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(not implemented in code)

Anomalous Heating (LHDI)

- Model Based on
- A. E. Robson, Phys. Fluid B3, 1481(1991)
- H. Bruzzone, Nucleonika 46, S3(2001)
- L. Bernal, H. Bruzzone, Plasma Phys.&Cont. Fusion, 41, 223(2002)
- Chittenden, Phys. Plasma, 2, 1242(1995).

Low-Hybrid-Drift Instability

- It is usually originated at the later stage of implosion
- It is triggered whenever the electron drift velocity (v_d)is greater or equal to the ion thermal speed (v_T)
- The main effect is modification of electron-ion collision frequency ν_{ei} through additional effects (due to wave particle interaction), introduces collision frequency ν^*
- It is important in all the situations wherever the magnetic field B is perpendicular the perturbations ($B \perp k$) or $k.B = 0$ and $k_{\|} = 0$.

- In order to cause electron scattering and contribute to the anomalous resistivity the wave length should be shorter or comparable to the electron gyro-radius i.e. $\lambda_{\square}(1/k_{\square}) \leq r_L$.
- The natural frequency of oscillation for the low-hybrid wave is

$$\omega_{LH} = \omega_{ce}\sqrt{\frac{z_{eff} m_e}{A m_p}}$$

- In high threshold (for $v_d = \beta\, v_{Ti}$ for $\beta=(2\text{–}4)$), with $T_e \gg T_i$ (even for larger effective charge $T_e > (7Ti/Z_{eff})$), the Ion-acoustic instability is important.

- If the electron drift velocity v_d is very much high than the ion speed ($\beta > 4$) and the ($T_e >> T_i$ or even $T_i > T_e$), the role of two-stream or Buneman instability become important.

The Role of LHDI :

The main effect of this wave-particle interaction is to enhance the effective collisional frequency (in addition to the usual electron-ion collisional frequency), causing electrical resistivity

$$\eta = \frac{m_e}{ne^2}(\nu_{ei} + \nu^*) = \eta_\perp + \eta^*$$

For the pinch environment (including plasma focus) with $\omega_{pe} > \omega_{ce}$ $\{\omega_{pe} \approx (4\pi ne/m_e)^{0.5} \sim 5.64 \times 10^4\ n_e^{1/2}$ (in cm^{-3}) and $\omega_{ce} \approx (eB/m_e c) \sim 1.76 \times 10^7 B$(in Gauss)$\}$.

The effective frequency can be written as

$$\nu^* = \xi \omega_{LH} \left(\frac{v_d}{v_{Ti}}\right)^2 \cdots where \cdots \xi = \frac{1}{2}\sqrt{\frac{\pi}{2}}$$

$$\omega_{LH} = (\omega_{ce}\omega_{ci})^{\frac{1}{2}} = \left(\frac{eB}{m_e}\frac{z_{eff}\,eB}{m_i}\right)^{0.5} = \omega_{ce}\left(\frac{z_{eff}\,m_e}{Am_p}\right)^{0.5}$$

- Where $m_i = Am_p$ (A is ionic mass and m_p is mass of proton), z_{eff} is the effective charge state. Then we can write

$$\nu^* = 0.5\sqrt{\frac{\pi}{2}}\omega_{ce}\sqrt{\frac{z_{eff}\,m_e}{Am_p}}\left(\frac{v_d}{v_{Ti}}\right)^2$$

  Where $v_{Ti} = (2KT/m_i)^{0.5} = (2KT/Am_p)^{0.5}$ is the ion thermal speed, $v_d = (J/en)$ is the electron drift velocity.

- The anomalous heating caused by LHDI not only heat up the electrons but mostly the ions .

- The Joule heating per unit volume goes like
- (1) $(0.13\eta^* + 0.87\,\eta_{\square})\,J^2$ to electron
- (2) $(0.87\eta^* + 0.87\,\eta_{\square})\,J^2$ to ions
- Therefore

$$\nu^* = 0.5\sqrt{\frac{\pi}{2}}\left(\frac{eB}{m_e}\right)\sqrt{\frac{Z_{eff}\,m_e}{m_p A}}\left(\frac{J/en}{\sqrt{\frac{2KT}{m_i}}}\right)^2$$

- Simplifying the above equation one gets

$$\nu^* = 0.5\sqrt{\frac{\pi}{2}}\left(\frac{42.9}{en^2}\right)\sqrt{\frac{A}{Z_{eff}}}\left(\frac{J^2 B}{2KT}\right)$$

Or using the numerical value of constants

$$\nu^* = 1.68\times 10^{20}\sqrt{\frac{A}{Z_{eff}}}\left(\frac{J^2 B}{2KT}\right)\frac{1}{n^2}$$

- Now using the value of ν^*, we have

$$\eta^* = \frac{m_e \nu^*}{e^2 n} = \frac{m_e}{e^2 n}(1.68\times10^{20} \sqrt{\frac{A}{Z_{eff}}\left(\frac{J^2 B}{2KT}\right)\frac{1}{n^2}})$$

- Simplifying and putting the values of constants the above equation becomes

$$\eta^* = 5.97\times10^{27} \sqrt{\frac{A}{Z_{eff}}\left(\frac{J^2 B}{2KT}\right)\frac{1}{n^3}}$$

- The total resistive heating power density becomes
- $P_{ohm} = (1+(\eta^* /\eta_{\square})) \eta_{\square} J^2$ where $\eta_{\square} = 65.3x (Z \ln\Lambda/T^{(3/2)})$ is the classical Spitzer resistivity. But power per unit length $dQ/dt = P_{ohm} \pi r^2$.

- Therefore

$$\frac{dQ}{dt} = \left(1 + 0.918\times10^{26}\frac{1}{Z\ln\Lambda}\sqrt{\frac{A}{Z_{eff}}}\left(\frac{T^{1/2}J^2B}{2n^3}\right)\right)\frac{65.3Z\ln\Lambda I^2}{\pi r^2T^{3/2}}$$

- Where J = ($I/\pi r^2$) is the current density. The power heating

$$\frac{dQ}{dt} = \left(1 + 0.918\times10^{26}\frac{1}{Z\ln\Lambda}\sqrt{\frac{A}{Z_{eff}}}\left(\frac{T^{1/2}J^2B}{2n^3}\right)\right)\frac{65.3Z_{eff}z_f\ln\Lambda I^2}{\pi r^2T^{3/2}}$$

Collisional Effects

- Ref. K. N. Koshelev and N.R. Pereira, JAP 69, R21(1991)
- Consider the case when the ions have two modes of excitation
- (1) Spontaneous excitation with de-excitation rate W_r
- (2) Collisional de-excitation (excitation due to collision with electron) rate depends upon the density of electron ($n_e \sim n_i \sim n$) i. e. $W_e \sim n_e w_e$
- The total de-excitation rate could be $W = W_r + n_e w_e$. In low density coronal case the power density of the line radiation $P_l = n_i n_e Z_{eff} X_l$, where $X_l = 4.6\ 10^{-29} (Z_n/T)$ is the coefficient of the line radiation, and Z_n is the nuclear charge and Z_{eff} is the effective charge state of the ion.

- In high density (collisional case) the energy loss coefficient X_l' due line radiation is reduce by $\frac{w_r}{(w_r + n_e w_e)} = \frac{1}{(1 + \frac{n_e w_e}{w_r})}$
- And thus

$$X_l^{'} = X_l \left(\frac{1}{(1 + \frac{n_e w_e}{w_r})} \right)$$

- Therefore, the power loss becomes

$$P_l = n_i n_e Z_{eff} X_l \left(\frac{1}{(1 + \frac{n_e w_e}{w_r})} \right)$$

- The radiated power per unit length $Q_l' = P_l' \pi r^2$. For a case of dense plasma ($n_e w_e \gg w_r$) the coefficient (w_e/w_r) (for K and L-ion radiation), for analytical modeling, can be
- $$\frac{w_e}{w_r} \approx 10^{-14} / T^{7/2} \ldots cm^3$$

- Therefore, for $n_e w_e \gg w_r$, the radiation power loss can be re-written as

$$P_l = n_i n_e Z_{eff} \left(4 \times 10^{-29} \frac{Z_n^{\,4}}{T} \left(\frac{1}{\frac{n_e w_e}{w_r}} \right) \right)$$

$$= n_i Z_{eff} \left(4 \times 10^{-29} \frac{Z_n^{\,4}}{T} \frac{1}{10^{-14} / T^{7/2}} \right)$$

$$= n_i Z_{eff} \left(4 \times 10^{-15} Z_n^{\,4} T^{5/2} \right)$$

in watt/cm^3. The line radiation power per unit length Q_l

$$Q_l = n_i Z_{eff} \left(4 \times 10^{-15} Z_n^{\,4} T^{5/2} \pi r^2 \right)$$

Opacity (radiation self-absorption)

- The radiations are emitted by the plasma if its mean free-path is larger than the size of the plasma *r*. If the optical depth $r/l_\nu \gg 1$(the mean free-path $l_\nu <$ the plasma size), the photon excited large number *M* ions before reaching the edge, thus $M = r/l_\nu$. But when *r* decreases the density increases ($N = n_i \pi r^2$), the mean free path decreases $l_\nu \sim (1/n_i)$, thus *M* increases (as $M \sim (1/r)$).
- Without collisional de-excitation these M ions re-emit photons in arbitrary direction. Multiple scattering together with collisional de-excitation deposit the photon energy back into the plasma.

- The probability that the photon is re-emitted in one cycle (including electron collisions) is

$$p = \frac{1}{(1 + \frac{n_e w_e}{w_r})}$$

- The probability of the photon to escape from the plasma without being deposited is $\beta = P^M$, where M is average number of collisions per photon.
- For the case of total frequency redistribution inside the doppler line contour an acceptable approximation is $M = \alpha(r/l_\nu)$; where α depends upon the geometry, optical depth (as $\ln(r/l_\nu)$) and line density N; typically $\alpha = 5$; so $M = 5(r/l_\nu)$.

- The mean free path $l_\nu = \frac{1}{n_i \sigma_\nu}$; where σ_ν is the cross-section of the absorption of the photon depending upon the *line width*, *photon energy*, and is proportional to the *statistical weight* ω_g of the ground state of the ion (likely to be present in the plasma) e.g. $\omega_g = 10$.
- Assuming the temperature $T = 2h\nu$ and $n_e = Z_{eff}\, n_i$; the mean free path becomes (in meter)

$$l_\nu = 3\times 10^{13}\,\frac{Z_{eff}\, T^{\frac{3}{2}}(eV)}{n_e Z_n^{\frac{1}{2}}}$$

- Now the power density due to line radiation $P_l' = P_l'\, p^{1+M}$ and radiation loss/unit length $Q' = P_l' \pi r^2$. The photonic excitation number M (in numbers) becomes

$$M = \frac{5 r Z_n^{\frac{1}{2}} n}{3\times 10^{11} Z_{eff} T^{\frac{3}{2}}} = 1.66\times 10^{-11}\,\frac{r Z_n^{\frac{1}{2}} n}{Z_{eff} T^{\frac{3}{2}}}$$

- The power loss (i.)due line radiation will become

$$\frac{dQ_l}{dt} = n_i n_e Z_{eff}\, z_f\, \pi r^2 \left(4\times 10^{-29}\, \frac{Z_n{}^4}{T} \left(\frac{1}{1+\dfrac{n_e w_e}{w_r}} \right)^{1+M} \right)$$

$$= n_i Z_{eff}\, z_f\, \pi r^2 \left(4\times 10^{-29}\, \frac{Z_n{}^4}{T} \left(\frac{1}{10^{-14}\,/\,T^{7/2}} \right)^{1+M} \right)$$

Opacity and Critical Radius

- The radius of the plasma is called critical radius when the plasma radiaton goes from *volume like* to *surface like*. This stage occur when a sizable fraction of photon is reabsorbed
- i.e. $p^{1+M} = 1/e = 0.37$. Assuming that in using $M >> 1$ and $n_e w_e / w_r \ll 1$ in $p = (1/(1 + (n_e w_e / w_r))$, and upon expansion $p^{1+M} \sim (M n_e w_e / w_r))$, give critical radius (in cm)

$$r_c \approx 1.2 \times 10^5 \frac{Z_n^{\frac{1}{6}} I^{\frac{4}{3}}}{Z_{eff}^{1/2} T^{7/3}}$$

- The radiation losses are assume to be surface like for plasma aize smaller than r_c and surface-like radiation loss per unit length Q_s (Watt/cm)

$$Q_s = 2.7 \times 10^{-2} r Z n^{7/2} Z_{eff}^{1/2} T^4$$

- Surface like radiations are much weaker than the black body emission. A black body of cylinder with radius *r* radiates

$$Q_{BB} = 2\pi r \sigma_{SB} T^4$$

- watts per unit length, where $\sigma_{SB} = 10^5$ (W/cm^2-eV^4) is Stefan Boltzman constant. The emissivity ε

$$\varepsilon = Q_s / Q_{BB} \approx 4 \times 10^{-7} Z_{eff}^{7/2}$$

- and is about 1/30 for iron.