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Institute for Plasma Focus Studies

Internet Workshop on Numerical Plasma Focus Experiments

(Supplementary Notes SP3 for Module 3)

[in part extracted from [file 2Theory.pdf](#) from:
<http://www.intimal.edu.my/school/fas/UFLF/>] and from various papers

Radiation Terms

The Bremsstrahlung loss term may be written as:

$$\frac{dQ_B}{dt} = -1.6 \times 10^{-40} N_i^2 (\pi r_p^2)_{z_f} T^{1/2} z^3$$

$$N_o = 6 \times 10^{26} \frac{\rho_o}{M}; \quad N_i = N_o f_{mr} \left(\frac{a}{r_p} \right)^2$$

Recombination loss term is written as:

$$\frac{dQ_{rec}}{dt} = -5.92 \times 10^{-35} N_i^2 Z^5 (\pi r_p^2)_{z_f} / T^{0.5}$$

The line loss term is written as:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z Z_n^4 (\pi r_p^2)_{z_f} / T$$

$$\text{and } \frac{dQ}{dt} = \frac{dQ_J}{dt} + \frac{dQ_B}{dt} + \frac{dQ_L}{dt} + \frac{dQ_{rec}}{dt}$$

where dQ/dt is the total power gain/loss of the plasma column.

By this coupling, if, for example, the radiation loss $\left(\frac{dQ_B}{dt} + \frac{dQ_L}{dt} \right)$ is severe, this would

lead to a large value of $\frac{dr_p}{dt}$ inwards. In the extreme case, this leads to radiation collapse, with r_p going rapidly to zero, or to such small values that the plasma becomes opaque to the outgoing radiation, thus stopping the radiation loss.

This radiation collapse occurs at a critical current of 1.6 MA (the Pease-Braginski current) for deuterium. For gases such as Neon or Argon, because of intense line radiation, the critical current is reduced to even below 100kA, depending on the plasma temperature.

Plasma Self Absorption and transition from volumetric emission to surface emission

Plasma self absorption and volumetric (emission described above) to surface emission of the pinch column have been implemented in the following manner.

The photonic excitation number (see [File 3 Appendix](#) by N A D Khattak) is written as follows:

$$M = 1.66 \times 10^{-15} r_p Z_n^{0.5} n_i / (Z T^{1.5}) \text{ with } T \text{ in eV, rest in SI units}$$

The volumetric plasma self-absorption correction factor A is obtained in the following manner:

$$A_1 = (1 + 10^{-14} n_i Z) / (T^{3.5})$$

$$A_2 = 1 / AB_1$$

$$A = A_2^{(1+M)}$$

Transition from volumetric to surface emission occurs when the absorption correction factor goes from 1 (no absorption) down to 1/e (e=2.718) when the emission becomes surface-like given by the expression:

$$\frac{dQ}{dt} = -const x Z_n^{3.5} Z^{0.5} \left(r_p \right) Z_f T^4$$

where the constant *const* is taken as 4.62×10^{-16} to conform with numerical experimental observations that this value enables the smoothest transition, in general, in terms of power values from volumetric to surface emission.

Where necessary another fine adjustment is made at the transition point adjusting the constant so that the surface emission power becomes the same value as the absorption corrected volumetric emission power at the transition point. Beyond the transition point (with A less than 1/e) radiation emission power is taken to be the surface emission power.

Neutron Yield

<http://www.intimal.edu.my/school/fas/UFLF/>

Adapted from the following papers (with modifications for erratum)

Pinch current limitation effect in plasma focus (This version includes an Erratum)

S. Lee and S. H. Saw, *Appl. Phys. Lett.* 92, 021503 (2008), DOI:10.1063/1.2827579

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<http://link.aip.org/link/?APPLAB/92/021503/1>

Neutron Scaling Laws from Numerical Experiments (This version includes an Erratum)

S Lee and S H Saw, *J of Fusion Energy*, DOI: 10.1007/s10894-008-9132-7

published first online 20 February 2008 at <http://dx.doi.org/10.1007/s10894-008-9132-7>

"The original publication is available at www.springerlink.com."

Neutron yield is calculated with two components, thermonuclear term and beam-target term.

The thermonuclear term is taken as:

$$dY_{th} = 0.5n_i^2 (3.142) r_p^2 z_f \langle \sigma v \rangle (\text{time interval})$$

where $\langle \sigma v \rangle$ is the thermalised fusion cross section-velocity product corresponding to the plasma temperature, for the time interval under consideration. The yield Y_{th} is obtained by summing up over all intervals during the focus pinch.

The beam-target term is derived using the following phenomenological beam-target neutron generating mechanism¹⁷, incorporated in the present RADPFV5.13. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modeling each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with proportionality constant. The yield is then calibrated against a known experimental point.

$$\text{The beam-target yield is written in the form: } Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$$

where n_b is the number of beam ions per unit plasma volume, n_i is the ion density, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n- branch¹⁸, v_b the beam ion speed and τ is the beam-target interaction time assumed proportional to the confinement time of the plasma column.

Total beam energy is estimated¹⁷ as proportional to $L_p I_{pinch}^2$, a measure of the pinch inductance energy, L_p being the focus pinch inductance. Thus the number of beam ions is $N_b \sim L_p I_{pinch}^2 / v_b^2$ and n_b is N_b divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that⁴ $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$ where U is the disruption-caused diode voltage¹⁷. Here 'b' is the cathode radius. We also assume reasonably that U is proportional to V_{max} , the maximum voltage induced by the current sheet collapsing radially towards the axis.

$$\text{Hence we derive: } Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 ((\ln b/r_p)) \sigma / V_{max}^{1/2} \quad (1)$$

where I_{pinch} is the current flowing through the pinch at start of the slow compression phase; r_p and z_p are the pinch dimensions at end of that phase. Here C_n is a constant which in practice we will calibrate with an experimental point.

The D-D cross-section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute σ . The code computes V_{max} of the order of 20-50 kV. However it is known¹⁷, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50-150keV¹⁷, and for smaller lower-voltage machines the relevant energy¹⁹ could be lower at 30-60keV. Thus to align with experimental observations the D-D cross section σ is reasonably obtained by using beam energy equal to 3 times V_{max} .

A plot of experimentally measured neutron yield Y_n vs I_{pinch} was made combining all available experimental data^{2,4,12,13,17,19-22}. This gave a fit of $Y_n = 9 \times 10^{10} I_{pinch}^{3.8}$ for I_{pinch} in the range 0.1-1MA. From this plot a calibration point was chosen at 0.5MA, $Y_n = 7 \times 10^9$ neutrons. The model code²³ RADPFV5.13 was thus calibrated to compute Y_{b-t} which in our model is the same as Y_n .

Notes on **The total current and I_{peak} , the plasma current and I_{pinch}**

Extracted From: [Computing Plasma Focus Pinch Current from Total Current Measurement](#)

S. Lee, S. H. Saw, P. C. K. Lee, R. S. Rawat and H. Schmidt, *Appl Phys Letters* **92**, 111501 (2008) DOI:10.1063/1.2899632

The total current I_{total} waveform in a plasma focus discharge is easily measured using a Rogowski coil. The peak value I_{peak} of this trace is commonly taken as a measure of the drive efficacy and is often used to scale the yield performance of the plasma focus. This is despite the fact that yields should more consistently be scaled to focus pinch current I_{pinch} , since it is I_{pinch} which directly powers the emission processes. The reason many researchers use I_{peak} instead of I_{pinch} for scaling is simply that while I_{peak} is easily measured, I_{pinch} , which is the value of the plasma sheath current I_p at time of pinch, is very difficult to measure even in large devices where it is possible to place magnetic probes near the pinch. This measurement is also inaccurate and perturbs the pinch. In a small device, there is no space for such a measurement.

The relationship between I_{pinch} and I_{peak} is not simple and has only been recently elaborated. It primarily depends on the value of the static inductance L_0 compared to the dynamic inductances of the plasma focus. As L_0 is reduced, the ratio $I_{\text{pinch}} / I_{\text{peak}}$ drops. Thus, yield laws scaled to I_{peak} will not consistently apply when comparing two devices with all parameters equal but differing significantly in L_0 . Better consistency is achieved when yield laws are scaled to I_{pinch} . In this paper, we propose a numerical method to consistently

Distinguishing the I_{total} waveform from the I_p waveform

A measured trace of I_{total} is commonly obtained with a Rogowski coil wrapped around the plasma focus flange through which is fed I_{total} discharged from the capacitor bank between the coaxial electrodes across the back wall. A part of I_{total} , being the plasma sheath current I_p , lifts off the back-wall insulator and drives a shock wave axially down the coaxial space. We denote f_c as the current fraction I_p / I_{total} for the axial phase and f_{cr} for the radial phases. In modeling it is found that a reasonable value for initial trial for f_c is 0.7 with a similar first trial value for f_{cr} . However in a DPF78 experiment f_c was found to vary from 0 at the start of the axial phase rising rapidly above 0.6 for the rest of the axial phase. In the radial phase f_{cr} was found to stay above 0.6 before dropping to 0.48 at the start of the pinch and then towards 0.4 as the pinch phase progressed. These Stuttgart results confirm a complex relationship between the waveforms of I_{total} and I_p .

The performance of a plasma focus is closely linked to the current I_{pinch} actually participating in the focus pinch phase rather than the total current flowing in the circuit. It is a common practice to take I_{peak} or some representative fraction of it as I_{pinch} . Another practice is to take the value of I_{total} at the time of the pinch as I_{pinch} . Whilst in their special cases this practice could be justifiable, the distinction of I_p from I_{total} should generally be clearly made. We emphasize that it should be the value of I_p at the time of pinch which is the relevant value for the purpose of yield scaling. The practice of associating yield

scaling with the total current waveform (i.e. taking I_{peak} or I_{total} at estimated pinch time) would be justifiable if there were a linear relationship between the waveforms of I_{total} and I_p . However as shown by the Stuttgart experiments the actual relationship is a very complex one which we may ascribe to the interplay of the various electro-dynamical processes including the relative values of static inductance L_o , tube inductance and the dynamic resistances which depend on the tube geometry and plasma sheath speeds. This relationship may change from one machine to the next. Whilst these electro-dynamical processes and other relevant ones such as radiation are amenable to modeling there are other machine effects such as back wall restriking (for example due to high induced voltages during the pinch phase) which can almost unpredictably affect the relationship between I_{total} and I_p during the crucial radial phases. Hence it is not only simplistic to discuss scaling in terms of the I_{total} waveform (i.e. taking I_{peak} or the value of I_{total} at the estimated time of pinch) but also inconsistent. One of the most important features of a plasma focus is its neutron production. The well-known neutron yield scaling, with respect to current, based on various compilations of experimental data, is $Y_n \sim I_{\text{pinch}}^x$ where x is varied in the range 3–5. In a recent paper, numerical experiments using a code was used to derive a scaling with $x = 4.7$. Difficulties in the interpretation of experimental data ranging across big and small plasma focus devices include the assignment of the representative neutron yield Y_n for any specific machine and the assignment of the value of I_{pinch} . In a few larger machines attempts were made to measure I_{pinch} using magnetic probes placed near the pinch region, with uncertainties of 20%. Moreover the probes would have affected the pinching processes. In most other cases related to yield scaling data compilation or interpretation I_{pinch} is simply assigned a value based on the measurement of peak total current I_{peak} or the value of total current at the observed current dip.

The difficulties in distinguishing I_{pinch} from I_{total} are obviated in numerical experiments using the Lee Model [In a typical simulation, the I_{total} trace is computed and fitted to a measured I_{total} trace from the particular focus. Three model parameters for fitting are used: axial mass swept-up factor f_m , current factor f_c and radial mass factor f_{mr} . A fourth model parameter, radial current factor, f_{cr} may also be used. When correctly fitted the computed I_{total} trace agrees with the measured I trace in peak amplitude, rising slope profile and topping profile which characterize the axial phase electro-dynamics. The radial phase characteristics are reflected in the roll-over of the current trace from the flattened top region, and the subsequent current drop or dip. Any machine effects, such as restrikes, current sheath leakage and consequential incomplete mass swept up, not included in the simulation physics is taken care of by the final choice of the model parameters, which are fine-tuned in the feature-by-feature comparison of the computed I_{total} trace with the measured I_{total} trace. Then there is confidence that the computed gross dynamics, temperature, density, radiation, plasma sheath currents, pinch current and neutron yield may also be realistically compared with experimental values.

A note on scaling:

Scaling of yields to say I_{pinch} should be carried out using yields which are at optimum, or at least near optimum. If one indiscriminately uses any data one may end up with

completely trivial or misleading results. For example if a point is used at too high or low pressure (away from the optimum pressure) then there may be zero yield ascribed to values of I_{pinch} .

References

- ¹ Lee S 1984 *Radiations in Plasmas* ed B McNamara (World Scientific) pp 978–87
Also: S. Lee in *Laser and Plasma Technology*, edited by S. Lee, B. C. Tan, C. S. Wong, & A. C. Chew. World Scientific, Singapore, (1985), pp. 387–420.
- ²S. Lee, T. Y. Tou, S. P. Moo, M. A. Elissa, A. V. Gholap, K. H. Kwek, S. Mulyodrono, A. J. Smith, Suryadi, W. Usala, & M. Zakaullah, *Am. J. Phys.* **56**, 62 (1988).
- ³T. Y. Tou, S. Lee, & K. H. Kwek, *IEEE Trans. Plasma Sci.* **17**, 311 (1989).
- ⁴S. Lee and A. Serban, *IEEE Trans. Plasma Sci.* **24**, 1101 (1996).
- ⁵D. E. Potter, *Phys. Fluids* **14**, 1911 (1971).
- ⁶M. H. Liu, X. P. Feng, S. V. Springham, and S. Lee, *IEEE Trans. Plasma Sci.* **26**, 135 (1998).
- ⁷S. Lee, P. Lee, G. Zhang, X. Feng, V. A. Gribkov, M. Liu, A. Serban, and T. Wong, *IEEE Trans. Plasma Sci.* **26**, 1119 (1998).
- ⁸S. Bing, “Plasma dynamics and x-ray emission of the plasma focus,” Ph.D. thesis, NIE, (2000) in ICTP Open Access Archive: <http://eprints.ictp.it/99/>
- ⁹S. Lee, in <http://ckplee.myplace.nie.edu.sg/plasmaphysics/> (2000 & 2007).
- ¹⁰S. Lee in ICTP Open Access Archive: <http://eprints.ictp.it/85/> (2005).
- ¹¹V. Siahpoush, M. A. Tafreshi, S. Sobhanian, and S. Khorram, *Plasma Phys. Controlled Fusion* **47**, 1065 (2005).
- ¹²S. Lee, Twelve Years of UNU/ICTP PFF-A Review (1998) IC, 98 (231); A.Salam ICTP, Miramare, Trieste (in ICTP OAA: <http://eprints.ictp.it/31/>).
- ¹³L. Soto, P. Silva, J. Moreno, G. Silvester, M. Zambra, C. Pavez, L. Altamirano, H. Bruzzone, M. Barbaglia, Y. Sidelnikov, and W. Kies, *Braz. J. Phys.* **34**, 1814 (2004).
- ¹⁴H. Acuna, F. Castillo, J. Herrera, and A. Postal, International Conference on Plasma Sci, 3–5 June 1996 (unpublished), p. 127.

- ¹⁵C. Moreno, V. Raspa, L. Sigaut, & R. Vieytes, *Appl. Phys. Lett.* **89**, 15 (2006).
- ¹⁶D. Wong, P. Lee, T. Zhang, A. Patran, T. L. Tan, R. S. Rawat, and S. Lee, *Plasma Sources Sci. Technol.* **16**, 116 (2007).
- ¹⁷V A Gribkov, A Banaszak, B Bienkowska, A V Dubrovsky, I Ivanova-Stanik, L Jakubowski, L Karpinski, R A Miklaszewski, M Paduch, M J Sadowski, M Scholz, A. Szydlowski, and K. Tomaszewski, *J. Phys. D* **40**, 3592 (2007).
- ¹⁸J. D. Huba, 2006 Plasma Formulary, p. 44.
http://wwwppd.nrl.navy.mil/nrlformulary/NRL_FORMULARY_07.pdf
- ¹⁹S. V. Springham, S. Lee, and M. S. Rafique, *Plasma Phys. Controlled Fusion* **42**, 1023 (2000).
- ²⁰W. Kies, in *Laser and Plasma Technology, Proceedings of Second Tropical College*, edited by S. Lee, B. C. Tan, C. S. Wong, A. C. Chew, K. S. Low, H. Ahmad, and Y. H. Chen World Scientific, Singapore, (1988), pp. 86–137.
- ²¹H. Herold, in *Laser and Plasma Technology, Proceedings of Third Tropical College*, edited by C. S. Wong, S. Lee, B. C. Tan, A. C. Chew, K. S. Low, and S. P. Moo World Scientific, Singapore, (1990), pp. 21–45.
- ²²A. Patran, R. S. Rawat, J. M. Koh, S. V. Springham, T. L. Tan, P. Lee, & S. Lee, 31st EPS Conf on Plasma Phys London, (2004), Vol. 286, p. 4.213.
- ²³S Lee **Radiative Dense Plasma Focus Computation Package (2008) RADPF**:
<http://www.intimal.edu.my/school/fas/UFLF/>

Reference to this course and the Lee model code should be given according to the following format:

Lee S. Radiative Dense Plasma Focus Computation Package (2008): RADPF
www.plasmafocus.net www.intimal.edu.my/school/fas/UFLF/