilloscope (DSO) signal acquisition system showed that noise frequencies below 200 MHz were removed by the numerical integration. The resultant waveform is clean and clearly shows an extended current dip with good depth and duration (see Fig. 3.11, the darker trace). The KSU DPF shows very consistent operation with more than 95% of the shots showing current dips with similar depth and duration.

Following the usual procedure of the Lee model code, an attempt was made to fit the computed current trace with the measured. The computed current trace has only a small dip as is characteristic of the computed current dip of a device with large static inductance $L_0$. All possible adjustments were made to the model parameters but the computed current dip could not be made to fit the whole measured current dip. The best fit is shown in Fig. 3.11; which shows that the computed dip does fit the first small part of the measured current dip. But the measured dip continues on in both depth and duration far beyond the computed dip.

![Fig. 3.11](image_url) Computed current trace (lighter trace) with best attempt to fit the measured current trace (darker trace) of the KSU PF. Reprinted from Lee et al. [110]. Copyright (2011), with permission from Springer Science+Business Media, LLC
3.2.5.2 Factors Distinguishing the Two Types of Plasma Focus Devices

The code models the electrodynamic situation using the slug model and a reflected shock for the radial phase, ending the radial phase in phase 4. Let’s call the radial phase modelled in that manner as the REGULAR radial phase. This REGULAR radial phase, in increasing sharply the inductance of the system (constituting also a dynamic resistance [21, 37, 44]) causes a dip on the current trace. Call this the regular dip, RD. At the end of the REGULAR radial phase, experimental observations point to another phase (phase 4a) of “instabilities” manifesting in anomalous resistance [1]. These effects would also extract energy from the magnetic field and hence produce further current dips. These effects are not modelled specifically in the code. Call this the extended current dip, ED.

However, it may be argued that as long as the model parameters can be stretched sufficiently to have the computed current dip agree with the measured current dip, then in a gross sense, the modelling is energetically and mass-wise equivalent to the physical situation. Then the resulting gross characteristics from the model would give a fair representation of the actual plasma properties, even though the model has not specifically modelled ED. In other words, RD is able to be stretched to also model ED, with equivalent energetics and mass implications. Whether RD can be stretched sufficiently to cover ED depends on the relative sizes of the two effects. If RD is already a big dip, then this effect may dominate and it is more likely that RD may be stretched sufficiently to cover the less prominent ED. If RD is only a miniscule dip and ED is a big dip, then it is unlikely that the RD can be stretched enough to encompass the ED.

We looked at the inductance \( L_0 \) and the ratio of \( L_0 \) with various inductances inherent in the system. We considered the inductance ratio \( R_L = (L_0 + L_a)/L_{\text{pinch}} \) where \( L_{\text{pinch}} \) is the inductance of the focus pinch at the end of the REGULAR radial phase, \( L_0 \) the bank static inductance and \( L_a \) the inductance of the axial part of the focus tube. We also considered the remnant energy ratio \( R_{EL} = (E_{L0} + E_{La})/E_{L_{\text{pinch}}} \) where \( E_{L0} = \text{energy stored in } L_0 \text{ at end of the RD} \), \( E_{La} = \text{energy stored in } L_a \text{ at end of the RD} \) and \( E_{L_{\text{pinch}}} = \text{energy stored inductively in the pinch at end of RD} \).

Computing the values of these two quantities [110] for PF1000, Poseidon, DPF78, NX2, PF400J, FMPF-1, FNII, UNU/ICTPPFF and KSU PF, we have a range of devices from very big (MJ) to rather small (sub-kJ) of which we have well-documented fittings.

The results show that the smaller is the ratio \( R_L \), the bigger is the regular current dip (RD). When this ratio is large (primarily due to a large \( L_0 \) in the numerator), like in the case of KSU PF, the REGULAR radial phase RD is minuscule. Likewise, the trend is also observed for the ratio \( R_{EL} \). The smaller this energy ratio, the bigger is the current dip.

On the basis of these two ratios, we have divided the plasma focus devices into two types: T1 and T2. Type T1 is for plasma focus devices with relatively small \( L_0 \) with large RD’s and with relatively small ratios \( R_L \) and \( R_{EL} \). These T1 focus devices are well-fitted using the Lee model code. The computed current traces (with radial phase computed only as a regular dip RD) are well-fitted to the whole measured
current trace. Type T2 is for plasma focus devices with relatively large $L_0$ with small RD’s and with relatively large ratios of $R_L$ and $R_{EL}$. These T2 focus devices are not well-fitted using the Lee model code. The computed current trace shows only a small dip which is fitted to the first portion of the measured current dip, but the measured current dip has an extended portion which is not well-fitted using the 5-phase Lee model code.

Next, we note that the magnetic energy per unit mass at the start of the radial phase is the same across the whole range of devices [137]. Thus T1 with a big RD drops the current a lot and strongly depletes the magnetic energy per unit mass at the end of the RD, leading to a small ED. Consequently, T1 are completely fitted using a model that computes only the RD, stretching the model parameters until the large RD covers also the small ED. Conversely, a T2 plasma focus has a small RD, consequently a large ED and cannot be completely fitted with the computed RD. Thus a big RD drops the current a lot and strongly depletes the magnetic energy per unit mass at the end of the REGULAR radial phase. Hence a device with small $R_L$ produces a big RD and ends up with relatively less energy per unit mass at the end of the REGULAR phase when compared to a device with a big value of $R_L$. Therefore a big RD generally tends to lead to a small ED; whereas a small RD is more conducive to lead to a larger ED.

From the above we summarized that T1 plasma focus has a big RD, consequently, a small ED and hence can be completely fitted using a model that computes only the RD, which is able to stretch its RD by stretching the model parameters until the large RD covers also the small ED. Moreover energetically and mass-wise the fitting is correct. On the other hand, T2 plasma focus has a small RD, consequently a large ED. T2 plasma focus cannot be completely fitted with the RD computed from the code, no matter how the model parameters are stretched. To fit the computed current trace to the measured current for T2, a phase 4a needs to be included into the model in order to progress the current dip beyond the small RD into the large ED part of the current dip.

One way to simulate the current ED is to assign the phase 4a period with an anomalous resistance term such as:

$$R = R_0 \left[ \exp(-t/t_2) - \exp(-t/t_1) \right]$$  \hspace{1cm} (3.56)

where $R_0$ could be of the order of 1 $\Omega$, $t_1$ is a characteristic time representative of the rise time of the anomalous resistance and $t_2$ is characteristic of the fall time of the anomalous resistance (Fig. 3.12).

We have applied this technique to the KSU current waveform (Fig. 3.11). We note that using the 5-phase code, the computed RD does not follow the measured current dip which goes on to an ED. Following that first current dip in this particular case, the dip continues in a second portion which is almost flat then followed by a third section which is less steep than the first dip but of slightly longer duration. We applied a resistance term to each of the 3 sections. We adjusted the parameters $R_0$, $t_2$ and $t_1$ for each of the section as well as a fraction (endfraction) which terminates the term. The fitted parameters are as follows in Table 3.1:
With these parameters, it is found that the computed current dip now fits the measured current dip all the way to the end of the current dip at 2.1 μs (see Fig. 3.11) and even beyond to 2.6 μs where the computation ends as we are not interested in the fitting beyond phase 4a. The fitting involved the fitting of the RD followed by the ED of the first dip, then the second and third dips treated as ED’s each requiring a separate anomalous resistance function of the type Eq. (3.56).

The resistance functions used for the fitting are also shown in Fig. 3.13 (dashed trace, with the resistance values magnified 200 times in order to be visible on the scale of Fig. 3.13). The computed voltage waveform is also shown (trace labelled 2) compared with the measured voltage waveform (trace labelled 1). The correspondence of the computed voltage waveform and the measured is seen clearly. The lower measured values of voltage may be attributed to the inadequate frequency response of the resistive divider voltage probe.

### 3.2.5.3 Procedure for Using 6-Phase Code—Control Panel for Adding Anomalous Phases

The code is extended to the 6-phase version RADPFV6.1b. The corresponding control is by way of an additional panel for the anomalous resistances (default...)

### Table 3.1 Anomalous resistances used for the fitting

<table>
<thead>
<tr>
<th>Dip</th>
<th>( R_0 ) (Ω)</th>
<th>( t_2 ) (ns)</th>
<th>( t_1 ) (ns)</th>
<th>Endfraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dip 1</td>
<td>1.0</td>
<td>70</td>
<td>15</td>
<td>0.53</td>
</tr>
<tr>
<td>Dip 2</td>
<td>0.2</td>
<td>70</td>
<td>40</td>
<td>0.4</td>
</tr>
<tr>
<td>Dip 3</td>
<td>0.5</td>
<td>70</td>
<td>25</td>
<td>1.0</td>
</tr>
</tbody>
</table>

With these parameters, it is found that the computed current dip now fits the measured current dip all the way to the end of the current dip at 2.1 μs (see Fig. 3.11) and even beyond to 2.6 μs where the computation ends as we are not interested in the fitting beyond phase 4a. The fitting involved the fitting of the RD followed by the ED of the first dip, then the second and third dips treated as ED’s each requiring a separate anomalous resistance function of the type Eq. (3.56).
design is a 3-step AR). The main sheet (sheet 1) of the 6-phase code is the same as the 5-phase code depicted in Fig. 3.8b; except for an additional panel inserted below block 6 (see Fig. 3.8a and b for the location of block 6).

3.2.6 Conclusion for Description of the Lee Model Code

We presented the complete 5-phase Lee Model code, which is found to be adequate for fitting the computed current waveform and the measured waveform of each low $L_0$ (Type T1) plasma focus by varying two pairs of mass and current factors, one each for the axial and radial phases. Once fitted the code outputs in tabular and graphical forms realistic axial and radial dynamics (positions and speeds), pinch length and minimum pinch radius, temperatures and densities, bremsstrahlung and line yields, thermonuclear and beam-target yields, fast ion beam flux and fluence, energy flux and fluence, power flow and damage factors, fast plasma stream energies and speeds. All tests to date of computed with measured quantities have shown good agreement. The next section reviews the considerable results of the code of designing and optimizing machines, providing expected neutron, soft X-rays (various gases) and ion beams (various gases) yields, giving insights into current and neutron yield limitations, deterioration of neutron scaling (neutron saturation), radiative collapse, speed-enhanced PF, current-stepped PF and

---

**Fig. 3.13** Computed current (dip region only and expanded to see details) fitted to measured current with the inclusion of Phase 4a. Note that the computed current trace is fitted so well to the measured current trace that the two traces lie very closely on top of each other, these being the topmost traces (overlapping). Note also that the computed trace is stopped at 2.6 µs which is beyond the end of the AR3. *Note: AR stands for “anomalous resistance”*. Reprinted from Lee et al. [110]. Copyright (2011), with permission from Springer Science+Business Media, LLC
extraction of diagnostic data and anomalous resistance data from current signals. Yield scaling laws for neutron, soft X-rays and ion beams are obtained from the code. In one respect the 5-phase code has been found wanting. For high \( L_0 \) (Type T2) PF devices it is able to fit the computed current trace to the first part of the measured current trace; but the measured current then exhibits a much larger ‘extended’ dip which the computed current cannot be fitted; although the 5-phase code does produce reasonable values of neutron and soft X-rays yields in comparison with measured yields even for the high \( L_0 \) cases. The section concludes with an extended 6-phase code in order to complete the current fitting for high \( L_0 \) machines. An important use of the 6-phase code is for gathering data on the anomalous resistance of the plasma focus.

### 3.3 Scaling Properties of the Plasma Focus Arising from the Numerical Experiments

#### 3.3.1 Various Plasma Focus Devices

In Fig. 3.14 (upper left) is shown the UNU ICTP PFF 3 kJ device [7–9, 57] mounted on a 1 m × 1 m × 0.5 m trolley, which was wheeled around the International Centre for Theoretical Physics (ICTP) for the 1991 and 1993 Plasma Physics Colleges during the experimental sessions. The single capacitor is seen in the picture mounted on the trolley. In contrast, the upper right image in Fig. 3.14 shows on approximately the same scale, the PF1000, the 1 MJ plasma focus device [12] at the International Centre for Dense Magnetized Plasmas (ICDMP) in Warsaw, Poland. Only the chamber and the cables connecting the plasma focus to the capacitors are shown. The capacitor bank with its 288 capacitors, switches and chargers are located in a separate hall.

The comparison of physical dimensions of the devices is shown, refer lower part of the image in Fig. 3.14, in the shadowgraphs of the focus pinches of the two devices, on the same scale for comparison of the pinch dimensions. Each dimension of each pinch scales according to the anode radius of the corresponding device. It will be shown later that the anode radius scales to the peak current available to drive the plasma focus.

#### 3.3.2 Scaling Properties (Mainly Axial Phase)

Table 3.2 shows the characteristics of three plasma focus devices including the two shown in Fig. 3.14 [7, 11, 12]. These characteristics are taken from the code after using model parameters resulting from the fitting of computed current waveform with the corresponding measured current waveform.
In Table 3.2 we look at the PF1000 and study its properties at typical operation with energy storage at 500 kJ level. We compare this big focus (MJ) with two small devices at the kJ and sub-kJ level. The storage energy of the three devices covers a range over four orders of magnitude. In a survey of existing machines in 1996, it was noted [137] that machines, when optimized for neutron yield all, had axial speeds close to 10 cm/μs. Observations since then have not changed this view. The reason for a lower limit of speed is due to ionization and the need for a sufficiently large magnetic Reynold number to achieve efficient electromagnetic drive. The upper limit has been postulated [137] as due to a separation of centre of force field from the centre of mass field which would render the electromagnetic drive to become inefficient as the dynamics transitions from axial to radial phase. From Table 3.2 we note:

![Image](image.png)

Above: A small PF (3 kJ UNU ICTP PFF) on same scale for comparison with a large PF (1 MJ PF1000)

Below: Shadowgraphs of the pinches of the two machines placed on the same scale.

**Fig. 3.14** Comparing physical sizes and pinch sizes for a small PF (3 kJ) and a large PF (1 MJ)

In Table 3.2 we look at the PF1000 and study its properties at typical operation with energy storage at 500 kJ level. We compare this big focus (MJ) with two small devices at the kJ and sub-kJ level. The storage energy of the three devices covers a range over four orders of magnitude. In a survey of existing machines in 1996, it was noted [137] that machines, when optimized for neutron yield all, had axial speeds close to 10 cm/μs. Observations since then have not changed this view. The reason for a lower limit of speed is due to ionization and the need for a sufficiently large magnetic Reynold number to achieve efficient electromagnetic drive. The upper limit has been postulated [137] as due to a separation of centre of force field from the centre of mass field which would render the electromagnetic drive to become inefficient as the dynamics transitions from axial to radial phase. From Table 3.2 we note:
1. Voltage and pressure do not have particular relationship to $E_0$.

2. Peak current $I_{\text{peak}}$ increases with $E_0$.

3. Anode radius ‘$a$’ increases with $E_0$.

4. ID (current per cm of anode radius), $I_{\text{peak}}/a$, is in a narrow range from 160 to 210 kA/cm.

5. SF (speed or drive factor), $(I_{\text{peak}}/a)/P_0^{0.5}$, is 82–100 kA cm$^{-1}$/Torr$^{0.5}$ deuterium gas [137].

6. Peak axial speed $v_a$ is in the narrow range 9–11 cm/µs.

7. Neutron yield $Y_n$ ranges from $10^6$ neutrons/shot for the smallest device to $10^{11}$ neutrons/shot for PF1000.

We stress that whereas the ID and SF are practically constant at around 180 kA/cm and (90 kA/cm)/Torr$^{0.5}$ for deuterium gas operation throughout the range of small to big devices, $Y_n$ changes over 5 orders of magnitude.

The data of Table 3.2 is generated from numerical experiments and most of the data has been confirmed by actual experimental measurements and observations.

We note that the speed factor SF is inherent in the equations of motion (see Sections “Normalizing the Generating Equations to Obtain Characteristic Axial Transit Time, Characteristic Axial Speed and Speed Factor S; and Scaling Parameters of Times, $\alpha$ and Inductances $\beta$” and “Normalizing the Generating Equations to Obtain Characteristic Radial Transit Time, Characteristic Radial Transit Speed and Speed Factor S; and Scaling Parameters for Times $\alpha_1$ and Inductances $\beta_1$; also Compare Axial to Radial Length Scale, Time Scale and Speed Scale”). $(SF)^2$ is a measure of the kinetic energy per unit mass. Soto [158] has proposed an empirical ‘stored energy per unit volume parameter’ $28E_0/a^3$ as another constant for plasma focus machines with a range of $(0.4–80) \times 10^{10}$ J m$^{-3}$. The wide range of that parameter is because it is a “storage energy density” which translates into plasma energy density with different efficiency depending on the widely differing performance of different machines. Thus to result in the necessary plasma energy density (which is found to be a near constant for optimized neutron production) requires widely differing initial storage density. It is important to distinguish that whereas the Speed Factor is a fundamental scaling quantity derived inherently from the axial and radial phase equations, the ‘stored energy per unit volume’ parameter is an empirical parameter with a wide range of values.

<table>
<thead>
<tr>
<th>PF devices</th>
<th>$E_0$ (kJ)</th>
<th>$A$ (cm)</th>
<th>$Z_0$ (cm)</th>
<th>$V_0$ (kV)</th>
<th>$P_0$ (Torr)</th>
<th>$I_{\text{peak}}$ (kA)</th>
<th>$v_a$ (cm/µs)</th>
<th>ID (kA/cm)</th>
<th>SF [(kA cm$^{-1}$)/Torr$^{0.5}$]</th>
<th>$Y_n$ ($10^8$ n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1000</td>
<td>486</td>
<td>11.6</td>
<td>60</td>
<td>27</td>
<td>4</td>
<td>1850</td>
<td>11</td>
<td>160</td>
<td>85 ([kA cm$^{-1}$]/Torr$^{0.5}$]</td>
<td>1100</td>
</tr>
<tr>
<td>UNUPFF</td>
<td>2.7</td>
<td>1.0</td>
<td>15.5</td>
<td>14</td>
<td>3</td>
<td>164</td>
<td>9</td>
<td>173</td>
<td>100 ([kA cm$^{-1}$]/Torr$^{0.5}$]</td>
<td>0.20</td>
</tr>
<tr>
<td>PF400J</td>
<td>0.4</td>
<td>0.6</td>
<td>1.7</td>
<td>28</td>
<td>7</td>
<td>126</td>
<td>9</td>
<td>210</td>
<td>82 ([kA cm$^{-1}$]/Torr$^{0.5}$]</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 3.2 Characteristics of three PF devices covering energy range over four orders of magnitude
3.3.3 Scaling Properties (Mainly Radial Phase)

Table 3.3 compares the properties of the same three plasma focus devices [159] related to the radial phase and pinch. From Table 3.3 we note:

1. The pinch temperature \( T_{\text{pinch}} \) is strongly correlated to the square of the radial pinch speed \( v_p \).
2. The radial pinch speed \( v_p \) itself is closely correlated to the value of \( v_a \) and \( c = b/a \); so that for a constant \( v_a \), \( v_p \) is almost proportional to the value of \( c \). Thus it is noted that whereas the PF1000 has largest \( v_a \) of the 3 machines, it has the lowest \( v_p \) of the 3 machines. This is entirely due to its low value of \( c \); as can be seen from Eq. (3.28).
3. The dimensions and lifetime of the focus pinch scale as the anode radius ‘\( a \)’.
4. The \( r_{\text{min}}/a \) is almost constant at 0.14–0.17.
5. The \( z_{\text{max}}/a \) is almost constant at 1.5. (Note: \( z_{\text{max}} \) is twice the visible length of the pinch.)
6. Pinch duration has a relatively narrow range of 8–14 ns per cm of anode radius.
7. The pinch duration per unit anode radius is correlated to the inverse of \( T_{\text{pinch}} \).

This summary shows that the computed dimensions and lifetime of the pinch agree with the measured values depicted in Fig. 3.14.

\( T_{\text{pinch}} \) itself is a measure of the energy per unit mass. It is quite remarkable that this energy density at the focus pinch varies so little (factor of 5) over a range of device energy four orders of magnitude.

This practically constant pinch energy density (per unit mass) is related to the constancy of the axial speed moderated by the effect of the values of \( c \) on the radial speed.

The constancy of \( r_{\text{min}}/a \) suggests that the devices also produce the same compression of ambient density to maximum pinch density; with the ratio (maximum pinch density)/(ambient density) being proportional to \((a/r_{\text{min}})^2\). So for two devices of different sizes starting with the same ambient fill density, the maximum pinch density would be the same.

Note that Table 3.3 is for operation in deuterium over the range of machines. The deuterium is fully ionized and behaves like an ideal gas under the high

### Table 3.3 Properties of the pinches of the three plasma focus devices operated in deuterium

<table>
<thead>
<tr>
<th>PF devices</th>
<th>( c = b/a )</th>
<th>( a ) (cm)</th>
<th>( T_{\text{pinch}} ) (10⁴/ K)</th>
<th>( v_p ) (cm/μs)</th>
<th>( r_{\text{min}} ) (cm)</th>
<th>( z_{\text{max}} ) (cm)</th>
<th>Pinch duration (ns)</th>
<th>( r_{\text{min}}/a )</th>
<th>( z_{\text{max}}/a )</th>
<th>Pinch duration/ ( a ) (ns/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PF1000</td>
<td>1.4</td>
<td>11.6</td>
<td>2</td>
<td>13</td>
<td>2.2</td>
<td>19</td>
<td>165</td>
<td>0.17</td>
<td>1.6</td>
<td>14</td>
</tr>
<tr>
<td>UNU PFF</td>
<td>3.4</td>
<td>1.0</td>
<td>8</td>
<td>26</td>
<td>0.13</td>
<td>1.4</td>
<td>7.3</td>
<td>0.14</td>
<td>1.4</td>
<td>8</td>
</tr>
<tr>
<td>PF400 J</td>
<td>2.6</td>
<td>0.6</td>
<td>6</td>
<td>23</td>
<td>0.09</td>
<td>0.8</td>
<td>5.2</td>
<td>0.14</td>
<td>1.4</td>
<td>9</td>
</tr>
</tbody>
</table>
temperature operation of the plasma focus. In gases that are freely ionizing the compressions are affected by the thermodynamics of ionization and also line radiation. These effects will be further discussed in Sect. 3.5.

3.3.4 Scaling Properties: Rules of Thumb

From the above discussion, we may put down as rule-of-thumb the following scaling relationships, subject to minor variations caused primarily by the variation in \( c \) (cathode to anode radius ratio).

1. Axial phase energy density (per unit mass): constant
2. Radial phase energy density (per unit mass): constant
3. Pinch to anode radius ratio: constant
4. Pinch to anode length ratio: constant
5. Pinch duration per unit anode radius: constant.

Summarizing

1. The dense hot plasma pinch of a small \( E_0 \) plasma focus and that of a big \( E_0 \) plasma focus have essentially the same energy density, and the same mass density.
2. The big \( E_0 \) plasma focus has a bigger physical size and a bigger discharge current. The size of the plasma pinch scales proportionately to the current and to the anode radius, as does the duration of the plasma pinch.
3. The bigger \( E_0 \), the bigger \( ‘a’ \), the bigger \( I_{\text{peak}} \), the larger the plasma pinch and the longer the duration of the plasma pinch. The larger size and longer duration of the big \( E_0 \) plasma pinch are essentially the properties leading to the bigger neutron yield compared to the yield of the small \( E_0 \) plasma focus. The well-known yield \( \sim I^4 \) rule-of-thumb may be ascribed to the following: yield \( \sim \) product of volume and lifetime of the hot plasma; and each dimension of the volume as well as the lifetime is proportional to the current.

Numerical experiments have provided the following summary in Table 3.4.

<table>
<thead>
<tr>
<th>Plasma focus pinch parameters</th>
<th>Deuterium</th>
<th>Neon (for SXR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum radius, ( r_{\text{min}} )</td>
<td>0.15( a )</td>
<td>0.05( a )</td>
</tr>
<tr>
<td>Max length (hollow anode), ( z )</td>
<td>1.5( a )</td>
<td>1.6( a )</td>
</tr>
<tr>
<td>Radial shock transit, ( t_{\text{comp}} )</td>
<td>( 5 \times 10^{-6} )( a )</td>
<td>( 4 \times 10^{-6} )( a )</td>
</tr>
<tr>
<td>Pinch lifetime, ( t_p )</td>
<td>( 10^{-6} )( a )</td>
<td>( 10^{-6} )( a )</td>
</tr>
</tbody>
</table>

Unit of time is in s when ‘\( a’ \) the anode radius is in m.
In Table 3.4, the times are in s, and the value of anode radius, $a$, is in m. For the neon calculations, radiative terms are included, and the stronger compression (smaller radius) is due to thermodynamic and radiation effects.

The above description of the plasma focus combines data from numerical experiments and is consistent with laboratory observations some of which is summarized and depicted in Fig. 3.14.

### 3.3.5 Designing an Efficient Plasma Focus: Rules of Thumb

The Lee Model code may be used to aid the design of a new conventional machine. First, use the following rule of thumb procedure [use SI units].

1. What capacitance ($C_0$) are you planning?
2. How low is the inductance ($L_0$) you expect to attain?
3. What maximum voltage ($V_0$) do you expect to operate?
4. For the stray (circuit) resistance, take 1/4 the value of ($L_0/C_0$)\(^{1/2}\).
5. Estimate the undamped peak current using the formula $I_0 = V_0/(L_0/C_0)^{1/2}$.
6. Use $(I_0/a) = 250$ kA max undamped current per cm to assign the value of centre electrode (anode) radius ‘$a$’.
7. Put in double this value for outer electrode radius ‘$b$’.
8. The length of the electrode may be assigned as 5 times the value of $1.6(L_0/C_0)^{1/2}$. This length is in cm when the value of ($L_0/C_0$)\(^{1/2}\) is expressed in $\mu$s. This gives a length which will provide an average axial speed of 5 cm/$\mu$s which typically gives a peak speed at end of axial phase of 8 cm/$\mu$s. For operation in H\(_2\) or D\(_2\) 20\% longer (electrode length) may be better; for neon operation to get suitable line radiation (12–13.5 Å) for SXR microlithography purposes, 20\% shorter may be better as we require a lower speed to get to the correct level of ionization stages. The focus is normally operated so that the start of current dip (signifying the end of the axial phase) occurs at or just after peak current. For argon to generate characteristic argon SXR a high speed (much higher than calculated above for deuterium or neon) is required so use the same length as for H\(_2\). For xenon, if the aim is for EUV (around 13 nm) for experiments for NGL (next generation lithography) the model has predicted a requirement for very low speeds, around 1.3 cm/$\mu$s. So it appears one needs very short anodes, at least 5 times shorter than that needed for D\(_2\). However, there is not much experimental experience accumulated so far for xenon.
9. For pressure values assign as follows: D: 4 Torr; Ne: 1.5 Torr; Ar: 0.7 Torr. For xenon, runs on the code suggest several Torr go with the very short anode length.
The above rule of thumb design gives the complete specifications for planning a plasma focus.

The rule of thumb can be checked and fine tuned with the Lee Model code. First ask the question: is your PF fat or thin? (According to the ratio length of the centre electrode divided by diameter; for NX2 this ratio is 1.2, “fat”; for UNU/ICTP PFF this ratio is 17, “thin”.)

If it is “fat” use the model parameters suggested for the NX2. These suggested values are tabulated at the top right of the active sheet which appears when you open the RADPF code which is available online at www.plasmafocus.net.

If it is “thin” assign the model parameters closer to the UNU/ICTP PFF which are also included in the online code.

Configure the standard code using the parameters you have from the above rule of thumb estimates. Run the computation and from results make an adjustment to ‘a’, ‘b’, length $z_0$ ($V_0$ may also easily be varied, especially reduced since we have started with max $V_0$; $C_0$ also, use more or less capacitors). We need to be careful with $L_0$, normally make $L_0$ as small as possible, but be realistic. For a single capacitor with internal inductance of 40 nH, the value of $L_0$ could be put as 100 nH since $L_0$ is the total static inductance which includes the inductance of the capacitor bank plus that of the switch and connecting plates and or cable bunch plus the inductance of the PF collector plates usually forming the head of the PF tube. Adjust parameters for best results over a range of pressures and gases. Best results could mean a number of different (not necessarily concurrent) things for example strong current dip or best neutron yield (when operating with deuterium) or biggest line emission in the case of neon, which is useful for developing microlithography SXR sources.

3.3.6 Tapered Anode, Curved Electrodes, Current-Stepped PF, Theta Pinch

3.3.6.1 Tapered Anode

The code has incorporated tapered anode. The default value of taper configuration panel (see Sect. 3.2.4, Fig. 3.8b-block 2) is zero for no taper. If the anode is tapered, set the value to 1 and input the taper parameters: the position where taper starts and the radius of the end of the anode (i.e. at end of taper). The code has a branch which computes the inductance of the tapered anode geometry. Computations give reasonable results when compared to measured results.
3.3.6.2 Curved Electrodes

Bora Plasma Focus

For curved anodes such as NX1 [74, 78] or Bora [160] or the spherical plasma focus (SPF) [161, 162] the standard code is used and an ‘equivalent straight length’ technique is adopted. In effect, the curved electrodes are treated as straight electrodes and an equivalent length is estimated so as to use the standard code. A properly selected ‘equivalent’ anode length will give the correct run-down time. However, the change of inductance with distance travelled will not be correct since the inductance of the curved electrodes will have dependence with axial distance which is not strictly linear as is the case of a straight anode. This effect appears to be secondary since our experience is that a reasonable fit is obtained between the computed and the measured current waveforms with necessary adjustments being made to the equivalent straight length.

For example, for Bora, some details are given in a report by Gribkov et al. [160] from which the following diagram (Fig. 3.15) of the Bora plasma focus tube is obtained. A current waveform is also gleaned from that report as is explained in the manual by Saw [163].

We use the following configuration shown in Table 3.5:

The final fitted parameters $L_0$, $r_0$, $z_0$, $f_m$, $f_c$, $f_{mr}$ and $f_{cr}$ are shown in the fitting table (taken from fitted control panel) above.

The computed current waveform using the configuration of Table 3.5 gives the computed current waveform which is shown in Fig. 3.16.

---

Fig. 3.15 The geometry of Bora [160]
The computed neutron yield is $1.9 \times 10^8$ with a peak current of 304 kA. These compare well with their measured peak current of 300 kA and their estimated neutron yield of $10^8$ neutrons.

Spherical Plasma Focus, KU200

We had also applied the ‘equivalent straightened electrode’ technique to the KPU200 SPF [161, 162]. A diagram is given in Fig. 3.17.

We use the configuration shown in Table 3.6.

The final fitted parameters $L_0, r_0, z_0, f_m, f_c, f_{mr}$, and $f_{cr}$ are shown in the fitting Table 3.6 (taken from fitted control panel) above.

The computed current waveform using the configuration of Table 3.6 gives the computed current waveform which is shown as follows:

Table 3.5 Fitted configuration of Bora corresponding to relevant shot-values

<table>
<thead>
<tr>
<th>$L_0$ (nH)</th>
<th>$C_0$ (µF)</th>
<th>$b$ (cm)</th>
<th>$a$ (cm)</th>
<th>$z_0$ (cm)</th>
<th>$r_0$ (mΩ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54$^a$</td>
<td>24.4</td>
<td>2.5</td>
<td>1.5</td>
<td>6$^a$</td>
<td>6$^a$</td>
</tr>
<tr>
<td>Massf</td>
<td>Currf</td>
<td>Massfr</td>
<td>Currrfr</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.201$^a$</td>
<td>0.7$^a$</td>
<td>0.55$^a$</td>
<td>0.69$^a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_0$ (kV)</td>
<td>$P_0$ (Torr)</td>
<td>MW</td>
<td>A</td>
<td>At = 1 mol = 2</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>7.6</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Fitted using measured $dl/dr$; other values are given.
The fit of computed current trace with the experimental current signal is shown in Fig. 3.18. The fitted peak circuit current is 1.6 MA giving a D–T \( Y_n = 1.44 \times 10^{13} \) neutrons per shot. These compare with measured values of the peak current of 1.5 MA and D–T \( Y_n = 1.26 \times 10^{13} \) as reported in [162].

At 17.3 Torr in D-T, we computed 1.66 MA and \( 1.34 \times 10^{13} \) D-T neutrons. These compare well with their measured values of 1.65 MA and \( 1.2 \times 10^{13} \) D-T neutrons.

Operating in 12 Torr D at 25 kV, we computed peak current of 1.5 MA and D–D \( Y_n = 7.5 \times 10^{10} \) neutrons per shot. These compared with their measured value of 1.4 MA and D–D \( Y_n = (7.5–8.0 \times 10^{10} \) D–D neutrons per shot). The results for these 3 cases are summarized and compared in Table 3.7. The agreement is reasonable.

---

**Table 3.6** Fitted configuration of KPU corresponding to relevant shot-values

<table>
<thead>
<tr>
<th>( L_0 )</th>
<th>( C_0 )</th>
<th>( B )</th>
<th>( A )</th>
<th>( z_0 )</th>
<th>( r_0 ) (m( \Omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>36(^a)</td>
<td>432</td>
<td>15</td>
<td>8</td>
<td>21.3(^a)</td>
<td>1.2(^a)</td>
</tr>
<tr>
<td>Massf</td>
<td>Currf</td>
<td>Massfr</td>
<td>Curfr</td>
<td>Model parameters</td>
<td></td>
</tr>
<tr>
<td>0.0635(^a)</td>
<td>0.7(^a)</td>
<td>0.14(^a)</td>
<td>0.7(^a)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_0 )</td>
<td>( P_0 )</td>
<td>MW</td>
<td>A</td>
<td>At = 1 mol = 2</td>
<td>Operational</td>
</tr>
<tr>
<td>25</td>
<td>14.3</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>Parameters</td>
</tr>
</tbody>
</table>

\(^a\)Fitted using measured current; other values are given
A Note on the 2-D Model of Abdul Al-Halim et al.

In relation to the spherical plasma focus SPF, Abd Al-Halim has used a 2-D MHD model to obtain the dynamics of the spherical plasma focus [164], essentially modifying the Lee Model code to include curvature in the current sheet with the ability to follow curved electrodes. More recently his group has [165] extended the code to obtain neutrons, radiation and beam ions properties of the SPF. For the KPU200 they reported a computed value of $1.13 \times 10^{13}$ D-T neutrons for 14.3 Torr. Their computed value also agrees reasonably with the measured value.

### 3.3.6.3 Current-Steped Plasma Focus

Lee [166] had predicted based on basic energy and pressure considerations that a linear Z-pinch would have improved compression characteristics if a current is stepped rather than increased in the usual near sinusoidal fashion, even if it were just a faster sinusoid placed near the top of a slower sinusoid. This concept was tested by Saw in the laboratory using a current-stepped pinch [167, 168]. This same energy and pressure considerations, being fundamental, should already be inbuilt in the Lee Model code which is energy- and momentum-consistent. A two-circuit
A plasma focus with $b = 23.4$ cm, $a = 16.9$ cm and $z_0 = 35$ cm operating at 10 Torr was designed. For a 50 kV, 1 MJ, 6 μs rise-time bank, the current-step bank with $C_2$, $L_2$ and $r_2$ is switched by $S_2$ onto the anode, with mesh current $I_2$. The total current flowing into the anode is $I = I_1 + I_2$. The current taking part in the plasma dynamics is $f_c I_1$ with $(1 - f_c)I_1$ being a leakage current. Just before peak $I_1$, the current-step bank with $C_2$, $L_2$ and $r_2$ is switched by $S_2$ onto the anode, with mesh current $I_2$. The total current flowing into the anode is $I = I_1 + I_2$. The current taking part in the plasma dynamics is $f_c(I_1 + I_2)$; shown in this figure flowing in the axial phase. The leakage path of remnant current $(1 - f_c)(I_1 I_2)$ is not shown. Mesh 2 has smaller $L_2-C_2$ time constant than $L_1-C_1$ and $C_2$ is charged to a higher voltage than $C_1$. Reprinted from Lee and Saw [105]. Copyright (2012) with permission from Springer Science+Business Media, LLC

3.3.6.4 Procedure to Use Lee Code for the Above Devices

For application of the ‘straight electrode equivalent’ technique the standard 5-phase code is used. For the current-stepped computations, a separate code is required designated as the CS-RADPF05.15d (available from either author).

3.3.6.5 Theta Pinch Version of the Code

There is a version of the code written for the radiative theta pinch [169] which has the capacitor current coupled into the plasma via a single turn air-core loop. With this difference in the drive, the rest of the code is adapted from the radial phase of the Lee Model code. This code for the theta pinch is designated (May 2014) theta002.
3.4 Insights and Scaling Laws of the Plasma Focus Arising from the Numerical Experiments

3.4.1 Using the Lee Model Code as Reference for Diagnostics

The Lee Model code [10, 37] is configured [22, 152] to work as any plasma focus by inputting:

- Bank parameters \( L_0, C_0 \) and stray circuit resistance \( r_0 \);
- Tube parameters \( b, a \) and \( z_0 \);
- Operational parameters \( V_0 \) and \( P_0 \) and the fill gas.

The computed total current waveform is fitted to the measured waveform by varying model parameters \( f_{in}, f_{cr}, f_{mr} \) and \( f_{ct} \) one by one until the computed waveform agrees with the measured waveform as described earlier in Sect. 3.2.4.

During every adjustment of each of the model parameters, the code goes through the whole cycle of computation. In the last adjustment, when the computed total current trace is judged to be reasonably well-fitted in all 5 waveform features, computed time histories are presented, in Fig. 3.20a–n as an example, as follows: for the NX2 operated at 11 kV, 2.6 Torr Ne.

Thus the code after fitting to a measured current trace provides the dynamics and energetics of the plasma focus and properties of the plasma pinch. One important use of the code is to provide values that may act as a reference for diagnostic purposes.

3.4.1.1 Comments on Computed Quantities by Lee Model Code

1. The computed total discharge current trace typically is fitted very well with the measured trace. The end of the radial phase is indicated in Fig. 3.20a. Plasma currents are rarely measured. We had done a comparison of the computed plasma current with measured plasma current for the Stuttgart PF78 which shows good agreement of our computed to the measured plasma current [106]. The computed plasma current in this case of the NX2 is shown in Fig. 3.20b.

2. The computed tube voltage is difficult to compare with measured tube voltages in terms of peak values, typically because of the poor response time of voltage dividers used for voltage measurements. However the computed waveform shape in Fig. 3.20c is general as expected.

3. The computed axial trajectory and speed (Fig. 3.20d), agree with experimentally obtained time histories. Moreover, the behaviour with pressure, running the code at different pressures, agrees well with experimental results. The radial trajectories and speeds are difficult to measure. The computed radial trajectories Fig. 3.20e agrees with the scant experimental data available. The length of the
Fig. 3.20  a. Fitted computed $I_{total}$. b Computed $I_{total}$ or $I_{discharge}$ and $I_{plasma}$. c Tube voltage. d Axial trajectory and speed. e Radial trajectories. f Length of elongating structure. g Speeds in radial phases. h Tube inductance-both phases. i Total inductive energy. j Piston work and DR energy; traces overlap. k Peak and averaged uniform $n_p$. l Peak and averaged uniform $n_e$. m Plasma temperature $T$. n Neon soft X-ray power.
radial structure is shown in Fig. 3.20f. Computed speeds, radial shock front and piston speeds and speed of the elongation of the structure, are shown in Fig. 3.20g.

4. The computed inductance (Fig. 3.20h) shows a steady increase of inductance in the axial phase, followed by a sharp increase (rising by more than a factor of 2 in a radial phase time interval about 1/10 the duration of the axial phase for the NX2).

5. The inductive energy \(0.5LI^2\) peaks at 70% of initial stored energy and then drops to 30% during the radial phase, as the sharp drop of current more than offsets the effect of sharply increased inductance (Fig. 3.20i).

In Fig. 3.20j is shown the work done by the magnetic piston, computed using force integrated over distance method. Also shown is the work dissipated by the dynamic resistance, computed using dynamic resistance power integrated over
time. We see that the two quantities and profiles agree exactly. This validates the concept of half $L_{\text{dot}}$ as a dynamic resistance $DR$. The piston work deposited in the plasma increases steadily to some 12% at the end of the axial phase and then rises sharply to just below 30% in the radial phase. The values of the $DR$ in the axial phase, together with the bank surge impedance, are the quantities that determine $I_{\text{peak}}$.

6. The ion number density has a maximum value derived from shock-jump considerations, and an averaged uniform value derived from overall energy and mass balance considerations. The computed number density is averaged over the assumed flat profile, hence can be expected to be considerably lower than measured peak density. The time profiles of these are shown in Fig. 3.20k. The electron number density (Fig. 3.20l) has similar profiles to the ion density profile but is modified by the effective charge numbers due to ionization stages reached by the ions.

7. Plasma temperature too has a maximum value and an averaged uniform value derived in the same manner; are shown in Fig. 3.20m. Computed neon soft X-ray power profile is shown in Fig. 3.20n. The area of the curve is the soft X-ray yield in J. Pinch dimensions and lifetime may be estimated from Fig. 3.20e, f.

8. The code also computes the neutron yield, for operation in deuterium, using a phenomenological beam-target mechanism [10, 37] added on to a much smaller thermonuclear component. The code does not compute a time history of the neutron emission, only a yield number $Y_n$.

Thus as demonstrated above, the Lee Model code when properly fitted is able to realistically model any plasma focus and act as a guide to diagnostics of plasma dynamics, trajectories, energy distribution and gross plasma properties. Radiation yields and properties of FIB fast ion beams and FPS fast plasma streams are also computed. These are treated in other sections later.

3.4.1.2 Correlating Computed Plasma Dynamics with Measured Plasma Properties—A Very Powerful Diagnostic Technique

A measured current waveform is usually available together with measured time profiles of plasma properties. In such cases, the fitted code has been used to compare the computed dynamics with a measured streak photograph [95] and to correlate the computed dynamics with neon SXR time profiles [55, 56, 76] and Faraday Cup signals [81]. An example of such a correlation is shown in Fig. 3.46 in Sect. 3.5.6.7. This is a very powerful technique particularly when incorporated into a customized template. More widespread use will generate new data to further the understanding of the plasma focus.

Besides providing reference quantities for diagnostics, series of numerical experiments have been systematically carried out to look for behavioural patterns of the plasma focus. Insights uncovered by the series of numerical experiments include:
1. Pinch current limitation effect, and associated yield limitation, as static inductance is reduced;
2. Scaling laws for neutron, SXR, fast ion beams and fast plasma streams; and
3. The nature and a fundamental cause of neutron saturation (being a misnomer for neutron scaling deterioration).

3.4.2 Insight 1—Pinch Current Limitation Effect as Static Inductance Is Reduced Towards Zero

There was expectation [12] that the large MJ plasma focus PF1000 in Warsaw could increase its discharge current, and its pinch current, and consequently neutron yield by a reduction of its external or static inductance $L_0$. To investigate this point, experiments were carried out using the Lee Model code. Unexpectedly, the results indicated that whilst $I_{\text{peak}}$ indeed progressively increased with a reduction in $L_0$, no improvement may be achieved due to a pinch current limitation effect [49]. Given a fixed $C_0$ powering a plasma focus, there exists an optimum $L_0$ for maximum $I_{\text{pinch}}$. Reducing $L_0$ further will increase neither $I_{\text{pinch}}$ nor $Y_n$. The numerical experiments leading to this unexpected result is described below.

A measured current trace of the PF1000 with $C_0 = 1332 \ \mu F$, operated at 27 kV, 3.5 Torr deuterium, has been published [12], with cathode and anode radii $b = 16 \ \text{cm}$ and $a = 11.55 \ \text{cm}$ and anode length $z_0 = 60 \ \text{cm}$. In the numerical experiments, we fitted external (or static) inductance $L_0 = 33.5 \ \text{nH}$ and stray resistance $r_0 = 6.1 \ \text{m}\Omega$ (damping factor $RESF = r_0/(L_0/C_0)^{0.5} = 1.22$). The fitted model parameters are $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The computed current trace [10, 49] agrees very well with the measured trace [12] through all the phases, axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase as shown in Fig. 3.21.

We carried out numerical experiments for PF1000 using the machine and model parameters determined from Fig. 3.21. Operating the PF1000 at 35 kV and

![Fig. 3.21 Fitting computed current to measured current traces to obtain fitted parameters $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The measured current trace was for the PF1000 at 27 kV, storage capacity of 1332 \ $\mu F$ and fitted static inductance of 33.5 nH.](image-url)
3.5 Torr, we varied the anode radius \( a \) with the corresponding adjustment to \( b \) to maintain a constant \( c = b/a = 1.39 \) and in order to keep the peak axial speed at 10 cm/\( \mu \text{s} \). The anode length \( z_0 \) was also adjusted to maximize \( I_{\text{pinch}} \) as \( L_0 \) was decreased from 100 nH progressively to 5 nH.

As expected, \( I_{\text{peak}} \) increased progressively from 1.66 to 4.4 MA. As \( L_0 \) was reduced from 100 to 35 nH, \( I_{\text{pinch}} \) also increased, from 0.96 to 1.05 MA. However, then unexpectedly, on further reduction from 35 to 5 nH, \( I_{\text{pinch}} \) stopped increasing, instead of decreased slightly to 1.03 MA at 20 nH, to 1.0 MA at 10 nH, and to 0.97 MA at 5 nH. \( Y_n \) also had a maximum value of \( 3.2 \times 10^{11} \) neutron per shot at 35 nH.

To explain this unexpected result, we examine the energy distribution in the system at the end of the axial phase (see Fig. 3.21) just before the current drops from peak value \( I_{\text{peak}} \) and then again near the bottom of the almost linear drop to the pinch phase indicated by the arrow pointing to ‘end of radial phase’. The energy equation describing this current drop is written as follows:

\[
0.5I_{\text{peak}}^2 (L_0 + L_a f_c^2) = 0.5I_{\text{pinch}}^2 (L_0/f_c^2 + L_a + L_p) + \delta_{\text{cap}} + \delta_{\text{plasma}},
\]

where \( L_a \) is the inductance of the tube at full axial length \( z_0 \), \( \delta_{\text{plasma}} \) is the energy imparted to the plasma as the current sheet moves to the pinch position and is the integral of \( 0.5(dL/dt)I^2 \). We approximate this as \( 0.5L_p I_{\text{pinch}}^2 \) which is an underestimate for this case. \( \delta_{\text{cap}} \) is the energy flow into or out of the capacitor during this period of current drop. If the duration of the radial phase is short compared to the capacitor time constant, the capacitor is effectively decoupled and \( \delta_{\text{cap}} \) may be put as zero. From this consideration we obtain

\[
I_{\text{pinch}}^2 = I_{\text{peak}}^2 (L_0 + 0.5L_a)/(2L_0 + L_a + 2L_p)
\]

where we have taken \( f_c = 0.7 \) and approximated \( f_c^2 \) as 0.5.

Generally, as \( L_0 \) is reduced, \( I_{\text{peak}} \) increases; \( a \) is necessarily increased leading [49] to a longer pinch length \( z_p \), hence a bigger \( L_p \). Lowering \( L_0 \) also results in a shorter rise time, hence a necessary decrease in \( z_0 \), reducing \( L_a \). Thus, from Eq. (3.58), lowering \( L_0 \) decreases the fraction \( I_{\text{pinch}}/I_{\text{peak}} \). Secondly, this situation is compounded by another mechanism. As \( L_0 \) is reduced, the \( L-C \) interaction time of the capacitor bank reduces while the duration of the current drop increases due to an increasing \( a \). This means that as \( L_0 \) is reduced, the capacitor bank is more and more coupled to the inductive energy transfer processes with the accompanying induced large voltages that arise from the radial compression. Looking again at the derivation of Eq. (3.58) from Eq. (3.57) a nonzero \( \delta_{\text{cap}} \), in this case, of positive value, will act to decrease \( I_{\text{pinch}} \) further. The lower the \( L_0 \) the more pronounced is this effect.

Summarizing this discussion, the pinch current limitation is not a simple effect but is a combination of the two complex effects described above, namely, the interplay of the various inductances involved in the plasma focus processes abetted by the increasing coupling of \( C_0 \) to the inductive energetic processes, as \( L_0 \) is reduced.
3.4.3 **Neutron Yield Limitations Due to Current Limitations as \( L_0 \) Is Reduced**

From the pinch current limitation effect, it is clear that given a fixed \( C_0 \) powering a plasma focus, there exists an optimum \( L_0 \) for maximum \( I_{\text{pinch}} \). Reducing \( L_0 \) further will increase neither \( I_{\text{pinch}} \) nor \( Y_n \). The results of the numerical experiments carried out are presented in Fig. 3.22 and Table 3.8.

With large \( L_0 = 100 \text{ nH} \) it is seen (Fig. 3.22) that the rising current profile is flattened from what its waveform would be if unloaded; and peaks at around 12 \( \mu \text{s} \) (before its unloaded rise time, not shown, of 18 \( \mu \text{s} \)) as the current sheet goes into the radial phase. The current drop, less than 25% of peak value, is sharp compared with the current rise profile. At \( L_0 = 30 \text{ nH} \) the rising current profile is less flattened, reaching a flat top at around 5 \( \mu \text{s} \), staying practically flat for some 2 \( \mu \text{s} \) before the radial phase current drop to 50% of its peak value in a time which is still short compared to the rise time. With \( L_0 \) of 5 \( \text{nH} \), the rise time is now very short, there is hardly any flat top; as soon as the peak is reached, the current waveform

![Fig. 3.22](image)

**Fig. 3.22** PF1000 current waveforms computed at 35 kV, 3.5 Torr D\(_2\) for a range of \( L_0 \) showing the changes in waveforms as \( L_0 \) varies. Reprinted from Lee et al. [102]. © IOP Publishing. Reproduced with permission. All rights reserved.

**Table 3.8** Currents and ratio of currents as \( L_0 \) is reduced-PF1000 at 35 kV, 3.5 Torr deuterium

<table>
<thead>
<tr>
<th>( L_0 ) (nH)</th>
<th>( b ) (cm)</th>
<th>( a ) (cm)</th>
<th>( z_0 ) (cm)</th>
<th>( I_{\text{peak}} ) (MA)</th>
<th>( I_{\text{pinch}} ) (MA)</th>
<th>( Y_n ) ((10^{11}))</th>
<th>( I_{\text{pinch}}/I_{\text{peak}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>15.0</td>
<td>10.8</td>
<td>80</td>
<td>1.66</td>
<td>0.96</td>
<td>2.44</td>
<td>0.58</td>
</tr>
<tr>
<td>80</td>
<td>16.0</td>
<td>11.6</td>
<td>80</td>
<td>1.81</td>
<td>1.00</td>
<td>2.71</td>
<td>0.55</td>
</tr>
<tr>
<td>60</td>
<td>18.0</td>
<td>13.0</td>
<td>70</td>
<td>2.02</td>
<td>1.03</td>
<td>3.01</td>
<td>0.51</td>
</tr>
<tr>
<td>40</td>
<td>21.5</td>
<td>15.5</td>
<td>55</td>
<td>2.36</td>
<td>1.05</td>
<td>3.20</td>
<td>0.44</td>
</tr>
<tr>
<td>35</td>
<td>22.5</td>
<td>16.3</td>
<td>53</td>
<td>2.47</td>
<td>1.05</td>
<td>3.20</td>
<td>0.43</td>
</tr>
<tr>
<td>30</td>
<td>23.8</td>
<td>17.2</td>
<td>50</td>
<td>2.61</td>
<td>1.05</td>
<td>3.10</td>
<td>0.40</td>
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<tr>
<td>20</td>
<td>28.0</td>
<td>21.1</td>
<td>32</td>
<td>3.13</td>
<td>1.03</td>
<td>3.00</td>
<td>0.33</td>
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<tr>
<td>10</td>
<td>33.0</td>
<td>23.8</td>
<td>28</td>
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<td>1.00</td>
<td>2.45</td>
<td>0.27</td>
</tr>
<tr>
<td>5</td>
<td>40.0</td>
<td>28.8</td>
<td>20</td>
<td>4.37</td>
<td>0.97</td>
<td>2.00</td>
<td>0.22</td>
</tr>
</tbody>
</table>
drops significantly. There is a small kink on the current waveform of both the
$L_0 = 5 \text{ nH}, z_0 = 20 \text{ cm}$ and the $L_0 = 5 \text{ nH}, z_0 = 40 \text{ cm}$. This kink corresponds to
the start of the radial phase which, because of the large anode radius, starts with a
relatively low radial speed, causing a momentary reduction in dynamic loading.
Looking at the three types of traces it is seen that for $L_0 = 100–30 \text{ nH}$, there is a
wide range of $z_0$ that may be chosen so that the radial phase may start at peak or
near peak current, although the longer values of $z_0$ tend to give better energy
transfers into the radial phase.

The optimized situation for each value of $L_0$ is shown in Table 3.8. The table
shows that as $L_0$ is reduced, $I_{\text{peak}}$ rises with each reduction in $L_0$ with no sign of any
limitation. However, $I_{\text{pinch}}$ reaches a broad maximum of $1.05 \text{ MA}$ around
$40–30 \text{ nH}$. Neutron yield $Y_n$ also shows a similar broad maximum peaking at
$3.2 \times 10^{11}$ neutrons. Figure 3.23 shows a graphical representation of this $I_{\text{pinch}}$
limitation effect. The curve going up to $4 \text{ MA}$ at low $L_0$ is the $I_{\text{peak}}$ curve. Thus $I_{\text{peak}}$
shows no sign of limitation as $L_0$ is progressively reduced. However, $I_{\text{pinch}}$ reaches a
broad maximum. From Fig. 3.23 there is a stark and important message. One must
distinguish clearly between $I_{\text{peak}}$ and $I_{\text{pinch}}$. In general, one cannot take $I_{\text{peak}}$ to be
representative of $I_{\text{pinch}}$.

We carried out several sets of experiments on the PF1000 for varying $L_0$, each
set with a different damping factor. In every case, an optimum inductance was
found around $30–60 \text{ nH}$ with $I_{\text{pinch}}$ decreasing as $L_0$ was reduced below the opti-
num value. The results showed that for PF1000, reducing $L_0$ from its present
$20–30 \text{ nH}$ will increase neither the observed $I_{\text{pinch}}$ nor the neutron yield, because of
the pinch limitation effect. Indeed, the $I_{\text{pinch}}$ decreases very slightly on further
reduction to very small values. We would add that we have used a set of model

\begin{figure}[ht]
\centering
\includegraphics[width=\textwidth]{Fig_3.23}
\caption{Currents and current ratio (computed) as $L_0$
is reduced PF1000, 35 kV, 3.5 Torr D$_2$. Reprinted from
[102]. © IOP Publishing. Reproduced with permission. All rights reserved.}
\end{figure}
parameters which in our experience is the most reasonable to be used in these numerical experiments. Variations of the model parameters could occur but we are confident that these variations are not likely to occur with such a pattern as to negate the pinch current limitation effect. Nevertheless, these variations should be actively monitored and any patterns in the variations should be investigated.

Similar series of numerical experiments have been carried out for SXR yields \([98, 103]\) which indicate that the same current limiting effect also applies to optimizing the yields of neon SXR, and likely to other yields including FIB and FPS as well.

### 3.4.4 Insight 2—Scaling Laws for Neutron—Scaling Laws for Neutrons from Numerical Experiments Over a Range of Energies from 10 kJ to 25 MJ

We apply the Lee Model code to the MJ machine PF1000 over a range of \(C_0\) to study the neutrons emitted by PF1000-like bank energies from 10 kJ to 25 MJ.

As shown earlier the PF1000 current trace has been used to fit the model parameters, with very good fitting achieved between the computed and measured current traces (Fig. 3.10). Once the model parameters have been fitted to a machine for a given gas, these model parameters may be used with some degree of confidence when operating parameters such as the charging voltage are varied \([10, 132]\). With no measured current waveforms available for the higher megajoule numerical experiments, it is reasonable to keep the model parameters that we have got from the PF1000 fitting.

The optimum pressure for this series of numerical experiments is 10 Torr and the ratio \(c = b/a\) is retained at 1.39. For each \(C_0\), anode length \(z_0\) is varied to find the optimum. For each \(z_0\), anode radius \(a_0\) is varied so that the end-axial speed is 10 cm/\(\mu\)s. The numerical experiments were carried out for \(C_0\) ranging from 14 to 39,960 \(\mu\)F corresponding to energies from 8.5 kJ to 24.5 MJ \([44]\).

For this series of experiments we find that the \(Y_n\) scaling changes from \(Y_n \sim E_0^{2.0}\) at tens of kJ to \(Y_n \sim E_0^{0.84}\) at the highest energies (up to 25 MJ) investigated in this series. This is shown in Fig. 3.24.

From Figs. 3.24 and 3.25, over wide ranges of energy, optimizing pressure, anode length and radius, the scaling laws for \(Y_n\) \([21, 44, 100, 114–116]\) obtained through numerical experiments are listed here:

\[
Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5}
\]

\[
Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8} \cdot I_{\text{peak}} (0.3–5.7) \text{ in MA, } I_{\text{pinch}} (0.2–2.4) \text{ in MA.}
\]

\[
Y_n \sim E_0^{2.0} \text{ at tens of kJ to}
\]

\[
Y_n \sim E_0^{0.84} \text{ at MJ level (up to 25 MJ).}
\]
These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as optimized neutron sources.
3.4.5 Insight 3—Scaling Laws for Soft X-ray Yield

3.4.5.1 Computation of Neon SXR Yield

We note that the transition from Phase 4 to Phase 5 is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These localized regions are not modelled in the code, which consequently computes only an average uniform density, and an average uniform temperature which is considerably lower than measured peak density and temperature. However, because the 4 model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modelled. Hence the computed gross features such as speeds and trajectories and integrated soft X-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

In the code [10, 37], neon line radiation $Q_L$ is calculated:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} n_i^2 Z Z \left( \frac{\pi r_p^2}{C} \right) z_f / T$$

(3.59)

where for the temperatures of our interest we take the SXR yield $Y_{sxr} = Q_L$, $Z_n$ is the atomic number.

Hence the SXR energy generated within the plasma pinch depends on the properties: number density $n_i$, effective charge number $Z$, pinch radius $r_p$, pinch length $z_f$ and temperature $T$. It also depends on the pinch duration since in our code $Q_L$ is obtained by integrating over the pinch duration.

This generated energy is then reduced by the plasma self-absorption which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the SXR yield. These effects are included in the modelling by computing volumetric plasma self-absorption factor $A$ derived from the photonic excitation number $M$ which is a function of $Z_n$, $n_i$, $Z$ and $T$. However, in our range of operation, the numerical experiments show that the self-absorption is not significant. It was first pointed out by Liu [73, 75] that a temperature around 300 eV is optimum for SXR production. Subsequent work [55–57, 76–79] and further experience through numerical experiments suggest that around $2 \times 10^6$ K (below 200 eV) or even a little lower could be better. Hence unlike the case of neutron scaling, for SXR scaling there is an optimum small range of temperatures ($T$ windows) to operate.
3.4.5.2 Scaling Laws for Neon SXR Over a Range of Energies from 0.2 kJ to 1 MJ

We next use the Lee Model code to carry out a series of numerical experiments over the energy range 0.2 kJ–1 MJ [117]. In this case, we apply it to a proposed modern fast plasma focus machine with optimized values for \( c \) the ratio of the outer to inner electrode radius and \( L_0 \) obtained from our numerical experiments.

The following parameters are kept constant: (i) the ratio \( c = b/a \) (kept at 1.5, which is practically optimum according to our preliminary numerical trials); (ii) the operating voltage \( V_0 \) (kept at 20 kV); (iii) static inductance \( L_0 \) (kept at 30 nH, which is already low enough to reach the \( I_{\text{pinch}} \) limitation regime [49] over most of the range of \( E_0 \) we are covering) and; (iv) the ratio of stray resistance to surge impedance RESF (kept at 0.1, representing a higher performance modern capacitor bank). The model parameters [132] \( f_m, f_c, f_{mr} \) and \( f_{cr} \) are also kept at fixed values 0.06, 0.7, 0.16 and 0.7. We choose the model parameters so they represent the average values from the range of machines that we have studied. A typical example of a current trace for these parameters is shown in Fig. 3.26.

The storage energy \( E_0 \) is varied by changing the capacitance \( C_0 \). Parameters that are varied are operating pressure \( P_0 \), anode length \( z_0 \) and anode radius \( a \). Parametric variation at each \( E_0 \) follows the order; \( P_0, z_0 \) and \( a \) until all realistic combinations of \( P_0, z_0 \) and \( a \) are investigated. At each \( E_0 \), the optimum combination of \( P_0, z_0 \) and \( a \) is found that produces the biggest \( Y_{sxr} \). In other words at each \( E_0 \), a \( P_0 \) is fixed, a \( z_0 \) is chosen and \( a \) is varied until the largest \( Y_{sxr} \) is found. Then keeping the same values of \( E_0 \) and \( P_0 \), another \( z_0 \) is chosen and \( a \) is varied until the largest \( Y_{sxr} \) is found. This procedure is repeated until for that \( E_0 \) and \( P_0 \), the optimum combination of \( z_0 \) and \( a \) is found. Then keeping the same value of \( E_0 \), another \( P_0 \) is selected. The procedure for parametric variation of \( z_0 \) and \( a \) as described above is then carried out for this \( E_0 \) and new \( P_0 \) until the optimum combination of \( z_0 \) and \( a \) is found. This procedure is repeated until for a fixed value of \( E_0 \), the optimum combination of \( P_0, z_0 \) and \( a \) is found.

The procedure is then repeated with a new value of \( E_0 \). In this manner after systematically carrying out some 2000 runs, the optimized runs for various energies are tabulated in Table 3.9. We plot \( Y_{sxr} \) against \( E_0 \) as shown in Fig. 3.27.

**Fig. 3.26** Computed total current versus time for \( L_0 = 30 \) nH and \( V_0 = 20 \) kV, \( C_0 = 30 \) \( \mu \)F, RESF = 0.1, \( c = 1.5 \) and model parameters \( f_m, f_c, f_{mr}, f_{cr} \) are fixed at 0.06, 0.7, 0.16 and 0.7 for optimized \( a = 2.285 \) cm and \( z_0 = 5.2 \) cm.
Table 3.9 Optimized configuration found for each $E_0$

<table>
<thead>
<tr>
<th>$E_0$ (kJ)</th>
<th>$C_0$ (µF)</th>
<th>$a$ (cm)</th>
<th>$z_0$ (cm)</th>
<th>$P_0$ (Torr)</th>
<th>$I_{\text{peak}}$ (kA)</th>
<th>$I_{\text{pinch}}$ (kA)</th>
<th>$v_a$ (cm/µs)</th>
<th>$v_s$ (cm/µs)</th>
<th>$v_p$ (cm/µs)</th>
<th>$Y_{\text{sxr}}$ (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>1</td>
<td>0.58</td>
<td>0.5</td>
<td>4.0</td>
<td>100</td>
<td>68</td>
<td>5.6</td>
<td>22.5</td>
<td>14.9</td>
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<tr>
<td>1</td>
<td>5</td>
<td>1.18</td>
<td>1.5</td>
<td>4.0</td>
<td>224</td>
<td>143</td>
<td>6.6</td>
<td>23.3</td>
<td>15.1</td>
<td>7.5</td>
</tr>
<tr>
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<td>1.52</td>
<td>2.1</td>
<td>4.0</td>
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<td>186</td>
<td>6.8</td>
<td>23.6</td>
<td>15.2</td>
<td>20</td>
</tr>
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<td>5.2</td>
<td>4.2</td>
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<td>294</td>
<td>8.1</td>
<td>24.5</td>
<td>15.6</td>
<td>98</td>
</tr>
<tr>
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<td>2.79</td>
<td>7.5</td>
<td>4.0</td>
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<td>8.7</td>
<td>24.6</td>
<td>15.7</td>
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<td>470</td>
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<td>4.55</td>
<td>20</td>
<td>3.5</td>
<td>1109</td>
<td>565</td>
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<td>24.7</td>
<td>16.2</td>
<td>1000</td>
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<tr>
<td>100</td>
<td>500</td>
<td>6.21</td>
<td>42</td>
<td>3.0</td>
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<td>727</td>
<td>11.2</td>
<td>24.8</td>
<td>16.4</td>
<td>2700</td>
</tr>
<tr>
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<td>1000</td>
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<td>63</td>
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<td>876</td>
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<td>24.8</td>
<td>16.5</td>
<td>5300</td>
</tr>
<tr>
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<td>2000</td>
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<td>1036</td>
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<td>24.9</td>
<td>16.5</td>
<td>9400</td>
</tr>
<tr>
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<td>2.9</td>
<td>2157</td>
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<td>25.1</td>
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</tr>
<tr>
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<td>5000</td>
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<td>2428</td>
<td>1261</td>
<td>11.4</td>
<td>25.2</td>
<td>16.7</td>
<td>18000</td>
</tr>
</tbody>
</table>

Optimisation carried out with RESF = 0.1, $c = 1.5$, $L_0 = 30$ nH and $V_0 = 20$ kV and model parameters $f_m$, $f_c$, $f_{mr}$, $f_{cr}$ are fixed at 0.06, 0.7, 0.16 and 0.7, respectively. The $v_a$, $v_s$ and $v_p$ are the peak axial, radial shock and radial piston speeds, respectively.
We then plot $Y_{sxr}$ against $I_{peak}$ and $I_{pinch}$ and obtain SXR yield scales as

\[
Y_{sxr} \sim I_{pinch}^{3.6} \quad \text{and} \\
Y_{sxr} \sim I_{peak}^{3.2}.
\]

The $I_{pinch}$ scaling has less scatter than the $I_{peak}$ scaling. We next subject the scaling to further test when the fixed parameters RESF, $c$, $L_0$ and $V_0$ and model parameters $f_m$, $f_c$, $f_{mr}$, and $f_{cr}$ are varied. We add in the results of some numerical experiments using the parameters of several existing plasma focus devices including the UNU/ICTP PFF (RESF = 0.2, $c = 3.4$, $L_0 = 110$ nH and $V_0 = 14$ kV with fitted model parameters $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.2$, $f_{cr} = 0.8$), the NX2 (RESF = 0.1, $c = 2.2$, $L_0 = 20$ nH and $V_0 = 11$ kV with fitted model parameters $f_m = 0.10$, $f_c = 0.7$, $f_{mr} = 0.12$, $f_{cr} = 0.68$), and PF1000 (RESF = 0.1, $c = 1.39$, $L_0 = 33$ nH and $V_0 = 27$ kV with fitted model parameters $f_m = 0.1$, $f_c = 0.7$, $f_{mr} = 0.15$, $f_{cr} = 0.7$). These new data points (white data points in Fig. 3.28) contain wide ranges of $c$, $V_0$, $L_0$ and model parameters. The resulting $Y_{sxr}$ versus $I_{pinch}$ log–log curve remains a straight line, with the scaling index 3.6 unchanged and with no more scatter than before. However, the resulting $Y_{sxr}$ versus $I_{peak}$ curve now exhibits considerably larger scatter and the scaling index has changed slightly (note the change is not shown/obvious here).

We would like to highlight that the consistent behaviour of $I_{pinch}$ in maintaining the scaling of $Y_{sxr} \sim I_{pinch}^{3.6}$ with less scatter than the $Y_{sxr} \sim I_{peak}^{3.2}$ scaling particularly when mixed-parameters cases are included, strongly supports the conclusion that

Fig. 3.27 $Y_{sxr}$ versus $E_0$. The parameters kept constants are: RESF = 0.1, $c = 1.5$, $L_0 = 30$ nH and $V_0 = 20$ kV and model parameters $f_m$, $f_c$, $f_{mr}$, and $f_{cr}$ at 0.06, 0.7, 0.16 and 0.7, respectively. The scaling deterioration observed in this figure is similar to that for neutron yield and is discussed in Sect. 3.4.7. Reprinted from Lee et al. [117]. © IOP Publishing. Reproduced with permission. All rights reserved.
I\(_{\text{pinch}}\) scaling is the more universal and robust one. Similarly, conclusions on the importance of I\(_{\text{pinch}}\) in plasma focus performance and scaling laws have been reported [106].

It may also be worthy of note that our comprehensively surveyed numerical experiments for Mather configurations in the range of energies 0.2 kJ–1 MJ produce an I\(_{\text{pinch}}\) scaling rule for Y\(_{\text{sxr}}\) not compatible with Gates’ rule [170]. However it is remarkable that our I\(_{\text{pinch}}\) scaling index of 3.6, obtained from a set of comprehensive numerical experiments over a range of 0.2 kJ–1 MJ, on Mather-type devices, is within the range of 3.5–4 postulated on the basis of sparse experimental data, (basically just two machines one at 5 kJ and the other at 0.9 MJ), by Filippov et al. [171], for Filippov configurations in the range of energies 5 kJ–1 MJ.

It must be pointed out that the results represent scaling for comparison with baseline plasma focus devices that have been optimized in terms of electrode dimensions. It must also be emphasized that the scaling with I\(_{\text{pinch}}\) works well even when there are some variations in the actual device from \(L_0 = 30 \, \text{nH}, V_0 = 20 \, \text{kV}\) and \(c = 1.5\).

**Summary of Soft X-ray scaling laws found by numerical experiments:**

Over wide ranges of energy, optimizing pressure, anode length and radius, the scaling laws for neon SXR found by numerical experiments are:

\[
Y_{\text{sxr}} = 8.3 \times 10^3 \times I_{\text{pinch}}^{3.6},
\]
\[
Y_{\text{sxr}} = 600 \times I_{\text{peak}}^{3.2} \cdot I_{\text{peak}} \text{ (0.1 to 2.4)}, \quad I_{\text{pinch}} \text{ (0.07 to 1.3)} \text{ in MA}.
\]
\[
Y_{\text{sxr}} \sim E_0^{1.6} \text{ (kJ range)} \quad \text{to} \quad Y_{\text{sxr}} \sim E_0^{0.8} \text{ (towards MJ)}.
\]

These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as neon SXR sources.

---

**Fig. 3.28** \(Y_{\text{sxr}}\) is plotted as a function of \(I_{\text{pinch}}\) and \(I_{\text{peak}}\). The parameters kept constant for the black data points are: RESF = 0.1, \(c = 1.5\), \(L_0 = 30 \, \text{nH}\) and \(V_0 = 20 \, \text{kV}\) and model parameters \(f_{mn}, f_{fr}, f_{ft}\) at 0.06, 0.7, 0.16 and 0.7, respectively. The white data points are for specific machines which have different values for the parameters \(c, L_0\) and \(V_0\). © IOP Publishing. Reproduced with permission. All rights reserved.
In a similar fashion, scaling laws for several other gases, e.g. nitrogen, oxygen, argon have been computed [98, 119, 120].

3.4.6 Insight 4—Scaling Laws for Fast Ion Beams and Fast Plasma Streams from Numerical Experiments

3.4.6.1 Computation of Beam Ion Properties

The Lee code has been extended (RADPFV5.15FIB) and computes the flux of the ion beams:

\[ J_b = n_b v_b \quad (3.60) \]

where \( n_b \) = number of beam ions \( N_b \) divided by the volume of plasma traversed is derived from pinch inductive energy considerations; and \( v_b \) = effective speed of the beam ions is derived from the accelerating voltage taken as diode voltage \( U \). All quantities are expressed in SI units, except where otherwise stated.

3.4.6.2 The Ion Beam Flux and Fluence Equations

We derive \( n_b \) from the beam kinetic energy BKE and pinch inductive energy PIE considerations.

The BKE is contributed from the total number of beam ions \( N_b \) where each beam ion has a mass \( M_{m_p} \) and speed \( v_b \) and is represented by \( BKE = (1/2)N_bM_{m_p}v_b^2 \). The mass of the proton \( m_p \) is \( 1.673 \times 10^{-27} \) kg and \( M \) is the mass number of ion, e.g. neon ion has a mass number \( M = 20 \).

This BKE is imparted by a fraction \( f_e \) of the PIE represented by \( PIE = (1/2)L_pI_{\text{pinch}}^2 \) where \( L_p = (\mu/2\pi)(\ln[b/r_p])z_p \) is the inductance of the focus pinch; \( \mu = 4\pi \times 10^{-7} \) Hm\(^{-1} \); \( b \) = outer electrode of the plasma focus carrying the return current; \( r_p \) = pinch radius carrying the current through the plasma; \( z_p \) = length of the pinch and \( I_{\text{pinch}} \) is the pinch current value taken at start of pinch.

Thus: \( (1/2)N_bM_{m_p}v_b^2 = f_e(1/2)(\mu/2\pi)(\ln[b/r_p])z_pI_{\text{pinch}}^2 \)

This gives:

\[ n_b = N_b/(\pi r_p^2 z_p) = (\mu/[2\pi^2 m_p])\{f_e/M \} \{((\ln[b/r_p])^2/r_p^2)\}(I_{\text{pinch}}^2/v_b^2) \quad (3.61) \]

Next, we proceed to derive \( v_b \) from the accelerating voltage provided by the diode voltage \( U \) to an ion. Each ion with effective charge \( Z_{\text{eff}} \) is given kinetic energy of \( (1/2)M_{m_p}v_b^2 \) by diode voltage \( U \). Thus:
\[ (1/2)M m_p v_b^2 = Z_{\text{eff}} e U \]

where \( e \) is the electronic (or unit) charge \( 1.6 \times 10^{-19} \text{ C} \);

Hence

\[ v_b = (2e/m_p)^{1/2} (Z_{\text{eff}}/M)^{1/2} U^{1/2} \quad (3.62) \]

Now, we substitute \( n_b \) and \( v_b \) from Eqs. (3.61) and (3.62) into Eq. (3.60); and noting that \( (\mu/[2.83\pi^2(e m_p)^{1/2}]) = 2.75 \times 10^{15} \), we have the flux equation:

\[
\text{Flux} = J_b = 2.75 \times 10^{15} \left\{ f_e/[M Z_{\text{eff}}]^{1/2}\right\} \left\{ \left(\ln[b/r_p]\right)/(r_p^2)\right\} \left(r^2_{\text{pinch}}\right)/U^{1/2} \quad (3.63)
\]

in units of \((\text{ions m}^{-2} \text{ s}^{-1})\)

where \( M \) = ion mass, \( Z_{\text{eff}} \) = average effective charge of the ion in the pinch, \( b = \) cathode radius, \( r_p = \) pinch radius and \( I_{\text{pinch}} = \) pinch current. The parameter \( f_e \) is the fraction of energy converted into beam energy from the inductive energy of the pinch. Analyzing neutron yield data [58, 59] and pinch dimensional-temporal relationships we estimate \( f_e = 0.14 \) and use the approximate scaling [137]: \( \tau = 10^{-6} z_p \). This condition \( f_e = 0.14 \) is equivalent to ion beam energy of 3–6% \( E_0 \) in the case when the pinch inductive energy holds 20–40% of \( E_0 \). Our extensive study of high performance low inductance PF classified [110] as Type T1 shows that this estimate of \( f_e \) is consistent with data.

The value of the ion flux is deduced in each situation by computing \( Z_{\text{eff}}, r_p, I_{\text{pinch}} \) and \( U \) from the code.

The fluence is the flux multiplied by pulse duration \( \tau \). Thus:

\[
\text{Fluence (ions m}^{-2}\text{)} = 2.75 \times 10^{15} \tau \left\{ f_e/[M Z_{\text{eff}}]^{1/2}\right\} \left\{ \left(\ln[b/r_p]\right)/(r_p^2)\right\} \left(r^2_{\text{pinch}}\right)/U^{1/2} \quad (3.64)
\]

For deuteron where \( M = 2 \) and \( Z_{\text{eff}} = 1 \); and if we take \( f_e = 0.14 \) (ie 14% of PIE is converted into BKE) then we have for deuterons:

\[
\text{Fluence (ions m}^{-2}\text{)} = J_b \tau = 8.5 \times 10^8 \left[Z_{\text{pinch}}\right]^{2} \left\{ \left(\ln[b/r_p]\right)/(\pi r_p^2 U^{1/2})\right\} \quad (3.65)
\]

Equation (3.65) is exactly equivalent to Eq. (3.5) first derived in [58].

In other words starting from first principles we have derived exactly the same equation using empirical formula derived with quantities all with proportional constants finally calibrated at a 0.5 MJ point of neutron yield. In this present derivation from first principles, we need only one additional condition \( f_e = 0.14 \) (the fraction of energy converted from PIE into BKE) and the approximate scaling \( \tau = 10^{-6} z_p \). This additional condition of \( f_e = 0.14 \) is equivalent to ion beam energy of 3–6% \( E_0 \) for cases when the PIE holds 20–40% of \( E_0 \) as observed for Type T1 or low inductance plasma focus device. We also conclude that the flux Eq. (3.63)
derived here is the more basic equation to use as it does not have to make any assumptions about the ion beam pulse duration.

According to Eqs. (3.63) and (3.64) the flux and fluence are dependent on $(MZ_{\text{eff}})^{-1/2}$, if all other pinch properties remain equal. From this simple dependency one would expect the flux and fluence to reduce as we progress from H$_2$ to D$_2$, He to Kr and Xe. However, the pinch properties, primarily the pinch radius do change drastically for different gases at different regimes of operation; due to thermodynamic and radiative effects. The change in $r_p$ and associated and consequential changes in pinch dynamics and other properties, as computed from the code we use in this chapter, have profound effects on modifying this simple dependence.

We summarize the assumptions:

1. Ion beam flux $J_b$ is $n_b v_b$ with units of ions m$^{-2}$ s$^{-1}$.
2. Ion beam is produced by diode mechanism [12].
3. The beam is produced uniformly across the whole cross section of the pinch.
4. The beam speed is characterized by an average value $v_b$.
5. The BKE is a fraction $f_e$ of the PIE, taken as 0.14 in the first instance; to be adjusted as numerical experiments indicate.
6. The beam ion energy is derived from the diode voltage $U$.
7. The diode voltage $U$ is $U = 3V_{\text{max}}$ taken from data fitting in extensive earlier numerical experiments [10, 100], where $V_{\text{max}}$ is the maximum induced voltage of the pre-pincho radial phase. However for cases exhibiting strong radiative collapse, the strong radiative collapse generates an additional induced voltage $V^{*}_{\text{max}}$. This voltage is very large and from extensive numerical experiments appears to be a reasonable estimate of the beam ion energy from the point of view of the various energy distributions including the ion beam energy relative to the fast plasma stream energy. Hence the feedback from our extensive examinations of the data suggests that we take, in such cases [58–60] $U = V^{*}_{\text{max}}$.

The value of the ion flux is deduced in each situation for specific machine using specific gas by computing the values of $Z_{\text{eff}}, r_p, I_{\text{pinch}}$ and $U$ by configuring the Lee Model code with the parameters of the specific machine and specific gas. The code and the procedure are discussed in more detail in a later section.

### 3.4.6.3 Consequential Properties of the Ion Beam [59]

Once the flux is determined, the following quantities are also computed:

1. Energy flux or power density flow (W m$^{-2}$) is computed from $J_b \times Z_{\text{eff}} U$ noting the need to multiply by $1.602 \times 10^{-19}$ to convert eV to J;
2. Power flow (W) is computed from Energy flux $\times$ pinch cross section;
3. Current density (A m$^{-2}$) is computed from $J_b \times$ ion charge $eZ_{\text{eff}}$;
4. Current (A) is computed from Current density $\times$ pinch cross section;
5. Ions per sec (ions s$^{-1}$) is computed from $J_b \times$ pinch cross section;

S. Lee and S.H. Saw
6. Fluence (ions m\(^{-2}\)) is computed from \(J_b \times \tau\);
7. Energy fluence (J m\(^{-2}\)) is computed from \(J_b \times \tau \times Z_{\text{eff}}U\);
8. Number of ions in beam (ions) is computed from Fluence \(\times\) pinch cross section;
9. Energy in beam (J) is computed from Number of ions in beam \(\times Z_{\text{eff}}U\);
10. Damage Factor (Wm\(^{-2}\) s\(^{0.5}\)) is computed from \(J_b \times Z_{\text{eff}}U \times \tau^{1/2}\);

Experimentally it is found that as the focus pinch starts to break-up a fast shock wave exits the plasma focus pinch in the axial direction preceding the ion beams which rapidly catches up and overtakes it. Associated with this fast post-pinch axial shock wave is a fast plasma stream (FPS) [59]. We estimate the energy of the FPS by computing the work done by the magnetic piston through the whole radial phase from which is subtracted twice the ion beam energy (the second count being for the oppositely directed relativistic electron beam which we assume to have the same energy as the ion beam) and from which is further subtracted the radiation yield of the plasma pinch.

3.4.6.4 Fast Ion Beam and Fast Plasma Stream Properties of a Range of Plasma Focus Devices—Investigations of Damage to Plasma Facing Wall Materials in Fusion Reactors

Each of twelve machines of an IAEA CRP (Coordinated Research Program F13013—“Investigations on materials under high repetition and intense fusion pulses”) [172] was fitted using parameters (bank, tube and operation) supplied with a measured current trace. Where necessary the fitting of the code output current waveform to the measured current waveform also entailed adjustments to the values of static inductance \(L_0\) and stray resistance \(r_0\). The results of the fitting are a set of model parameters \(f_{\text{m},c}\) for the axial phase and \(f_{\text{mr},c}\) for the radial phase. Once fitted the dynamics in terms of axial and radial speeds and trajectories are found, as are the plasma axial phase and radial phase and pinch plasma properties such as temperatures and densities and neutron yields (in deuterium). Also computed are the properties (number and energy fluence and flux, power flow and damage factors, ion energy and current) of the fast ion beam (FIB) and the energies and properties of the fast plasma stream (FPS). The most important of the computed properties of the IAEA CRP plasma focus machines are listed in Table 3.10.

The most important features regarding the scaling of ion beam properties that are observed from Table 3.10 are as follows:

- FIB Properties independent of machine size: Number fluence, damage factor and speed factor
- FIB Properties dependent on machine size: Beam size (footprint-cross sectional radius), beam pulse length, number of ions per shot, beam current, total beam energy per shot and power flow (W)
Table 3.10 The parameters of the IAEA CRP PF machines

<table>
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<tr>
<th>Machine</th>
<th>PF1000</th>
<th>NX3</th>
<th>BARC</th>
<th>PF6</th>
<th>Bora</th>
<th>INTI</th>
<th>NX2T</th>
<th>PF12</th>
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<td>PF1000</td>
<td>NX3</td>
<td>BARC</td>
<td>PF6</td>
<td>Bora</td>
<td>INTI</td>
<td>NX2T</td>
<td>PF12</td>
<td>Sofia</td>
<td>PF5M</td>
<td>PF400J</td>
<td>FMPF3</td>
</tr>
</tbody>
</table>
• FIB properties with minor dependence on machine size: Number flux, number fluence, energy flux and energy per ion
• FPS properties, practically independent of machine size: equivalent damage factor, FPS energy as fraction of $E_0$ and flow speed.

Validation of our computations in regards to FIB and FPS properties in relation to the damage testing is presented in reference [60]. In particular, our computed values of damage factor, power flow density and FPS energy [58] for the case of PF-400J agree with those reportedly measured by Soto et al. [173]. Related recent work on target interaction and damage testing has been reported [113, 174–176]. Numerical experiments [58, 59] have already established that the fluence and flux and energy fluence and flux and damage factors have similar values within a narrow range for all plasma focus whether big or small. Thus small plasma focus devices produce as much damage as a big plasma focus; except that the damage produced in a small plasma focus is over a smaller area compared to the bigger target area that the big plasma focus irradiate on a per shot basis. However, exposure frequencies of $>1$ Hz are achieved in small plasma focus devices aggregating thousands of shots in a few minutes [74]. Thus the damage accumulated over a number of shots can be achieved much more rapidly in a small plasma focus fired repetitively than in a big focus which is a single shot. Therefore, important progress could be achieved in materials damage testing for plasma facing walls of fusion reactors using small plasma focus devices as plasma sources.

3.4.6.5 Slow Focus Mode SFM Versus Fast Focus Mode

FFM-Advantage of SFM for Fast Plasma Stream
Nano-materials Fabrication: Selection of Energy of Bombarding Particles by Pressure Control [63]

As a source of neutrons, X-rays and charged-particle beams the plasma focus PF is typically operated in the time-matched regime (TMR) where maximum energy is pumped into the radial shock waves and compression, resulting in large inductive voltages, high temperatures and copious multi-radiations. In this Fast Focus Mode (FFM) of operation, targets placed in front of the anode are subjected to strong bursts of fast ion beams (FIB), post-pinch fast plasma streams (FPS) followed by materials exploded off the anode by relativistic electron beams (REB); in that order of time sequence. In the INTI PF in hydrogen, as the operational pressure is increased beyond the TMR, the dynamics slows, the minimum pinch radius ratio increases, peak inductive voltages $V_{\text{max}}$ decreases, the FIB reduces in energy per ion $U$, in beam power flow $P_{\text{FIB}}$ and in damage factor $D_{\text{FIB}}$, as operation moves away from FFM into the Slow Focus Mode (SFM). This is the same pattern for D, He, N and Ne; but for the highest radiative gases Ar, Kr and Xe, radiative collapse becomes dominant, past the time-matched point; and the points of highest $V_{\text{max}}$, $P_{\text{FIB}}$ and $D_{\text{FIB}}$ shift to relatively higher pressures. However in all gases in all machines, as operational pressure is increased further, there comes a point (slowest
SFM or SSFM point) where compression is so weak that outgoing reflected shock barely reaches the incoming piston ie the focus pinch barely forms (see Fig. 3.29, case of 24.4 Torr). We consider this point (at 24.4 Torr) as hypothetical and would rather take the case of 22 Torr for discussion. In any case, because we have taken the model parameters as fixed throughout the whole range of pressure we expect that in practice the choice of SFM regime would likely be shifted towards higher pressure since from experiments we find the tendency is for $f_m$ and $f_{mr}$ to be larger as pressure moves towards the SFM regime. Thus the approach the SSFM point would be reached at a lower pressure than shown in this discussion.

At this SSFM point the pinch radius ratio is at its largest (typically $>2$ times that of the FFM), the $V_{\text{max}}$ (see Fig. 3.30), $P_{\text{FIB}}$ and $D_{\text{FIB}}$ (see Fig. 3.31) are very low and we expect a great reduction of anode boil-offs due to reduction of REB’s [63].

However, FPS energy is near its highest level. Operation near this SSFM point reduces ion beam damage and anode materials on-target and allows the largest area of interaction, primarily with the FPS (Fig. 3.32).

The above is surmised from results using RADPF FIB code. Recent laboratory experiments with targets in INTI PF [81] confirm experimental indications [177] that such high pressure operations produce a bigger area of more uniform target interaction. This should produce better results in production of nano-materials such as carbon nano-tubes on graphite substrate. Moreover, numerical experiments suggest that operational pressure may be used to select FPS particle energy (see Fig. 3.33) [63].
This ability to control and select bombarding particle energy, particularly in the range from tens to hundreds of eV will contribute to making PF materials technology more of a science than the present state-of-the-art.
In the above section we have demonstrated operating FFM and SFM in the INTI PF, and in similar fashion can demonstrate both regimes in any other plasma focus. However, it is more efficient, given a capacitor bank, to have two sets of electrodes, one designed for FFM (smaller anode radius) and the other set designed for SFM (larger anode radius).

The key to the production of these two distinct regimes of operation of the plasma focus, the intense pinch regime and the plasma flow regime, is the speed parameter which may be expressed as \((\frac{I_{\text{peak}}}{a})/\sqrt{P}\) where ‘\(a\)’ is the radius and \(P\) is the pressure in Torr. For plasma focus operated in intense neutron-optimized regime in deuterium the speed factor is known to be in the region of \((90 \text{ kA/cm})/\text{Torr}^{0.5}\) [137]. At typical operation of 4 Torr deuterium the required current density may be taken to be 180 kA/cm of ‘\(a\)’. On the other hand we expect that at a speed factor < \((50 \text{ kA/cm})/\text{Torr}^{0.5}\) a PF will typically be not operating at optimized intense pinch. The radiation and ion beam emission from the low speed parameter pinch will be reduced. The design of a plasma focus that operate interchangeably in both regimes will hinge on designing it to operate efficiently in two different speed factors one of which is large of the order of \((90 \text{ kA/cm})/\text{Torr}^{0.5}\) and the other less than half that value.

Starting with available \(6 \times 450 \mu\text{F}\) capacitors rated at 11 kV (10% reversal), numerical experiments indicate safe operation at 9 kV, more than 1 Torr deuterium with FFM anode of 5 cm radius; producing intense ion beam and streaming plasma pulses which would be useful for studies of potential fusion reactor wall materials. On the other hand operating at 5 kV, near 10 Torr deuterium with SFM anode of 10 cm radius leads to long duration uniform flow of larger interacting cross sections with low damage factors which could be more suitable for synthesis of nano-materials.
The results shown in Tables 3.11 and 3.12 confirm that the FFM configuration of the DuPF in the range 2–12 Torr produces intense pulses suitable for damage testing whilst the SFM configuration in the range 3–10 Torr produces less damaging larger area streams with characteristics likely to be suitable for advanced materials fabrication. Schematics of the DuPF and the interchangeable electrodes for SFM and FFM are shown in Figs. 3.34, 3.35 and 3.36 [178]. Typical current waveforms [178] of the DuPF in FFM and in SFM are shown in Fig. 3.37.

<table>
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<tr>
<th>$P_0$ (Torr)</th>
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<td>2.73</td>
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Table 3.11 Results of numerical on 160 kJ DuPF operated with deuterium at different pressures during FFM operation; 9 kV deuterium, $b = 8$ cm, $a = 5$ cm, $z_0 = 70$ cm, $C_0 = 2700$ µF, $L_0 = 50$ nH, $r_0 = 1$ mΩ [81]
Besides being accurately descriptive and related to wide-ranging experimental reality, desirable characteristics of a model include predictive and extrapolative scaling. Moreover, a useful model should be accessible, usable and user-friendly and should be capable of providing insights. Insight, however, cannot be a characteristic of the model in isolation but is the interactive result of the model with the modeler or model user.

It was observed early in plasma focus research [1, 179] that neutron yield $Y_n \sim E_0^2$ where $E_0$ is the capacitor storage energy. Such scaling gave hopes of possible development as a fusion energy source. Devices were scaled up to higher $E_0$. It was then observed that the scaling deteriorated, with $Y_n$ not increasing as much as suggested by the $E_0^2$ scaling. In fact, some experiments were interpreted as

<table>
<thead>
<tr>
<th>$P_0$ (Torr)</th>
<th>3</th>
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<td>$I_{\text{peak}}$ (kA)</td>
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<td>714</td>
<td>736</td>
<td>753</td>
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<td>$I_{\text{pinch}}$ (kA)</td>
<td>330</td>
<td>328</td>
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<td>1848</td>
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<td>993</td>
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<tr>
<td>FIB energy flux ($\times 10^{11}$ W m⁻²)</td>
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<tr>
<td>FIB damage flux ($\times 10^{10}$ W m⁻² s⁰.⁵)</td>
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<td>27.8</td>
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<tr>
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<tr>
<td>Ion current (kA)</td>
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</table>

### 3.4.7 Insight 5—Neutron Saturation

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</tr>
<tr>
<td>FPS energy % $E_0$</td>
<td>16.2</td>
<td>16.8</td>
<td>17.5</td>
<td>18.3</td>
<td>19.1</td>
</tr>
<tr>
<td>PS energy/FIB energy</td>
<td>2.9</td>
<td>2.9</td>
<td>3.2</td>
<td>4.1</td>
<td>6.5</td>
</tr>
<tr>
<td>FPS speed to $v_a$ ratio</td>
<td>1.8</td>
<td>1.6</td>
<td>1.4</td>
<td>1.1</td>
<td>0.8</td>
</tr>
</tbody>
</table>
evidence of a neutron saturation effect [1] as $E_0$ approached several hundreds of kJ. As recently as 2006 Krauz [180] and 2007 Scholz [181] have questioned whether the neutron saturation was due to a fundamental cause or to avoidable machine effects such as the incorrect formation of plasma current sheath arising from impurities or sheath instabilities. We should note here that the region of discussion (several hundreds of kJ approaching the MJ region) is in contrast to the much higher energy region discussed by Schmidt at which there might be expected to be a decrease in the role of beam-target fusion processes [1].
3.4.7.1 The Global Neutron Scaling Law

Recent extensive numerical experiments \[10, 21, 22, 44, 100\] also showed that whereas at energies up to tens of kJ the \(Y_n \sim E_0^2\) scaling held, deterioration of this scaling became apparent above the low hundreds of kJ. This deteriorating trend worsened and tended towards \(Y_n \sim E_0^{0.8}\) at tens of MJ. The results of these numerical experiments are summarized in Fig. 3.38, with the solid line representing results from numerical experiments. Experimental results from 0.4 kJ to MJ, compiled from several available published sources are also included. The combined experimental and numerical experimental results \[10, 21, 44\] (see Sect. 3.4.4) appear to have general agreement particularly with regards to the \(Y_n \sim E_0^2\) at energies up to 100 kJ, and the deterioration of the scaling from low hundreds of kJ to the 1 MJ level. The global data of Fig. 3.38 suggests that the apparently observed neutron saturation effect is overall not at significant variance with the deterioration of the scaling shown by the numerical experiments.
3.4.7.2 The Dynamic Resistance

A simple yet compelling analysis of the cause of this neutron saturation has been published [21]. In Fig. 3.1 (see Sect. 3.1.1) on the left side is shown a schematic of the plasma dynamics in the axial phase of the Mather-type plasma focus with the current sheet shown to go from the anode to the cathode perpendicularly. Experimental observations show that there is actually a canting of the current sheet and also that only a fraction (typically 0.7) of the total current participates in driving the current sheet. These points are accounted for in the modelling by model parameters $f_m$ and $f_c$. We have represented the plasma focus circuit in Fig. 3.5.

We consider only the axial phase. By surveying published results of all Mather-type experiments we find that all deuterium plasma focus devices operate at practically the same speeds [137] and are characterized by a constancy of energy density (per unit mass) over the whole range from the smallest sub-kJ to the largest MJ devices. The time-varying tube inductance is $L = (\mu / 2\pi \ln(c))z$ where $c = b/a$ and $\mu$ is the permeability of free space. The rate of change of inductance is $dL/dt = 2 \times 10^{-7} \ln(c)(dz/dt)$ in SI units. Typically on switching, as the capacitor discharges, the current rises towards its peak value, the current sheet is accelerated, quickly reaching nearly its peak speed and continues accelerating slightly towards its peak speed at the end of the axial phase. Thus for most of its axial distance, the current sheet is travelling at a speed close to the end-axial speed. In deuterium, the end-axial speed is observed to be about 10 cm/µs over the whole range of devices [6]. This fixes the rate of change of inductance $dL/dt$ as $1.4 \times 10^{-2}$ H/s for all the devices, if we take the radius ratio $c = b/a = 2$. This value of $dL/dt$ changes by at most a factor of 2, taking into account the variation of $c$ from low values of 1.4 (generally for larger machines) to 4 (generally for smaller machines). The typical value of $dL/dt$ is about 14 mΩ.

We need now to inquire into the nature of the change in the inductance $L(t)$. 

---

**Fig. 3.38** $Y_n$ scaling deterioration observed in numerical experiments from 0.4 kJ to 25 MJ (solid line) using the Lee model code, compared to measurements compiled from publications (squares) of various machines from 0.4 kJ to 1 MJ. Reprinted from Lee [21]
Consider instantaneous power $P$ delivered to $L(t)$ by a change in $L(t)$:

Induced voltage:

$$V = \frac{d}{dt}(LI) = I(dL/dt) + L(dI/dt) \quad (3.66)$$

Hence instantaneous power into $L(t)$:

$$P = VI = I^2(dL/dt) + LI(dI/dt) \quad (3.67)$$

Next, consider instantaneous power associated with the inductive energy ($\frac{1}{2}LI^2$):

$$P_L = \frac{d}{dt}(\frac{1}{2}LI^2) = I^2(dL/dt) + LI(dI/dt) \quad (3.68)$$

We note that $P_L$ of Eq. (3.68) is not the same as $P$ of Eq. (3.67). The difference $P - P_L = (\frac{1}{2})(dL/dt)I^2$ is not associated with the inductive energy stored in $L$. We conclude that whenever $L(t)$ changes with time, the instantaneous power delivered to $L(t)$ has a component that is not inductive. Hence this component of power ($\frac{1}{2})(dL/dt)I^2$ must be resistive in nature; and the quantity ($\frac{1}{2})(dL/dt)$ also denoted as half $L_{dot}$ is identified as a resistance, due to the motion associated with $dL/dt$; which we call the dynamic resistance $DR$ [10, 21, 37, 44]. Note that this is a general result and is independent of the actual processes involved. In the case of the plasma focus axial phase, the motion of the current sheet imparts power to the shock wave structure with consequential shock heating, Joule heating, ionization, radiation, etc. The total power imparted at any instant is just the amount ($\frac{1}{2})(dL/dt)I^2$, with this amount powering all consequential processes. We denote the dynamic resistance of the axial phase as $DR_0$.

We have thus identified for the axial phase of the plasma focus a typical dynamic resistance of 7 mΩ due to the motion of the current sheet at 10 cm/μs. It should be noted here that similar ideas of the role of $dL/dt$ as a resistance were discussed by Bernard et al. [1]. In that work, the effect of $dL/dt$ was discussed only for the radial phase. In our opinion, the more important phase for the purpose of neutron saturation is actually the axial phase for the Mather-type plasma focus.

### 3.4.7.3 The Interaction of a Constant Dynamic Resistance with a Reducing Generator Impedance Causes Deterioration in Current Scaling

We now resolve the problem into its most basic form as follows. We have a generator (the capacitor charged to 30 kV), with an impedance of $Z_0 = (L_0/C_0)^{0.5}$ driving a load with a near constant resistance of 7 mΩ. We also assign a value for stray resistance of 0.1Z$_0$. This situation is shown in Table 3.11 where $L_0$ is given a typical value of 30 nH. We also include in the last column the results from a circuit (L–C–R) computation, discharging the capacitor with initial voltage of 30 kV into a fixed resistance load of 7 mΩ simulating the effect of the $DR_0$ and a stray resistance of value 0.1Z$_0$ (Table 3.13).
Plotting the peak current as a function of $E_0$ we obtain Fig. 3.39, which shows the tendency of the peak current towards saturation as $E_0$ reaches large values; the deterioration of the curve becoming apparent at the several hundred kJ level. This is the case for $I_{\text{peak}} = \frac{V_0}{Z_{\text{total}}}$ and also for the L–C–R discharge with simulated value of the DR$0$. In both cases it is seen clearly that a capacitor bank of voltage $V_0$ discharging into a constant resistance such as DR$0$ will have a peak current $I_{\text{peak}}$ approaching an asymptotic value of $I_{\text{peak}} = \frac{V_0}{DR_0}$ when the bank capacitance $C_0$ is increased to such large values that the value of $Z_0 = (L_0/C_0)^{0.5} \ll DR_0$. Thus DR$0$ causes current ‘saturation’.

3.4.7.4 Deterioration in Current Scaling Causes Deterioration in Neutron Scaling

In Sect. 3.4.4 we had shown the following relationships between $Y_n$ and $I_{\text{peak}}$ and $I_{\text{pinch}}$ as follows:

### Table 3.13 Discharge characteristics of equivalent PF circuit, illustrating the ‘saturation’ of $I_{\text{peak}}$ with an increase of $E_0$ to very large values

<table>
<thead>
<tr>
<th>$E_0$ (kJ)</th>
<th>$C_0$ (μF)</th>
<th>$Z_0$ (m Ω)</th>
<th>DR$0$ (m Ω)</th>
<th>$Z_{\text{total}}$ (m Ω)</th>
<th>$I_{\text{peak}} = \frac{V_0}{Z_{\text{total}}}$ (kA)</th>
<th>$I_{\text{peak}}$, L–C–R (kA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.45</td>
<td>1</td>
<td>173</td>
<td>7</td>
<td>197</td>
<td>152</td>
<td>156</td>
</tr>
<tr>
<td>4.5</td>
<td>10</td>
<td>55</td>
<td>7</td>
<td>67</td>
<td>447</td>
<td>464</td>
</tr>
<tr>
<td>45</td>
<td>100</td>
<td>17</td>
<td>7</td>
<td>26</td>
<td>1156</td>
<td>1234</td>
</tr>
<tr>
<td>135</td>
<td>300</td>
<td>10</td>
<td>7</td>
<td>18</td>
<td>1676</td>
<td>1819</td>
</tr>
<tr>
<td>450</td>
<td>1000</td>
<td>5.5</td>
<td>7</td>
<td>12.9</td>
<td>2321</td>
<td>2554</td>
</tr>
<tr>
<td>1080</td>
<td>2400</td>
<td>3.5</td>
<td>7</td>
<td>10.8</td>
<td>2781</td>
<td>3070</td>
</tr>
<tr>
<td>4500</td>
<td>10000</td>
<td>1.7</td>
<td>7</td>
<td>8.8</td>
<td>3407</td>
<td>3722</td>
</tr>
<tr>
<td>450 000</td>
<td>100 000</td>
<td>0.55</td>
<td>7</td>
<td>7.6</td>
<td>4209</td>
<td>4250</td>
</tr>
</tbody>
</table>

The last column presents results using circuit (L–C–R) computation, with a fixed resistance load of 7 mΩ, simulating the effect of the DR$0$ and a stray resistance of value 0.1Z$0$.

**Fig. 3.39** $I_{\text{peak}}$ versus $E_0$ on log–log scale, illustrating $I_{\text{peak}}$ ‘saturation’ at large $E_0$.

Reprinted from Lee [21]
\[ Y_n \sim I_{\text{pinch}}^{4.5} \]
\[ Y_n \sim I_{\text{peak}}^{3.8} \]

Hence saturation of \( I_{\text{peak}} \) will lead to saturation of \( Y_n \).

At this point, we note that if we consider that only 0.7 of the total current takes part in driving the current sheet, as typically agreed upon from experimental observations, then there is a correction factor which reduces the axial dynamic resistance by some 40%. That would raise the asymptotic value of the current by some 40%; nevertheless, there would still be ‘saturation’.

Thus we have shown that current ‘saturation’ is inevitable as \( E_0 \) is increased to very large values by an increase in \( C_0 \), simply due to the dominance of the axial phase dynamic resistance. This makes the total circuit impedance tend towards an asymptotic value which approaches the dynamic resistance at infinite values of \( E_0 \). The ‘saturation’ of current inevitably leads to a ‘saturation’ of neutron yield. Thus the apparently observed neutron ‘saturation’ which is more accurately represented as a neutron scaling deterioration is inevitable because of the dynamic resistance. In line with current plasma focus terminology, we will continue to refer to this scaling deterioration as ‘saturation’. The above analysis applies to the Mather-type plasma focus. The Filippov-type plasma focus does not have a clearly defined axial phase. Instead, it has a lift-off phase and an extended pre-pinch radial phase which determine the value of \( I_{\text{peak}} \). During these phases, the inductance of the Filippov discharge is changing, and the changing \( L(t) \) will develop a dynamic resistance which will also have the same current ‘saturation’ effect as the Filippov bank capacitance becomes big enough.

The same scaling deterioration is also observed in the yield of Neon SXR (see Fig. 3.27) and we expect the same for other radiation yields as well. The speed restriction for a plasma focus operating in neon is not the same as that in deuterium. Nevertheless, there is a speed window related to the optimum temperature window. This again requires fixing the dynamic resistance of the axial phase for the neon plasma focus within certain limits typically the dynamic resistance equivalent to an axial speed range of 5–8 cm/µs. This dynamic resistance and its interaction with the capacitor bank impedance, as storage energy is increased, is again the cause of the scaling deterioration.

### 3.4.7.5 Beyond Presently Observed Neutron Saturation Regimes

Moreover, the ‘saturation’ as observed in presently available data is due also to the fact that all tabulated machines operate in a narrow range of voltages of 15–50 kV. Only the SPEED machines, most notably SPEED II [182] operated at low hundreds of kV. No extensive data have been published from the SPEED machines. Moreover, SPEED II, using Marx technology, has a large bank surge impedance of 50 mΩ which itself would limit the current. If we operate a range of such high voltage machines at a fixed high voltage, say 300 kV, with ever larger \( E_0 \) until the
surge impedance becomes negligible due to the very large value of $C_0$, then the ‘saturation’ effect would still be there, but the level of ‘saturation’ would be proportional to the voltage. Moreover operation at higher pressures beyond 60 Torr [44] would further increase the neutron yield. In this way we can go far above presently observed levels of neutron ‘saturation’; moving the research, as it were into presently beyond-saturation regimes.

Could the technology be extended to 1 MV? That would raise $I_{\text{peak}}$ to beyond 15 MA and $I_{\text{pinch}}$ to over 6 MA. Also multiple Blumleins at 1 MV, in parallel, could provide driver impedance of 100 mΩ, matching the radial phase dynamic resistance and provide fast-rise currents peaking at 10 MA with $I_{\text{pinch}}$ value of perhaps 5 MA. Bank energy would be several MJ. The push to higher currents may be combined with proven neutron yield enhancing methods such as doping deuterium with low % of krypton [183]. Further increase in pinch current might be by fast current injection near the start of the radial phase. This could be achieved with charged-particle beams or by circuit manipulation such as current-stepping [105]. The Lee model is ideally suited for testing circuit manipulation schemes.

### 3.4.7.6 Neutron Scaling—Its Relationship with the Plasma Focus Properties

In Sect. 3.4.7.1 we had discussed the global scaling law for neutron yield as shown in Fig. 3.1 which was compiled with data from experiments and numerical experiments. Figure 3.38 shows that whereas at energies up to tens of kJ the $Y_n \sim E_0^2$ scaling held, deterioration of this scaling became apparent above the low hundreds of kJ. This deteriorating trend worsened and tended towards $Y_n \sim E_0^{0.8}$ at tens of MJ. The global data of Fig. 3.38 suggests that the apparently observed neutron saturation effect is overall not in significant variance with the deterioration of the scaling shown by the numerical experiments.

### 3.4.7.7 Relationship with Plasma Focus Scaling Properties

Now we link up this neutron scaling law deterioration and subsequent saturation with the scaling properties of the plasma focus discussed in Sect. 3.3. This scaling law deterioration and saturation are due to the constancy of the speed factor SF and energy density, as $E_0$ increases. The constancy of the axial speed or SF causes the deterioration of current scaling, requiring that the anode radius ‘$a$’ is not increased as much as it would have been increased if there were no deterioration. This implies that the size and duration of the focus pinch are also restricted by the scaling deterioration. Ultimately at high tens of MJ, $I_{\text{peak}}$ saturates, the anode radius of the focus should not be increased anymore with $E_0$. The size and duration of the focus pinch no longer increase with $E_0$ and $Y_n$ also saturates. We now have the complete picture.
We may consider the other effects such as the current limiting effect as inductance is reduced and the scaling laws of plasma focus for SXR yield. These are all related to the behaviour of the scaling properties and the interaction of these scaling properties, particularly the dynamic resistance with the capacitor bank impedance.

### 3.4.8 Summary of Scaling Laws

Numerical experiments carried out using the universal plasma focus laboratory facility based on the Lee Model code give reliable scaling laws for neutrons production and neon SXR yields for plasma focus machines. The scaling laws obtained are summarized in Table 3.14.

These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as an optimized neutron or neon SXR sources. More recently, the scaling of $Y_n$ versus $E_0$ as shown above has been placed in the context of a global scaling law with the inclusion of available experimental data. From that analysis, the cause of scaling deterioration for neutron yield versus energy as shown in Fig. 3.38 (which has also been given the misnomer ‘neutron saturation’) has been uncovered as due to a current scaling deterioration caused by an almost constant axial phase ‘dynamic resistance’ interacting with a reducing bank impedance as energy storage is increased by increasing capacitance of energy bank at essentially constant voltage. The deterioration of soft X-ray yield with storage energy as shown in Fig. 3.27 could also be ascribed to the same axial phase ‘dynamic resistance’ effect. This deterioration of scaling will also appear in the scaling trends (with stored energy) of beam ions.

<table>
<thead>
<tr>
<th>Table 3.14 Summary of radiation scaling laws for the plasma focus</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For neutron yield: (yield in number of neutrons per shot)</strong></td>
</tr>
<tr>
<td>$Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{0.5}$, $Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8}$, $I_{\text{peak}}$ (0.3–5.7), $I_{\text{pinch}}$ (0.2–2.4) in MA</td>
</tr>
<tr>
<td>$Y_n \sim E_0^{0.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at MJ level (up to 25 MJ)</td>
</tr>
<tr>
<td><strong>For neon soft X-rays: (yield in J per shot)</strong></td>
</tr>
<tr>
<td>$Y_{\text{sxr}} = 8.3 \times 10^2 I_{\text{pinch}}^{3.6}$, $Y_{\text{sxr}} = 6 \times 10^2 I_{\text{peak}}^{3.2}$, $I_{\text{peak}}$ (0.1–2.4), $I_{\text{pinch}}$ (0.07–1.3) in MA</td>
</tr>
<tr>
<td>$Y_{\text{sxr}} \sim E_0^{1.6}$ (kJ range) to $Y_{\text{sxr}} \sim E_0^{0.8}$ (towards MJ)</td>
</tr>
<tr>
<td><strong>For beam ions at exit of a deuterium plasma pinch: (yield in J per shot)</strong></td>
</tr>
<tr>
<td>$Y_{\text{beamions}} = 4.8 \times 10^{-7} I_{\text{pinch}}^{3.6}$, $Y_{\text{beamions}} = 9.7 \times 10^{-7} I_{\text{peak}}^{3.2}$ where $Y_{\text{beamions}}$ is in J; currents in kA</td>
</tr>
<tr>
<td>$Y_{\text{beamions}} = 18.2 E_0^{1.2}$ where $Y_{\text{beamions}}$ is in J and $E_0$ is in kJ; averaged over 1 kJ–1 MJ</td>
</tr>
</tbody>
</table>
We emphasise here that the scaling laws with $I_{\text{pinch}}$ are the more fundamental and robust one compared to $I_{\text{peak}}$. This is because although the PF is reasonably consistent in its operations, there will be occasions when even the best-optimized machines may not focus or poorly focused although having a high $I_{\text{peak}}$ with no neutrons. However, $I_{\text{pinch}}$ being the current actually flowing in the pinch is more consistent in all situations.

The numerical experiments give robust scaling laws for PFs covering a wide range of energies from sub-kJ to tens of MJ. It supplements the limited (non-existent in the case of beam ions) scaling laws available to predict PF radiations yields. Now, we have on stronger footing the useful scaling laws for neutron, SXR and ion yields from PF machines.

3.5 Radiative Cooling and Collapse in Plasma Focus

3.5.1 Introduction to Radiative Cooling

The Plasma Focus has wide-ranging applications due to its intense radiation of SXR, XR, electron and ion beams and fusion neutrons [1]. The use of gases such as Ne and Xe for generation of specific SXR or EUV lines for microlithography applications [1, 2, 74, 78] has been widely discussed in the literature as has the use of N and O to generate the lines suitable for water-window microscopy [184, 185]. Recently Ar has been considered for micro-machining due to the harder characteristic line radiation [186]. Various gases including Kr have been discussed and used for fusion neutron yield enhancement [183] due arguably to mechanisms such as thermodynamically enhanced pinch compressions.

In a Z-pinch, compressed by large electric currents to high densities and temperatures [187], an equilibrium state may be envisaged when the plasma kinetic pressure rises to balance the compressing magnetic pressure, resulting in the pinch achieving an equilibrium pinch radius. This is the pressure balance basis of the Bennett equation [188]. During the compression, work is done on the column leading to a rise in internal energy. By applying energy balance additionally to pressure balance the equilibrium radius of the pinch may in principle be computed [189], as might also the density ratio of a compression driven by radiation pressure [190]. This minimum pinch radius was computed to be 0.3 [189] for a deuterium Z-pinch compared to Imperial College observation of 1/3 [191]. For Ar, the energy balance and pressure balance method [134] computed the radius ratio as 0.18, compared to observations of 0.17 at temperatures of $2 \times 10^7$ K for the Imperial College low-pressure high-speed Ar Z-pinch. The radius ratio is somewhat temperature-dependent due to the compressibility of the gas dependent on the specific heat ratio $\gamma$ of the plasma. The above is for the situation in which the pinch is assumed to be purely electromagnetic with energy input into the pinch arising only through electromagnetic motional effect. When Joule heating and radiation
emission are considered, these will modify pinch dynamics and pinch configuration. Joule heating will increase internal energy allowing a bigger equilibrium pinch radius whilst radiation emission will oppose this trend. The power loss due to emitted radiation may exceed the gain due to Joule heating. In such a situation the magnetic pressure associated with the electric current continues to exert a radially inward squeezing (pinching) force, but the kinetic (resisting) pressure drops due to the excess radiation power loss (emitted radiation power minus the Joule power gain). This radiation cooling effect, if sufficient, will lead to a sharp enhancement of compression to the very small radius, which could be far smaller than envisaged in the case of the electromagnetic pinch.

In the case of hydrogen pinch, the plasma is typically far above fully ionized temperature and the dominant radiation is free-free transitions (bremsstrahlung). The bremsstrahlung power $P_{\text{brem}}$ is proportional to $T^{1/2}$ whilst plasma resistive heating $P_{\text{joule}}$ is proportional to $T^{-3/2}$; implying an increase in $P_{\text{brem}}$ and decrease in $P_{\text{joule}}$ with increasing plasma temperature. Thus as pinch current is increased and pinch temperature rises, there comes a point when $P_{\text{brem}}$ exceeds $P_{\text{joule}}$. Pease [143] and Braginskii [144] separately showed that in hydrogen this point may be defined by a critical pinch current referred to as $I_{\text{P-B}}$ of 1.4 MA. In such a pinch at equilibrium when pinch current is raised above 1.4 MA, radiation collapse may occur.

As the temperature drops due to excessive emitted radiation the kinetic pressure is reduced and hence the pinch compressed density increases, the plasma self-absorption [146, 147] sets in limiting the emission of radiation. Radiation collapse will stop. This mechanism will place a lower limit on the radius of the pinch.

The possibility of intense radiation leading to extreme compressions in a Z-pinch and the implications of such a mechanism for the development of radiation sources has recently been reviewed [187]. Shearer [142] considered an equilibrium model of the Z-pinch based on Bennett relation, radiation losses and Ohmic heating to explain the highly localized X-ray sources observed in plasma focus experiments. Vikhrev [192] considered the dynamics of a Z-pinch contraction in deuterium with appreciable radiative loss taking into account decreased current due to pinch inductance and resistance; the viscous heat and anomalous resistive heat release; transition of plasma bremsstrahlung into blackbody surface radiation; the pressure of the degenerate electron gas and the thermonuclear heat release. A neutron yield of $10^{14}$ is found from a highly compressed plasma of a 2 MJ system. With a 1% mixture of xenon with a fully ionized plasma at 10 MA, the enhanced compression to a density of $10^{27}$ cm$^{-3}$ leads to a neutron yield of $1.5 \times 10^{16}$. Using a mixture of deuterium and tritium the neutron yield reached $10^{18}$ per discharge with an input energy of 2 MJ, reaching breakeven according to their calculations. Koshelev et al. [193] considered the formation of radiation enhanced micropinches as a source of highly ionized atoms. It is known that in gases undergoing intense line radiation the radiation-cooled threshold current is considerably lowered [194].

We show that the equations of the Lee Model code [10, 37] may be used to compute this lowering. The model is correctly coupled between the plasma
dynamics and the electrical circuit which is an advantageous feature when compared to computations which use a fixed current or a current which is not correctly associated with the electric circuit interacting with the plasma dynamics. The model treats the pinch as a column. Our computations show that the radial collapse of the column is significantly enhanced by the net energy loss due to radiation and joule heating with consideration of plasma opacity. This radiatively enhanced compression of the plasma column would, in reality, mean that as the column breaks up into localized regions (hot spots) the radiative collapse would be further enhanced. Thus the calculated radiative collapse of the column would be an underestimate of the more realistic ‘line of hot spots’ situation [195]. Nevertheless, the Lee Model code does give useful information since it incorporates the time history of the axial and radial phases. Earlier work has already suggested that the neutron enhancement effect of seeding [183] could at least in part be due to the enhanced compression caused by radiation cooling.

### 3.5.2 The Radiation-Coupled Dynamics for the Magnetic Piston

The code uses Eq. (3.38) for the piston position $r_p$ derived from the first law of thermodynamics applied to the pinch volume (For convenience of the readers we reproduce this equation here):

$$\frac{dr_p}{dt} = \frac{-rf_p}{\gamma} \frac{dI}{dr} - \frac{1}{\gamma + 1} \frac{r_p}{Z_f} \frac{dz_f}{dr} + \frac{4\pi(\gamma - 1)}{\mu_0 c} f_c r_p \frac{dQ}{dt} \frac{1}{\gamma - 1} \frac{L_c}{\gamma}$$  \(3.38\)

where $I$ is the total discharge current in the circuit, $f_c$ is the fraction of current flowing into the pinch, $z_f$ is the time-varying length of the PF pinch and $\gamma$ is the specific heat ratio (SHR) of the plasma. When $dQ/dt$ (sum of Joule heating and radiation energy loss) is negative, energy is lost from the plasma adding a negative component to $dr_p/dt$ which tends to reduce the radius $r_p$.

### 3.5.3 The Reduced Pease-Braginskii Current

Following Lee et al. [50, 52] we write the reduced P-B current $I_{P-B\text{-reduced}}$ as:

$$I_{P-B\text{-reduced}}^2 = I_{P-B}^2 \times \frac{1}{K} \times Z'$$  \(3.69\)
with

\[ Z' = (1/4) \frac{(1+Z_{\text{eff}})^2}{Z_{\text{eff}}^2} \]

\[ K = \left[ \frac{(dQ_{\text{line}}/dt) + (dQ_{\text{Brem}}/dt)}{dQ_{\text{Brem}}/dt} \right] \]

(3.70)

We consider the following powers (all quantities in SI units unless otherwise stated): respectively, Joule heating, Bremsstrahlung and Line radiation generated in a plasma column of radius \( r_p \), length \( l \) at temperature \( T \) (rewriting Eqs. (3.32)–(3.45) so that the powers are functions of the pinch current written as \( I \) from this point onwards without reference to fractions \( f_{cr} \)):

\[ \frac{dQ_J}{dt} = C_J T^{-3/2} \frac{l}{\pi r_p^2} Z_{\text{eff}} I^2 \quad \text{where } C_J \approx 1300 \text{ and } T \text{ is in Kelvin} \] (3.71)

\[ \frac{dQ_{\text{Brem}}}{dt} = C_1 T^{1/2} n_i^2 Z_{\text{eff}}^2 \pi r_p^2 l \quad \text{where } n_i \text{ is in } \text{m}^{-3} \text{ and } C_1 = 1.6 \times 10^{-40} \] (3.72)

\[ \frac{dQ_{\text{line}}}{dt} = C_2 T^{-1} n_i^2 Z_{\text{eff}}^4 \pi r_p^2 l \quad \text{where } C_2 = 4.6 \times 10^{-31} \] (3.73)

So that we write the total power adding the three terms as follows:

\[ \frac{dQ}{dt} = -\pi \left[ C_1 b^{1/2} \right] \frac{Z_{\text{eff}}^2}{(1+Z_{\text{eff}})^{1/2}} n_i^{3/2} \pi r_p I + \frac{C_J}{\pi b^{3/2}} (1+Z_{\text{eff}})^{3/2} Z_{\text{eff}} n_i^{3/2} r_p \frac{l}{I} \] (3.74)

For He the factor \( Z' = 0.56 \). This factor alone reduces the Pease-Braginskii current to 1.2 MA, even if we assume that He is completely ionized with insignificant line radiation so that \( K = 1 \). When line radiation becomes dominant the calculation of \( K \) is complicated by the dependence of \( P_{\text{line}} \) on density and temperature; so that there is no one value for reduced Pease-Braginskii current, \( I_{\text{P-Breduced}} \).

### 3.5.3.1 The Reduced Pease-Braginskii Current for PF1000 at 350 kJ

We take some likely points of operation in PF1000 for the gases Ne, Ar, Kr and Xe and estimate typical values of \( I_{\text{P-Breduced}} \) for these gases; shown in Table 3.15. In the example for Ne we take a typical point of operation for intense line radiation at \( Z_{\text{eff}} \approx 9 \) so that \( Z' \approx 0.31 \). At this point of operation \( P_{\text{line}} \) is found to be 20 \( P_{\text{brem}} \); so we have \( I_{\text{P-Breduced}} \approx 190 \) kA. It is emphasized that unlike the value for H or D which is derived by balancing \( P_{\text{Joule}} \) and \( P_{\text{brem}} \) resulting in a value dependant only on the pinch current, when higher \( Z \) gases are considered with line radiation that needs to be included in the factor \( K \), then there is no one value for the \( I_{\text{P-Breduced}} \). Table 3.15 thus gives only indicative values of \( I_{\text{P-Breduced}} \) with the trend as the
Z-number increases, a lower value of $I_{\text{P-Breduced}}$ may be expected. In particular, He may have a smaller $I_{\text{P-Breduced}}$ than indicated in Table 3.15 which for simplicity has only considered bremsstrahlung for He.

We note that in deriving Table 3.15 the radiation powers are considered at the source. The derived $I_{\text{P-Breduced}}$ is indicative of the situation when the plasma is assumed to be completely transparent to the radiation.

### 3.5.3.2 The Reduced Pease-Braginskii Current for INTI PF at 2 kJ

Similarly, we compute indicative values of $I_{\text{P-Breduced}}$ for INTI PF at 12 kV [196]. We select some possible points of operation for the gases Ne, Ar, Kr and Xe and estimate typical values of $I_{\text{P-Breduced}}$ for these gases in Table 3.16. In the example for Ne we take a typical point of operation for intense line radiation at $Z_{\text{eff}} \sim 8.5$ so that $Z' \sim 0.31$. At this point $P_{\text{line}}$ is found to be 136 $P_{\text{brem}}$, so we have $I_{\text{P-Breduced}} \sim 76$ kA. Note that this much smaller value of $I_{\text{P-Breduced}}$ (compared to the corresponding neon value for PF1000 in Table 3.15) is obtained by selecting a lower operating temperature conducive to a higher ratio of $P_{\text{line}}$ to $P_{\text{brem}}$. Table 3.16 gives attainable values of $I_{\text{P-Breduced}}$ in INTI PF.

### 3.5.4 Effect of Plasma Self-absorption

We also note that the above consideration has not taken into account the effect of plasma self-absorption. Taking that into consideration the emission power will be

<table>
<thead>
<tr>
<th>Table 3.15 Reduced Pease-Braginskii current for various gases; PF1000 operating conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gases</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>D2</td>
</tr>
<tr>
<td>He</td>
</tr>
<tr>
<td>Ne</td>
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<td>Ar</td>
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<tr>
<td>Kr</td>
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<td>Xe</td>
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<table>
<thead>
<tr>
<th>Table 3.16 Reduced Pease-Braginskii current for various gases; at typical INTI PF operating conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gases</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>He</td>
</tr>
<tr>
<td>Ne</td>
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<tr>
<td>Ar</td>
</tr>
<tr>
<td>Kr</td>
</tr>
<tr>
<td>Xe</td>
</tr>
</tbody>
</table>
reduced, effectively reducing the value of $K$ thus raising the threshold current from that value computed in Eq. (3.69).

Plasma self-absorption included in the code is already discussed in Section “Plasma Self-Absorption and Transition from Volumetric Emission to Surface Emission” which shows the method for computing the plasma self-absorption correction factor $A$.

When there is no plasma self-absorption $A = 1$. When $A$ goes below 1, plasma self-absorption starts. When a sizeable fraction of the photons is re-absorbed, e.g. value of $A$ reaches $1/e$, plasma radiation is considered to switch over from volume radiation to surface radiation and is computed accordingly in the model.

Summarizing: The code computes the amount of radiation emitted, computes plasma self-absorption effects and incorporates these effects into the plasma dynamics.

3.5.5 Characteristic Times of Radiation

In a recent paper, Lee et al. [52] argue that $I_{P-B}$ or $I_{P-Breduced}$ is only one condition for the occurrence of radiative collapse. Another condition would be the magnitude of the excess radiative power $dQ/dt$ (which we call $Qdot$, where $Q = $ total energy radiated out of the pinch plasma less Joule heat released in the pinch plasma) acting to reduce the energy in the pinch $E_{pinch}$. We define a characteristic radiative time as $t_{rad} \sim E_{pinch}/Qdot$ which is the time required for all the pinch energy to be radiated away at the rate $Qdot$.

We preface our argument by reviewing the work of Robson on the Z-pinch. Robson [146] considered this situation for the case of the hydrogen and helium Z-pinch including the effects of opacity. Robson considered a circuit which provided a constant voltage until the pinch collapsed to its minimum radius limited by opacity; at which point the voltage is set to zero. Robson assigned line densities of $10^{17}$, $10^{18}$ and $10^{19}$ ions per cm at applied voltages of 65–380 kV per cm of pinch length with initial established fully ionized pinch of radius 1 mm. For a typical case in hydrogen of $10^{18}$ ions per cm, $L_0 = 25$ nH with applied 190 kV per cm driving initial $dI/dt$ of 5 kA per ns, the current reaches 1.81 MA in 440 ns. The radius which has reduced over the current rise time relatively ‘gradually’ to $10^{-3}$ cm at this time, abruptly plunges to $2 \times 10^{-5}$ cm in a time of 0.06 ns whilst the current drops precipitously from 1.8 to 0.8 MA.

According to Table 3.15, our calculations show that for He the reduced P-B current ($I_{P-Breduced}$) is 1.2 MA considering only the charge factor; though there may be a further reduction due to line radiation. However running the code for PF1000 at 40 kV (in principle the maximum operating voltage of PF1000) in He the pinch current exceeds 1.2 MA but there is no sign of radiative collapse. Even hypothetically increasing the PF1000 operating voltage to 100 kV when the pinch current exceeds 2 MA, there is still no sign of a sharp drop in pinch radius ratio which is the most indicative sign of radiative collapse. To explain this we develop
an expression for the characteristic time required to radiate away all the pinch energy through bremsstrahlung and also for the characteristic time for line radiation. The numerical experiments show that the pinch duration has to be of the order (typically at least 0.1) of the characteristic time of radiation ($t_{\text{rad}}$) in order for that radiation to cause significant radiative cooling resulting in radial collapse.

### 3.5.5.1 Definition-Pinch Energy/Radiation Power

We write down the thermal energy in the pinch as the total number of particles in the pinch multiplied by the thermal energy per particle:

$$E_{\text{pinch}} = \frac{kT}{(\gamma - 1)} n_i (1 + Z_{\text{eff}}) \pi r_p^2 c, \quad (3.75)$$

where $\gamma$ is the specific heat ratio which may be written in terms of the degree of freedom $f$ as $\gamma = (2+f)/f$, so that $1/(\gamma - 1) = f/2$.

In Eq. (3.75) the energy of the pinch is written in a form suitable for high-Z gases in which the energy expended in ionization is not insignificant when compared to the translational modes even at the high temperatures concerned. Note that for a fully ionized gas at such a high temperature that the expanded ionization energies are already insignificant compared to the translation energy then $f = 3$ and $[kT/(\gamma - 1)] = 3(kT/2)$ per particle, $k = 1.38 \times 10^{-23}$ J/K being the Boltzmann constant. As examples: for gases such as Ne in the PF pinch, the temperature may typically be high enough for it to be approaching full ionization; the specific heat ratio computes [134] to be 1.5 so that $f = 4$ and $[kT/(\gamma - 1)] = 4(kT/2)$ per particle. In Kr, operating at a temperature of $10^6$ K, $Z_{\text{eff}} \sim 14$, $\gamma \sim 1.3$, $f \sim 6.7$ and $[kT/(\gamma - 1)] = 6.7(kT/2)$ per particle.

We divide the pinch energy by the radiation power to give us a measure of the characteristic time it would take the pinch to have its energy radiated away by that radiation power taken as constant over the whole duration. We call this the characteristic depletion time of radiation.

### 3.5.5.2 Characteristic Depletion Time for Bremsstrahlung

From Eqs. (3.75) and (3.72) we derive $t_{\text{brem}}$:

$$t_{\text{brem}} = \frac{E_{\text{pinch}}}{P_{\text{brem}}} = \left[ \frac{kT^{1/2}/(C_1 n_0 f_n)}{(1 + Z_{\text{eff}})/[Z_{\text{eff}}^3 (\gamma - 1)]} \right],$$

$$t_{\text{brem}} = \left( \frac{kb^{1/2}/C_1}{f} \right) \left[ I/\left( n_0^3 f_n^{3/2} r_p \right) \right] \left( 1 + Z_{\text{eff}} \right)^{1/2}/[Z_{\text{eff}}^3 (\gamma - 1)]. \quad (3.76)$$

Here we have eliminated $T$ by using Bennett equation for a pinch in which magnetic pressure balances the kinetic pressure:
\[ T = b \frac{I^2}{(n_i r_p^2)(1 + Z_{\text{eff}})} \], where \( b = \mu/(8\pi^2 k) \) and \( \mu = 4\pi \times 10^{-7} \) H/m is the permeability of free space so that \( b = 1.15 \times 10^{15} \).

The pinch number density \( n_i \) is written in terms of the initial number density \( n_0 \) by writing:

\[ n_i = n_0 f_n \text{, where } f_n = \left( \frac{a}{r_p} \right)^2 f_{\text{ms}} f_g \text{, accounting not only for the area compression } \left( \frac{a}{r_p} \right)^2 \text{ but also for the mass fraction swept-in } f_{\text{ms}} \text{ and a geometrical factor } f_g \text{ due to the elongation of the radial collapse.} \]

To get an estimate of the size of \( t_{\text{brem}} \) we put in typical numbers for operation at pinch current higher than the Pease-Braginskii current for D into Eq. (3.75) as follows:

\[ I = 2.1 \times 10^6 \text{ operated at 3 Torr D so that } n_0 = 10^{23}, a = 0.2, r_p = 3 \times 10^{-2} \text{ (i.e. } k_{\text{min}} = 0.15), f_m = 0.2, f_g = 1/3, \text{ so that } f_n \sim 3; \gamma = 5/3 \text{ and } Z_{\text{eff}} = 1. \]

For these parameters, \( t_{\text{brem}} \sim 1 \times 10^{-3} \) s. This means that the magnitude of \( P_{\text{brem}} \) at a constant value is such that it would take \( 10^{-3} \) s to radiate away all the pinch thermal energy. Even to radiate away 10% would take 100 \( \mu \)s. The lifetime of such a PF pinch (e.g. PF1000) may typically be estimated as 0.2 \( \mu \)s. Thus in the lifetime of such a plasma focus, it is unlikely that the radiation would affect the dynamics. Looking at Eq. (3.75) we could possibly increase the effect of bremsstrahlung by increasing the ambient pressure within a range suitable for operation. Careful examination of a large range of numerical experiments shows no sign of radiative cooling in D in which the radiation is dominated by bremsstrahlung, although the code includes bremsstrahlung, line and recombination radiation.

### 3.5.5.3 Characteristic Depletion Time for Line Radiation

From Eqs. (3.75) and (3.73) we derive:

\[ t_{\text{line}} = E_{\text{pinch}}/P_{\text{line}} = \left( k/C_2 \right) \left( T^2/(n_0 f_n) \right) (1 + Z_{\text{eff}}) \left( Z_{\text{eff}} Z_{\text{n}}^4 (\gamma - 1) \right) \]

and eliminating \( T \):

\[ t_{\text{line}} = \left( k b^2/C_2 \right) I^4 \left[ n_0^3 f_n^3 r_p^4 \right] (1 + Z_{\text{eff}}) Z_{\text{eff}} Z_{\text{n}}^4 (\gamma - 1) \quad (3.77) \]

The above equation shows how depletion times for \( t_{\text{line}} \) for typical plasma focus operation may be computed.

### 3.5.5.4 Characteristic Depletion Time \( t_Q \) for PF1000

In the same way, the nett depletion time \( t_Q \) may also be computed from Eqs. (3.75) and (3.74) where \( t_Q \) is the ratio \( E_{\text{pinch}}/Q_{\text{dot}} \) where \( Q_{\text{dot}} \) or \( dQ/dt = P_{\text{line}} + P_{\text{brem}} - P_J \). The latter is the time which is more applicable. In Table 3.17 we show an example of computations of depletion times in D, He, Ne, Ar, Kr and
Xe for some conditions shown to be practicable PF operation in the numerical experiments. We model the PF configuration after the PF1000. For D and He, we operate at 90 kV in order to reach pinch current in excess of 2 MA. For the other gases, we operate the numerical experiments at 23 kV which is a voltage that is currently used in actual PF1000 operation.

In Table 3.17 we calculate depletion times $t_Q$ and also $t_Q^*$ which is $t_Q$ expressed in units of a characteristic pinch time $\tau_{\text{pinch}}$. We take the pinch time as proportional to anode radius [137] with a figure of 10 ns per cm (rounding $\tau_{\text{pinch}}$ to 100 ns). From Table 3.17 it may be surmised that even though the PF is operated with currents above the reduced P-B, nevertheless there would be no radiative collapse to be expected from operation in H and He. In Ne with a significant proportion of pinch energy radiated away within one $\tau_{\text{pinch}}$, radiative cooling should be expected, leading to considerable reduction in minimum radius ratio. In Ar, Kr and Xe one would expect a strong radiative collapse. It is stressed that these numbers act only as a rough guide since the pinch system is non-static and the various properties are interacting continuously. Moreover, all the above estimates are based on radiative terms at source without consideration of plasma opacity which in those cases when the plasma is not completely transparent would reduce the energy loss from the plasma.

3.5.5.5 Characteristic Depletion Time $t_Q$ for INTI PF

For comparison, we also calculate indicative values of the depletion times for the 2 kJ INTI PF [196] in Table 3.18.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$a$ (cm)</th>
<th>$V_0$ (kV)</th>
<th>$P_0$ (Torr)</th>
<th>$I_{\text{pinch}}$ (kA)</th>
<th>$Z_{\text{eff}}$</th>
<th>SHR</th>
<th>$t_Q$ (ns)</th>
<th>$t_Q^*$ ($\tau_{\text{pinch}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>20.0</td>
<td>90</td>
<td>3.5</td>
<td>2125</td>
<td>0.80</td>
<td>1.0</td>
<td>1.67</td>
<td>3200</td>
</tr>
<tr>
<td>He</td>
<td>20.0</td>
<td>90</td>
<td>3.5</td>
<td>2094</td>
<td>0.91</td>
<td>2.0</td>
<td>1.64</td>
<td>88</td>
</tr>
<tr>
<td>Ne</td>
<td>5.0</td>
<td>14</td>
<td>1.0</td>
<td>514</td>
<td>0.99</td>
<td>8.3</td>
<td>1.49</td>
<td>0.26</td>
</tr>
<tr>
<td>Ar</td>
<td>11.6</td>
<td>23</td>
<td>0.5</td>
<td>674</td>
<td>0.65</td>
<td>11.9</td>
<td>1.36</td>
<td>0.028</td>
</tr>
<tr>
<td>Kr</td>
<td>11.6</td>
<td>23</td>
<td>0.3</td>
<td>670</td>
<td>0.89</td>
<td>14.2</td>
<td>1.33</td>
<td>0.0024</td>
</tr>
<tr>
<td>Xe</td>
<td>11.6</td>
<td>23</td>
<td>0.2</td>
<td>657</td>
<td>0.56</td>
<td>16.6</td>
<td>1.27</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 3.18 Depletion times in Ne, Ar, Kr and Xe for various conditions ($Ab$ absorption correction factor at peak emission) in INTI PF

<table>
<thead>
<tr>
<th>Gas</th>
<th>$a$ (cm)</th>
<th>$V_0$ (kV)</th>
<th>$P_0$ (Torr)</th>
<th>$I_{\text{pinch}}$ (kA)</th>
<th>$Z_{\text{eff}}$</th>
<th>SHR</th>
<th>$t_Q$ (ns)</th>
<th>$t_Q^*$ ($\tau_{\text{pinch}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne</td>
<td>0.95</td>
<td>12</td>
<td>2.5</td>
<td>79</td>
<td>0.72</td>
<td>8</td>
<td>1.35</td>
<td>70</td>
</tr>
<tr>
<td>Ar</td>
<td>0.95</td>
<td>12</td>
<td>1.1</td>
<td>84</td>
<td>0.30</td>
<td>16</td>
<td>1.33</td>
<td>30</td>
</tr>
<tr>
<td>Kr</td>
<td>0.95</td>
<td>12</td>
<td>0.47</td>
<td>87</td>
<td>0.13</td>
<td>23</td>
<td>1.40</td>
<td>0.7</td>
</tr>
<tr>
<td>Xe</td>
<td>0.95</td>
<td>12</td>
<td>0.25</td>
<td>92</td>
<td>0.16</td>
<td>30</td>
<td>1.43</td>
<td>0.15</td>
</tr>
</tbody>
</table>
From Table 3.18 it may be surmised that in INTI PF at the selected attainable point of operation in Ne, with less than 2% of pinch energy radiated away within one $t_{\text{pinch}}$, radiative cooling should be hardly apparent leading to at most a small reduction in minimum radius ratio. In Ar, Kr and Xe one expects a strong radiative collapse in the small INTI PF as likely as in the big PF1000.

To confirm these indicative results we next carry out numerical experiments with the code in which $Q$ and $Q_{\text{dot}}$ and plasma self-absorption effect are all included with a smoothened transition from opacity-corrected volume emission to surface emission when opacity effects exceed a set limit. The code models all these effects and properties in properly coupled interactive fashion.

### 3.5.6 Numerical Experiments on PF1000 and INTI PF

#### 3.5.6.1 Fitting for Model Parameters in PF1000

We have a recently measured current waveform for the PF1000 operated at 23 kV at 1.5 Torr deuterium. In order to obtain the model parameters we use the following configuration for the PF1000:

- **Bank parameters:** $L_0 = 33$ nH (fitted), $C_0 = 1332$ μF, $r_0 = 3$ mΩ (fitted),
- **Tube parameters:** $b = 16$ cm, $a = 11.55$ cm, $z_0 = 60$ cm,
- **Operating parameters:** $V_0 = 23$ kV, $P_0 = 1.5$ Torr deuterium

We achieved a reasonably good fit [52] (Fig. 3.40), confirming the above bank and tube parameters and obtaining the following model parameters: $f_m = 0.11$, $f_c = 0.7$, $f_{\text{mr}} = 0.26$, $f_{\text{cr}} = 0.68$.

We then used these model parameters and the above-mentioned configuration for a series of numerical experiments. For all the gases we operated the numerical experiments at 23 kV which is a voltage that is currently used in actual PF1000 operation.

#### 3.5.6.2 PF 1000 in Deuterium and Helium—Pinch Dynamics Showing no Sign of Radiative Cooling or Collapse

Figure 3.41a shows the total discharge current rising to a peak value of 1836 kA. The pinch current $I_{\text{pinch}}$ at the start of pinch (time of start of pinch is shown with the right-pointing arrow) is calculated as 853 kA dropping to 796 kA at the end of the pinch (left-pointing arrow). Figure 3.41b shows the trajectories in the radial phase. The piston trajectory delineates the pinch radius after the piston meets the reflected shock (RS). For this shot, the pinch lasts for 206 ns. The code computes the radial trajectory up to this point. Figure 3.41b shows a very slow compression (radius decreases barely perceptibly), typical of an efficiently operated pinch with no
radiation compression or significant cooling. A careful study of the computed properties agrees with Table 3.15 (and Table 3.17) showing that the radiation power is too small to affect the trajectory. The minimum radius is 22.2 mm, with radius ratio $r_{min}/a = 0.19$. In an extension to this exercise we have increased the

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**Fig. 3.40** Fitting the computed current trace to the measured current trace of PF1000 at 23 kV, 1.5 Torr deuterium. *Note* the two curves have a close fit except after the bottom of the current dip. Fitting is done only up to the bottom of the dip, so any agreement or divergence of the computed and measured traces after the bottom of the dip has no significance. Reprinted from Lee et al. [52]. Copyright (2012) with permission from IEEE.

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**Fig. 3.41** a) Computed total current of PF1000 at 23 kV, 3 Torr D; *right-pointing arrow* shows start of pinch and *left-pointing arrow* shows end of pinch. Reprinted from Lee et al. [52]. Copyright (2012) with permission from IEEE. b) Radial dynamics on PF1000 plasma focus at 23 kV, 3 Torr D. Reprinted from Lee et al. [52]. Copyright (2012) with permission from IEEE.
charging voltage in this experiment to a hypothetical PF1000-like configuration of 90 kV with $a = 20$ cm, $b = 28$ cm and $P_0 = 3.5$ Torr. The pinch current is 2.1 MA. This experiment and other numerical experiments carried out earlier confirmed that despite the P-B current being far exceeded, there is no sign of radiative collapse in the D focus pinch in PF1000.

The results for He (at 23 kV 3 Torr and hypothetical 90 kV) are very similar to the case of D. The minimum radius is 20.5 mm with $r_{min}/a = 0.18$ which is a little smaller than that achieved in the very similar D discharge. This is in agreement with Table 3.15 (and Table 3.17) showing that the radiation power in He is not sufficient to severely affect the pinch compression but is larger than that of D and perhaps enough to reduce the minimum radius slightly from that of the case of D.

3.5.6.3 PF 1000 in Neon 23 kV, 1 Torr—Pinch Dynamics Showing Signs of Radiative Cooling and Enhanced Compression

In Ne, the effect of radiation on the radial compression of the pinch is unmistakable in both the current waveform (Fig. 3.42a) and the radial piston trajectory (Fig. 3.42b). The total discharge current shows an additional steepening in the final part of the dip (perceptible even without magnifying the relevant region) from a pinch current value of 819 kA at 9.021 µs (right-pointing arrow) to a value of 673 kA at 9.243 µs (left-pointing arrow). Detailed study of the code outputs shows

![Fig. 3.42 a](image1.png) Computed total current of PF1000 at 23 kV, 1 Torr Ne. Reprinted from Lee et al. [52]. Copyright (2012) with permission from IEEE.

![Fig. 3.42 b](image2.png) Radial dynamics of PF1000 at 23 kV, 1 Torr Ne. Reprinted from Lee et al. [52]. Copyright (2012) with permission from IEEE.
that over this pinch period of 222 ns the pinch compressed from 15.8 to 7.3 mm, reaching $r_{\text{min}}/a = 0.06$. These features also correlate with emission power time profile. Undoubtedly in PF1000, strong radiative cooling is exhibited in the Ne pinch plasma leading to a substantial reduction of pinch radius.

### 3.5.6.4 PF1000 in Argon, Krypton and Xenon—Pinch Dynamics Showing Strong Radiative Collapse

At 23 kV, in argon, krypton and xenon, respectively, at 0.5, 0.3 and 0.2 Torr, the numerical experiments show current waveform, radial trajectories and net power $dQ/dt$ or $Q_{\text{dot}}$ emissions consistent with strong radiative collapse. For illustration, we show here the case of xenon obtained from the numerical experiments. An analysis of all the gases is summarized in Table 3.19.

The current and radial trajectories for operation in Xe are shown in Fig. 3.43a, b. Figure 3.43b shows dramatically the collapse of the radius of the column.

These traces are presented in magnified scale in Fig. 3.43c to show details of the 50 ns which includes the radiative phase up to maximum compression and a little beyond. Figure 3.43c correlates the time profiles of current $I$, piston position $r_p$ and $Q_{\text{dot}}$. Each of these is normalized (as described in the caption of Fig. 3.43c) so that the 3 traces may be presented in the one figure. At the start of the pinch, $r_p$ is 16.1 mm at 1272 ns dropping sharply to 1.43 mm at 1274 ns and then further dropping less sharply to 0.48 mm at 1275 ns and then to a minimum radius of 0.388 mm ($r_{\text{min}}/a = 0.0034$) at 1278 ns. Over this time period, the pinch current drops from 843 to 483 kA. The value of $Q_{\text{dot}}$ rises from $3.1 \times 10^{12}$ W at the start of pinch to peak value of $6.0 \times 10^{13}$ W at 1274 ns and then drops sharply as plasma self-absorption which has been rising rapidly causes the emission to transition from volumetric emission to surface emission. The value has dropped to $4 \times 10^{12}$ W at 1275 ns and further to $2 \times 10^{11}$ W at 1278 ns. The value of $Q$ at 1278 ns is 33.4 kJ (9.5%) and at end of pinch 1480 ns is 37.0 kJ (10.5%); whilst $r_p$

<table>
<thead>
<tr>
<th>Gas</th>
<th>$I_{p_s}$ (kA)</th>
<th>$I_{p_e}$ (kA)</th>
<th>$t_{\text{min}}$ (ns)</th>
<th>$t_p$ (ns)</th>
<th>$r_{p_s}$ (mm)</th>
<th>$k_{\text{min}}$ ($r_p/a$)</th>
<th>$-Q_{\text{dot peak}}$ (10$^{11}$ W)</th>
<th>$-Q$ (%)</th>
<th>$-E_{\text{rad}}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>853</td>
<td>789</td>
<td>206</td>
<td>206</td>
<td>23.8</td>
<td>0.192</td>
<td>$-0.0005$</td>
<td>$-0.00002$</td>
<td>0.000003</td>
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<tr>
<td>He</td>
<td>833</td>
<td>768</td>
<td>190</td>
<td>190</td>
<td>21.9</td>
<td>0.178</td>
<td>0.0008</td>
<td>0.00004</td>
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<tr>
<td>Ne</td>
<td>819</td>
<td>650</td>
<td>222</td>
<td>222</td>
<td>15.4</td>
<td>0.063</td>
<td>0.72</td>
<td>2.63</td>
<td>2.76</td>
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<tr>
<td>Ar</td>
<td>820</td>
<td>530</td>
<td>130</td>
<td>208</td>
<td>14.8</td>
<td>0.016</td>
<td>7.6</td>
<td>6.8</td>
<td>9.2</td>
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<td>Kr</td>
<td>848</td>
<td>307</td>
<td>20</td>
<td>206</td>
<td>16.4</td>
<td>0.007</td>
<td>116</td>
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<tr>
<td>Xe</td>
<td>847</td>
<td>168</td>
<td>6</td>
<td>209</td>
<td>16.1</td>
<td>0.003</td>
<td>600</td>
<td>10.5</td>
<td>22.9</td>
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Key $I_{p_s}$ current at start of pinch, $I_{p_e}$ current at end of pinch, $t_{\text{min}}$ time to min radius, $t_p$ time to end of pinch, $r_{p_s}$ pinch radius at start of pinch, $k_{\text{min}}$ min radius ratio, $-Q_{\text{dot peak}}$ peak value of $-dQ/dt$, $Q$ energy radiated from pinch less Joule heat energy deposited in pinch; $E_{\text{rad}}$ energy radiated for whole pinch duration
has expanded to 2.6 mm (not shown in Fig. 3.37c, see Fig. 3.37b). The resistive heat liberated in the pinch over the pinch period is 43.6 kJ whilst 80.6 kJ (22.9% of $E_0$) of radiation is emitted from the pinch over the period of the pinch, one-third of this amount within the first 3 ns.
3.5.6.5 PF 1000 in Various Gases—Summary of Radiative Pinch Dynamics

The data [52] is summarized in Table 3.19.

In this table, the values of peak $-Q_{dot}$, $-Q$ and $-E_{rad}$ are shown in the last three columns. Positive values in the $-Q_{dot}$ and $-Q$ columns indicate that the radiation exceeds the Joule heating in the pinch so that the net power works in the direction of radiation cooling and collapse. The values of these two quantities for D are negative indicating that joule heating exceeds radiation. For all the other gases these terms act to radiatively cool the pinch although in the case of He the power is so small and the heat loss is such a small percentage of bank energy (also of pinch energy) that the effect is almost negligible; although the combined effect of SHR and $dQ/dt$ in the He pinch does show a perceptible increased compression of the pinch with pinch radius ratio $k_{min}$ of 0.178 compared to that of D of 0.192. In Ne the radiative cooling is unmistakable with $k_{min}$ of 0.063. Argon with $k_{min}$ of 0.016 shows a time $t_{min}$ of 130 ns to minimum radius and then a small expansion over the rest of the pinch period. In Kr, $t_{min}$ is only 20 ns to a $k_{min}$ of 0.007. In Xe, $t_{min}$ reduces further to 6 ns with $k_{min} = 0.003$, pinch radius of 0.35 mm; and in the rest of the pinch duration over some 203 ns the pinch expands back to almost 2 mm.

3.5.6.6 Comparison of $r_{min}$ from Experiments and Simulation in PF1000

Estimates of minimum radius $r_{min}$ were obtained of the PF1000 pinch from multiframe interferometric measurements of the plasma column employing the second harmonic (527 nm) of a Nd:YLF laser of less than 1 ns duration. The laser pulse was split by mirrors into fifteen separated beams which passed through a Mach-Zehnder interferometer [54, 197, 198]. The experimental results consisting of $r_{min}$ in neon operated in a narrow range of pressures are compared with our numerical experiments of $r_{min}$, with and without (hypothetically) radiative losses (see Fig. 3.44).

3.5.6.7 Six Regimes of the PF Pinch Characterized by Relative Dominance of Joule Heating Power, Radiative Power and Dynamic Power Terms

As the PF is operated at different pressure, the significance of Joule heating power, radiative power and dynamic power terms relative to each other varies. To characterize the pressure ranges at which each combination of power terms dominates the following four scenarios for the total radiative power $Q_{dot}$ may be computed:
With the total radiative power $Q_{dot} = P_J + PRAD$

Without Joule heating and radiative losses $Q_{dot} = 0$

With the Joule heating effect only $Q_{dot} = P_J$

With radiative losses only $Q_{dot} = PRAD$.

Plotting the curves corresponding to these four scenarios on the same chart, there are found to be six possible combinations distinguishable by the order of the magnitude of values calculated with the four scenarios.

1. Both $P_J$ and $PRAD$ significant (mod $P_J >$ mod $PRAD$): order of values: S3 highest, then S1, then S2, then S4 lowest.
2. Both $P_J$ and $PRAD$ significant (mod $P_J <$ mod $PRAD$): order of values: S3 highest, then S2, then S1, then S4 lowest.
3. Both $P_J$ and $PRAD$ significant (mod $P_J =$ mod $PRAD$): order of values: S3 highest, then S2 = S1 (or very close together), then S4 lowest.
4. $P_J$ significant and $PRAD$ insignificant: order of values: S3 = S1 (or very close together) these being higher than S2 = S4 (or S2 slightly greater than S4)
5. $P_J$ insignificant and $PRAD$ significant: order of values: S3 = S2 (or S3 slightly greater than S2) these being higher, then S1 = S4 (or these values being very close to each other).
6. Both $P_J$ and $PRAD$ insignificant: order of values: S1 = S2 = S3 = S4 (all 4 values being the same).

Thus by looking at the relative positions of the 4 curves plotted from S1 to S4, not only can we obtain information about the pressures at which radiative cooling and collapse occur, but we can differentiate further the six regimes of operation across the pressure range. This is another application of the code on which work has just commenced with the publication of a paper [199].
3.5.6.8 Experiments of INTIPF Showing Radiative Collapse and High-Energy Density (HED)

We have carried out series of experiments in INTI PF in various gases to obtain information on radiative collapse from the current waveform [200]. A fitting of the computed current waveform to the measured is carried out. Once fitted, the radial trajectory of the piston is obtained giving the value of $r_{\text{min}}$. An example is given here [196] for comparison with the numerical experiments of the PF1000 given above in Sect. 3.5.6.4.

We use a measured current waveform for the INTI PF operated at 12 kV 0.5 Torr Kr (Shot 631). We fitted the current waveform using Lee 6-phase Radiative code:

Bank parameters: $L_0 = 124$ nH (fitted), $C_0 = 30$ μF, $r_0 = 13$ mΩ (fitted),

Tube parameters: $b = 3.4$ cm, $a = 0.95$ cm, $z_0 = 16$ cm,

Operating parameters: $V_0 = 12$ kV, $P_0 = 0.5$ Torr and gas parameters (for Kr) are 84 (molecular weight), 36 (atomic number) and 1 (for atomic gas).

Fitted model parameters: $f_m = 0.0434$, $f_{mr} = 0.11$ and $f_c = f_{cr} = 0.7$ and fitted anomalous resistance parameters are as follows (shown in Table 3.20).

The fitted waveforms are shown in Fig. 3.45. An expanded view is shown in Fig. 3.46, which also correlates the expanded current waveform with computed dynamics, fitted anomalous resistances and measured quantities including tube voltage and Faraday cup waveforms.

Having fitted the computed current trace to the measured current trace, the resulting radial trajectory indicates strong radiative collapse, as shown in Fig. 3.47.

The peak compression region is magnified and shown in Fig. 3.48. The current values are normalized by 145 kA, the $P_{\text{line}}$ is normalized by $3.7 \times 10^{12}$ W and the radius ratio $k_p = r_p/a$ is multiplied by 20 for a good display. The pinch compresses to a radius of 0.0013 cm corresponding to a radius ratio (pinch radius normalized to anode radius) of 0.0014. The radiative collapse is ended when plasma self-absorption attenuates the intense line radiation. The rebound of the pinch radius is also evident in Fig. 3.48. The line radiation leaving the plasma is correlated to the trajectory in order to show the effect of the radiation on the compression. This intense compression, despite the low mass swept in the factor of $f_{mr} = 0.11$ (fitted), reaches $3.7 \times 10^{26}$ ions m$^{-3}$, which is 15 times atmospheric density (starting from less than 1/1000 of an atmospheric pressure). Moreover, the energy pumped into the pinch is 250 J whilst 41 J is radiated away in several ns, most of the radiation occurring in a tremendous burst over 50 ps at peak compression with a peak

<p>| Table 3.20 Anomalous resistance parameters of fit |</p>
<table>
<thead>
<tr>
<th>-----------------------------------</th>
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</tr>
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<tr>
<td>$R_0$ (Ω)</td>
<td>Ran1</td>
<td>Ran2</td>
<td>Ran3</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
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<td>$\tau_2$ (ns)</td>
<td>80.0</td>
<td>100.0</td>
<td>280.0</td>
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<td>10.0</td>
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<tr>
<td>End time</td>
<td>2.80</td>
<td>0.10</td>
<td>3.50</td>
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</table>
radiation power of almost $4 \times 10^{12}$ W. The energy density at peak compression is $4 \times 10^{13}$ J m$^{-3}$ or 40 kJ mm$^{-3}$. Thus even in this 2 kJ plasma focus intense HED is achieved with impressive radiation power. This radiation power is $\frac{1}{4}$ of that of PF1000 for Kr discussed in Sect. 3.5.6.4 and summarized in Table 3.19; and $\frac{1}{15}$
that of PF1000 for Xe discussed above. The outperformance in emission power of this PF storing 2 kJ compared to the PF1000 storing 350 kJ is due to the greater compression (fitted radius ratio of 0.0014) compared to the compression of PF1000 (computed radius ratio of 0.007 in Kr and 0.003 in Xe using assumed model parameters).

Fig. 3.47  Radial trajectory corresponding to the fitting of the current waveform of Fig. 3.45 for INTI PF 12 kV, 0.5 Torr Kr. Reprinted from Saw and Lee [200]. Copyright (2012) with permission from Springer Science+Business Media, LLC

Fig. 3.48  Normalized pinch current, piston radius ratio and $P_{\text{line}}$ at peak compression region. Reprinted from Saw and Lee [200]. Copyright (2012) with permission from Springer Science +Business Media, LLC
3.5.7 Conclusion for Section on Radiative Collapse

In this section, we have derived indicative values of the reduced P-B currents for various gases from D to Xe. We have also derived radiation levels from PF operations in these gases and from these we have estimated characteristic depletion times of pinch energy due to radiation less Joule heating. These depletion times indicate that D and He will have little or no radiation cooling; that Ne will show radiation cooling leading to some radiative compression for both large (PF1000) and small (INTI PF) plasma focus machines; and that Ar, Kr and Xe will have a severe radiative collapse. These results (Tables 3.15, 3.16, 3.17 and 3.18) are estimated without considering plasma opacity (plasma self-absorption). The numerical experiments, summarized in Table 3.19 include self-absorption demonstrating substantial moderating effects of self-absorption, nevertheless, confirm the indications of these tables. Some experimental results are presented. We note that the code assumes that the pinch is compressed as a column. In actual operation, break-up of the column into a line of spots have been observed particularly, but not exclusively in the heavier gases [195]. Such break-ups and indeed more detailed structures as described recently [201] will likely lead to localized enhanced compression and may tend to make it easier for the radiative collapse to occur. Moreover, the action of beams will also remove energy from the pinch [58, 59]. If beams are emitted even partially within the pinch time, this could also lead to beam-enhanced radiative collapse.

3.6 Conclusion

In this chapter, we have reviewed our experience in numerical experiments using the Lee Model code. This review describes the contributions made by this code in the past 30 years in the light of overall work on computations and simulations already carried out and documented in the area of plasma focus. The plasma focus is indeed a multi-faceted device with interesting phenomena ranging from electromagnetically driven dynamics to copious radiation including ions, electrons, X-rays, characteristic soft X-rays, fusion neutrons to fast ion beams (FIB) and fast plasma streams (FPS) to anomalous resistivity resulting from a range of plasma instabilities to plasma states of extreme high-energy density (HED) achieved in the focus pinch through radiative cooling and collapse. The Lee Model code is successful in modelling most of these multi-faceted aspects of the plasma focus. The Lee Model code developed originally as a simple code to complement the AAAPT inspired UNU ICTP PFF 3 kJ plasma focus has over the past 30 years been continuously developed. It is still a relatively simple tool. Its simplicity and sound fundamental grounding enable it to have a long and wide reach to compute gross plasma dynamics and properties, to obtain data on anomalous resistivity, to pick out fundamental scaling properties and design rules-of-thumb, recognize scaling trends
and scaling laws and develop insights into optimum inductance, current saturation, deterioration of neutron and radiation scaling in relation to energy and to understand some conditions for radiative collapse.

The physics and equations of the code are explained in detail in this chapter, as are the contributions of the code. A survey of the results of myriad simulations including MHD and kinetic methods show that the scope of the Lee Model code in the range of results and insights is unmatched. It is desirable to ask the following question. Why has such a simple model been so successful in interpreting so many aspects of plasma focus behaviour? The following may be the answer.

Its success on so many fronts is due to its use of 4 parameters (fitted to a measured current waveform) which in one sweep incorporates all the mechanisms and effects occurring in the plasma focus including mechanisms and effects difficult to compute or even as yet unrecognized. The simple premise is that the sum total effect of all these mechanisms and phenomena is represented in net result by mass field and force field distributions which in the gross sense are represented by a mass swept-up factor \( f_m \) and a effective current factor \( f_c \) in the axial phase and two corresponding factors in the radial phase, up to the end of the focus pinch.

Continuing this argument, the exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. There are many underlying mechanisms in the axial phase such as shock front and current sheet structure, porosity and inclination, boundary layer effects and current shunting and fragmenting which are not simply modelled; likewise in the radial phase mechanisms such as current sheet curvatures and necking leading to axial acceleration and ejection of mass, and plasma/current disruptions. These effects may give rise to localized regions of high density and temperatures. The detailed profile of the discharge current is influenced by these effects and during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase, the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely, all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current.

It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. The simplification of the model is that all these processes may in the gross sense be incorporated into the four model parameters \( f_m, f_c, f_{mr} \) and \( f_{cr} \) up to the end of the pinch indicated on the current waveform by the regular dip and beyond that in the extended of the current waveform by anomalous resistance functions.
This explains the importance attached to matching the computed total current trace to the measured total current trace in the procedure adopted by the Lee model code. Once matched, the fitted model parameters assure that the computation proceeds with all physical mechanisms accounted for, at least in the gross energy and mass balance sense.

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