

A Selection of published/presented papers

**Volume V (3)
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Papers 1-18

S Lee

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Volume V Part 3 (2010)
Papers 1-18
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1. S H Saw and S Lee, [Scaling the Plasma Focus for Fusion Energy Considerations](#)- Int. J. Energy Res. (2010); Published online 2 August 2010 Int. J. Energy Res. 2011; 35:81–88

2. Sing Lee and Sor Heoh Saw, [Numerical Experiments providing new Insights into Plasma Focus Fusion Devices](#) *Invited Review Paper: Special edition on “Fusion Energy”*
Energies **2010**, **3**, 711-737; doi:10.3390/en3040711-Published online 12 April 2010

3. S H Saw and S Lee :
[Scaling laws for plasma focus machines from numerical experiments](#)
Energy and Power Engineering, **2010**, **65-72** doi:10.4236/epe.2010.21010
Published Online February 2010 (<http://www.scirp.org/journal/epe>)

4. S H Saw, S Lee, F Roy, PL Chong, V Vengadeswaran, ASM Sidik, YW Leong & A Singh
[In-situ determination of the static inductance and resistance of a plasma focus capacitor bank](#) —
REVIEW OF SCIENTIFIC INSTRUMENTS **81**, 053505 (2010) published online 21 May 2010

5 M. Shahid Rafique, P. Lee, A. Patran, R. S. Rawat, S. Lee
[Radiation Emission Correlated with the Evolution of Current Sheath from a Deuterium Plasma Focus](#)
J Fusion Energ DOI 10.1007/s10894-010-9276-0, published online: 23 January 2010

6 M Akel, S Hawat, S Lee
[Pinch Current and Soft x-ray yield limitation by numerical experiments on Nitrogen Plasma Focus](#)- J
Fusion Energy (2010) 29:page 94
J Fusion Energ DOI 10.1007/s10894-009-9238-6 First online 21 August 2009

7 M. Akel, Sh. Al-Hawat, S. H. Saw and S. Lee
[Numerical Experiments on Oxygen Soft X-Ray Emissions from Low Energy Plasma Focus Using Lee Model](#) . *J Fusion Energ* (2010) 29:223–231
DOI 10.1007/s10894-009-9262-6 Published online: 22 November 2009

8 A E Abdou, R Verma, S Lee, R S Rawat, P Lee, S V Springham
Characterization of the Kansas State University Dense Plasma Focus as a Multi-Radiation Source,
presented at IEEE Conference on Plasma Science ICOPS 2010, Virginia, USA June 2010

9 S Lee and S H Saw
[Plasma Nuclear Fusion Energy- Mankind’s Giant Step Forward](#)
Invited Keynote paper: International Conference on Nuclear and Renewable Energy Resources,
Ankara 4-7 July 2010: Procs. pg 2-7

10 S Lee and S H Saw
[The Plasma Focus- Trending into the Future](#)
Invited paper: International Conference on Nuclear and Renewable Energy Resources, Ankara 4-7 July
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- 11 S Lee and S H Saw
[Plasma Diagnostics: Use of Current Measurements to Interpret Plasma Focus Properties and Mechanisms:](#)
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- 12 S H Saw and S Lee
[Plasma Diagnostic 2: In-situ measurement of Bank Parameters Using Lee model code.](#)
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- 13 S Lee and S H Saw
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- 18 S Lee and S H Saw
Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and Plasma Diagnostics, 15-26 November 2010, Trieste, Italy- [Course on Plasma Focus Numerical Experiments](#) Manual 200 pages –**ICTP Series e-manual 2**
<http://cdsagenda5.ictp.trieste.it/askArchive.php?base=agenda&categ=a09172&id=a09172s6t14/Manual>

Scaling the plasma focus for fusion energy considerations

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SUMMARY

Using the Lee model code for dense plasma focus, series of numerical experiments were systematically carried out to determine the scaling of bank energies with total current and focus pinch current and the scaling of neutron yields with energies and currents. The numerical experiments were carried out over a range of bank energies from 8 kJ extending up to 24 MJ on the PF1000 and a proposed less damped modern bank. It also includes a study on the effects of increasing bank energies by increasing bank charging voltage and capacitance of the bank for a practical optimum plasma focus machine. The results provide convincing data to show that it is possible to scale up the plasma focus machine at just 3 MJ for D-D neutron yield of 10^{13} per shot and 10^{15} neutrons per shot when it is converted to operate in D-T. Copyright © 2010 John Wiley & Sons, Ltd.

KEY WORDS

dense plasma focus; neutrons source; focus pinch current; fusion energy; neutron scaling law

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1. INTRODUCTION

Plasma focus machines consistently produce considerable amounts of neutrons. The scalability of the device to fusion reactor conditions remains an area of research [1]. Even a simple machine such as the UNU ICTP PFF 3 kJ machine consistently produces 10^8 neutrons when operated in deuterium [2]. A big machine such as the PF1000 typically produces 10^{11} neutrons per shot [3]. Gribkov *et al.* [4] had pointed out that $Y_n = 10^{13}$ in Deuterium is a desired landmark to achieve in a plasma focus device; from the point of view of possible exploitation as a powerful source of fusion neutrons for testing of prospective materials for the first wall components and construction elements in magnetic confinement fusion and also in inertial confinement fusion reactors. Converting such a plasma focus yield to operation in D-T, with $Y_n = 10^{15}$ could produce, during a 1-year run, an overall fluence-affecting materials to the order of 0.1–1.0 displacements per atom (DPA) (1 DPA is equal to a mean neutron flux of 4.5×10^{16} neutrons $\text{m}^{-2}\text{s}^{-1}$ for 1 year) for such testing purposes, at a very low cost relative to

other methods currently being considered. We now examine the requirements to reach this landmark.

This paper presents the results from a series of numerical experiments systematically carried out using the Lee model code [5] to investigate the scalability of the plasma focus to achieve $Y_n = 10^{13}$ D-D yield. In relation to this, it was necessary to determine the scaling laws between bank energies and peak total current and peak pinch current; and between Y_n and peak total current and peak pinch current [6–9].

2. THE LEE MODEL CODE

2.1. Description of the model

The Lee model code couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation, enabling realistic simulation of all gross focus properties. The basic model, described in 1984 [10] was successfully used to assist several projects [11–13]. Radiation-coupled dynamics was included in the five-phase code leading to numerical experiments on

radiation cooling [14]. The vital role of a finite small disturbance speed discussed by Potter in a Z-pinch situation [15] was incorporated together with real gas thermodynamics and radiation-yield terms. Before this 'communication delay effect' was incorporated, the model consistently over-estimated the radial speeds. This is serious from the point of view of neutron yields. A factor of 2 in shock speeds gives a factor of 4 in temperatures leading to a difference in fusion cross-sections of ~ 1000 at the range of temperatures we are dealing with. This version of the code assisted other research projects [16–21] and was web-published in 2000 [22] and 2005 [23]. Plasma self-absorption was included in 2007 [22] improving SXR yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [2,11,16,17,19,20,24,25], NX2 [18,21,26], NX1 [26,27] and adapted for the Filippov-type plasma focus DENA [28]. A recent development is the inclusion of the neutron yield, Y_n , using a beam-target mechanism [3,6–8,29], incorporated in recent versions [5] of the code (versions later than RADPFV5.13), resulting in realistic Y_n scaling with I_{pinch} [7,8]. The versatility and utility of the model is demonstrated in its clear distinction of I_{pinch} from I_{peak} [30] and the recent uncovering of a plasma focus pinch current limitation effect [6,29]. The description, theory, code and a broad range of results of this 'Universal Plasma Focus Laboratory Facility' are available for download from [5].

2.2. A brief description of the code

The five phases are summarized as follows:

1. Axial phase: Described by a snowplow model with an equation of motion coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c , respectively. The mass swept-up factor f_m accounts not only for the porosity of the current sheet but also for the inclination of the moving current sheet-shock front structure and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor, f_c , accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall and current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.
2. Radial inward shock phase: Described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, also called the magnetic piston, allowing the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock front and current sheath due to the finite small disturbance speed is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors, f_{mr} and f_{cr} , are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms that have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor, f_{cr} , accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.
3. Radial reflected shock (RS) phase: When the shock front hits the axis, because the focus plasma is collisional, a RS develops which moves radially outward, whereas the radial current-sheath piston continues to move inward. Four coupled equations are also used to describe this phase, these being for the RS moving radially outward, the piston moving radially inward, the elongation of the annular column and the circuit. The same model parameters, f_{mr} and f_{cr} , are used as in the previous radial phase. The plasma temperature behind the RS undergoes a jump by a factor approximately 2.
4. Slow compression (quiescent) or pinch phase: When the out-going RS hits the in-coming piston, the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects are incorporated into this phase. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.
5. Expanded column phase: To simulate the current trace beyond this point, we allow the column to suddenly attain the radius of the anode and use the expanded column inductance for further integration. In this final phase, the snowplow model is used, and two coupled equations are used; similar to the axial phase aforementioned. This phase is not considered important as it occurs after the focus pinch.

2.3. Computation of neutron yield

The neutron yield is computed using a phenomenological beam-target neutron-generating mechanism described recently by Gribkov *et al.* [3] and adapted to yield the following equation. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. The beam-target yield is derived [6] as:

$$Y_{b-t} = C_n n_i I_{\text{pinch}}^2 z_p^2 (\ln(b/r_p)) \sigma / U^{0.5} \quad (1)$$

where n_i is the ion density, b is the cathode radius, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n -branch [31] and U , the beam energy. C_n is treated as a calibration constant combining various constants in the derivation process.

The D-D cross-section is sensitive to the beam energy in the range 15–150 keV; hence, it is necessary to use the appropriate range of beam energy to compute σ . The code computes induced voltages (due to current motion inductive effects) V_{max} of the order of only 15–50 kV. However, it is known from experiments that the ion energy responsible for the beam-target neutrons is in the range 50–150 keV [3], and for smaller lower-voltage machines the relevant energy could be lower at 30–60 keV [24]. Thus, in line with experimental observations the D-D cross-section σ is reasonably obtained by using $U = 3V_{\text{max}}$. This fit was tested by using U equal to various multiples of V_{max} . A reasonably good fit of the computed neutron yields to the measured published neutron yields at energy levels from sub-kJ to near MJ was obtained when the multiple of 3 was used; with poor agreement for most of the data points when, for example, a multiple of 1 or 2 or 4 or 5 was used. The model uses a value of $C_n = 2.7 \times 10^7$ obtained by calibrating the yield [5,6] at an experimental point of 0.5 MA.

The thermonuclear component is also computed in every case and it is found that this component is negligible when compared with the beam-target component.

3. PROCEDURES FOR NUMERICAL EXPERIMENTS

The Lee model code is configured to work as any plasma focus by inputting the bank parameters, L_0 , C_0 and stray circuit resistance r_0 ; the tube parameters b , a and z_0 and operational parameters V_0 and P_0 and the fill gas. The standard practice is to fit the computed total current waveform to an experimentally measured total current waveform [5–8,23,24,29,30] using four model parameters representing the mass swept-up

factor f_m , the plasma current factor f_c for the axial phase and factors f_{mr} and f_{cr} for the radial phases.

From experience, it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace.

The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The detailed profile of the discharge current during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase, the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely, all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed current trace to the measured current trace in the procedure adopted by the Lee model code.

For this series of experiments, we configure the Lee model code using a published current trace measured from the PF1000 with $C_0 = 1332 \mu\text{F}$, operated at 27 kV, 3.5 torr deuterium, with cathode/anode radii $b = 16 \text{ cm}$, $a = 11.55 \text{ cm}$ and anode length $z_0 = 60 \text{ cm}$ [3]. In the numerical experiments, we fitted external (or static) inductance $L_0 = 33.5 \text{ nH}$ and stray resistance $r_0 = 6.1 \text{ m}\Omega$ (damping factor $\text{RESF} = \text{stray resistance} / (L_0/C_0)^{0.5} = 1.22$). The fitted model parameters are $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. Figure 1 shows the computed current trace [5,6,29] agreeing very well with the measured trace through all the phases, axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase. This agreement confirms the model parameters for the PF1000. Once the model parameters have been fitted to a machine for a given gas, these model parameters may be used with some degree of confidence when operating parameters such as the voltage are varied [5]. With no measured current waveform available for the higher megajoule numerical experiments, it is reasonable to keep the model parameters that we have got from the PF1000 fitting.

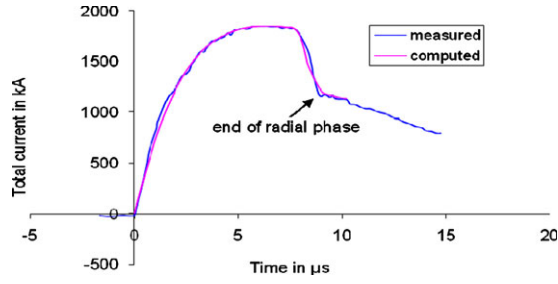


Figure 1. Fitting computed current trace to measured current trace to obtain fitted parameters $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. In this example for PF1000, the fitting is found to be good for the phases up to the end of the radial phase.

3.1. Determining the scaling laws for neutrons from numerical experiments over a range of energies from 8.5 kJ to 24.5 MJ

Using the model parameters determined in the above for PF1000, we run the first series of numerical experiments on PF1000-like machines at $V_0 = 35$ kV, $P_0 = 10$ torr, $L_0 = 33.5$ nH, RESF = 1.22 and the $c = b/a = 1.39$ for varying bank energies from 8.5 kJ to 24.5 MJ. This series of numerical experiments is operated at optimum pressure of 10 torr deuterium, and the ratio b/c retained at 1.39.

The numerical experiments were carried out for C_0 ranging from 14 to 39 960 μ F corresponding to energies from 8.5 kJ to 24.5 MJ. For each C_0 , we parametrically varied z_0 and then ‘ a ’ to find the optimum combination of z_0 and ‘ a ’ for each given C_0 corresponding to an end axial speed of 10 cm/ μ s. The optimum combinations for each C_0 are summarized in Table I together with the peak total current, peak pinch current and neutron yield computed for the range of bank energies from 8.5 kJ to 24.5 MJ.

The results show no saturation of the peak total current and peak pinch current as the energies of the bank is increased. However, it is clear that the scaling deteriorates with increasing bank energies.

Figure 2 shows the computed I_{peak} as a function of C_0 , which shows no saturation; although there is a scaling shift from $I_{peak} \sim E_0^{0.41}$ to $I_{peak} \sim E_0^{0.22}$ which is seen when plotted on log–log scale (Figure 3).

Similarly, the I_{pinch} scaling with E_0 slows down from $I_{pinch} \sim E_0^{0.41}$ to $I_{pinch} \sim E_0^{0.22}$ (Figure 3), but again no saturation. We would like to emphasize that the findings in earlier papers [6,7,29,30] concluded that it is the I_{pinch} scaling, rather than I_{peak} which directly affects the neutron yield scaling.

For this series of experiments, we find that the Y_n scaling decreases from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 24.5 MJ). This is shown in Figure 4.

Because of the way Y_n versus E_0 scaling slows down at the megajoule level and the corresponding way I_{peak}

Table I. Numerical experiments to find the scaling of I_{peak} , I_{pinch} and Y_n with C_0 .

E_0 (kJ)	C_0 (μ F)	a (cm)	z_0 (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_n (10^{10})
24 476	39 960	23.55	500.0	5678	2368	1320
18 765	30 636	22.80	400.0	5479	2267	1094
14 685	23 976	21.90	350.0	5268	2165	906
11 422	18 648	21.00	300.0	5046	2062	737
6527	10 656	18.73	220.0	4506	1827	438
4895	7992	17.66	160.0	4229	1721	329
4079	6660	16.88	140.0	4043	1650	273
3263	5328	15.90	120.0	3812	1563	215
2448	3996	14.53	86.6	3510	1461	159
1632	2664	12.69	65.0	3101	1316	102
816	1332	9.95	45.0	2458	1086	45.4
748	1221	9.62	42.0	2383	1059	40.9
680	1110	9.18	38.0	2295	1032	36.3
612	999	8.82	36.0	2208	1000	31.8
544	888	8.47	35.0	2116	965	27.3
476	777	8.05	33.0	2012	926	22.9
408	666	7.60	31.0	1898	882	18.6
340	555	7.04	28.0	1766	833	14.5
272	444	6.38	24.0	1613	778	10.8
204	333	5.59	20.0	1425	706	7.1
136	222	4.74	17.0	1205	613	3.8
68	111	3.47	12.0	888	476	1.21
34	56	2.48	8.0	644	363	0.346
17	28	1.72	5.0	457	272	0.085
8.5	14	1.28	4.0	335	202	0.017

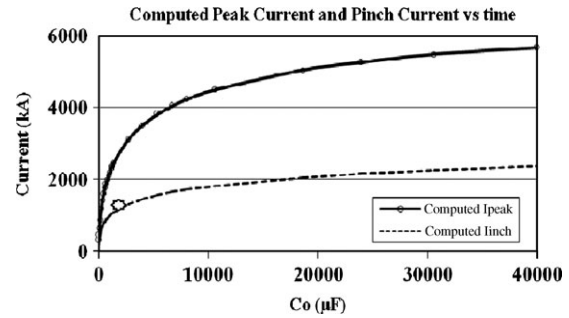


Figure 2. I_{peak} (top trace) computed from numerical experiments as a function of C_0 . Also shown is the I_{pinch} curve (lower trace). The single point at the 2 MA level is an experimental PF1000 point.

and I_{pinch} scaling with E_0 also slow down, the scaling of Y_n with I_{peak} and I_{pinch} over the whole range of energies investigated up to 24.5 MJ (Figure 5) are as follows:

$$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5};$$

$$Y_n = 1.8 \times 10^{10} I_{peak}^{3.8} \text{ where } I_{pinch} (0.2 - 2.4) \text{ and } I_{peak} (0.3 - 5.7) \text{ are in MA.}$$

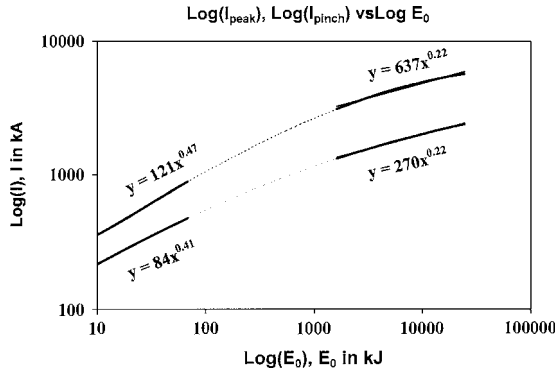


Figure 3. $\log I_{\text{peak}}$ (top curve) and $\log I_{\text{pinch}}$ versus $\log E_0$, showing no saturation for E_0 up to 24.5 MJ.

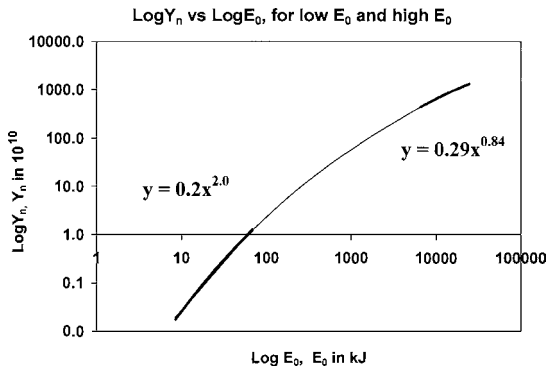


Figure 4. Y_n plotted as a function of E_0 in log-log scale, showing no saturation of neutron yield up to 24.5 MJ, the highest energy investigated.

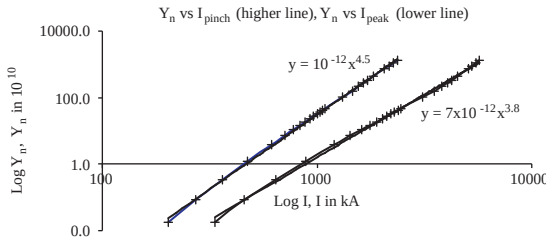


Figure 5. $\log(Y_n)$ scaling with $\log(I_{\text{peak}})$ and $\log(I_{\text{pinch}})$, for the range of energies investigated, up to 24.5 MJ.

In the above series of numerical experiments, we have shown that as the bank energies of the plasma focus increase from 10 kJ to 25 MJ:

$$I_{\text{peak}} \sim E_0^x \text{ where } x = 0.47 \text{ at tens of kJ and } x = 0.22 \text{ at the highest energies (up to 24.5 MJ)}$$

$$I_{\text{pinch}} \sim E_0^x \text{ where } x = 0.41 \text{ at tens of kJ and } x = 0.22 \text{ at the highest energies (up to 24.5 MJ)}$$

$$Y_n \sim E_0^x, \text{ where } x = 2.0 \text{ at tens of kJ and } x = 0.84 \text{ at the highest energies (up to 24.5 MJ)}$$

Y_n does not saturate with increasing E_0 at the megajoule level and the resultant scaling laws for Y_n in relation to the total peak current, I_{peak} and the total pinch current I_{pinch} are

$$Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5} \text{ where } I_{\text{pinch}} (0.2 - 2.4) \text{ in MA;}$$

and

$$Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8} \text{ where } I_{\text{peak}} (0.3 - 5.7) \text{ in MA}$$

This series of numerical experiments show that it is possible to scale up the plasma focus machine to a yield of 10^{13} D-D neutrons. This is achieved at bank energy $E_0 = 18.7$ MJ with $I_{\text{peak}} = 5.5$ MA and $I_{\text{pinch}} = 2.3$ MA, corresponding to the focus anode length $z_0 = 4$ m and anode radius $a = 22.8$ cm and cathode radius $b = 31.7$ cm.

3.2. Investigating the effect of RESF on Y_n yield

The PF1000 has an unusually high damping factor represented by $\text{RESF} = 1.22$. As it is practically possible for a modern capacitor bank system to be very much less damped with $\text{RESF} = 0.12$, we proceed to run the second series of numerical experiments with RESF changed to 0.12. Again for each C_0 , we parametrically varied z_0 and then 'a' to find the optimum combination of z_0 and 'a' for each given C_0 corresponding to an end axial speed of 10 cm/ μ s. The optimum combinations for each C_0 and with the peak total current, peak pinch current and neutron yield computed for the range of bank energies from 68 kJ to 24.5 MJ are tabulated in Table II. Figure 6 shows the summary of the results.

These results show that using a less resistive modern bank, $\text{RESF} = 0.12$ reduces the E_0 required to reach $Y_n = 10^{13}$ in Deuterium to some 8 MJ with corresponding $I_{\text{peak}} = 6$ MA, $I_{\text{pinch}} = 2.3$ MA, the focus length $z_0 = 2.6$ m, anode radius $a = 25.3$ cm and cathode radius $b = 35.2$ cm; as compared with 19 MJ with corresponding $I_{\text{peak}} = 5.5$ MA, $I_{\text{pinch}} = 2.3$ MA, the focus length $z_0 = 4$ m, anode radius $a = 22.8$ cm and cathode radius $b = 31.7$ cm for the earlier heavily damped bank with $\text{RESF} = 1.22$.

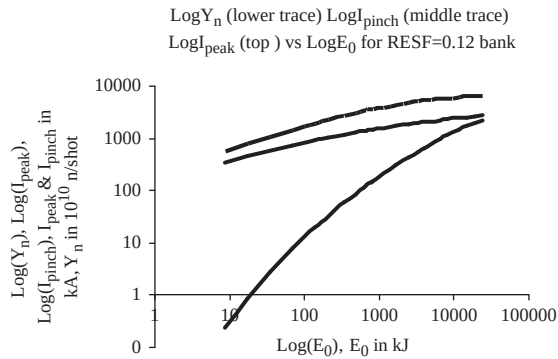
3.3. Investigating the effect on Y_n as operating voltage is increased from 35 to 90 kV, at $C_0 = 777 \mu\text{F}$

We run a third series of numerical experiments for a practical optimum configuration [8] with $c = b/a = 1.39$, $L_0 = 36$ nH, $P_0 = 10$ torr and $C_0 = 777 \mu\text{F}$, and vary V_0 from 35 to 90 kV. The results are summarized in Table III and plotted in Figure 7 in log-log scale.

Figure 7 shows that $Y_n \sim V_0^{2.8}$ over the range of voltages examined from 35 to 90 kV. Looking at this scaling, it may at first sight be tempting to think in terms of increasing the voltage further. However, it is then necessary to look more closely at that prospect.

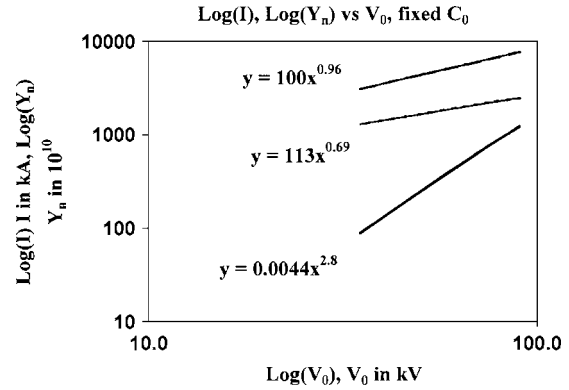
Table II. Numerical experiments with less resistive bank of RESF = 0.12.

E_0 (kJ)	C_0 (μ F)	a (cm)	z_0 (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_n (10^{10})
24 476	39 960	28.25	450.0	6774	2720	2255
18 765	30 636	27.44	430.0	6612	2622	1968
14 685	23 976	27.01	350.0	6488	2541	1711
11 421	18 646	26.34	300.0	6327	2449	1460
8159	13 320	25.26	260.0	6077	2314	1157
6527	10 656	24.62	210.0	5909	2236	974
4895	7992	23.61	160.0	5663	2130	777
4079	6660	22.83	140.0	5487	2060	668
3263	5328	21.72	110.0	5252	1976	558
2448	3996	20.36	93.0	4948	1866	437
1632	2664	18.40	75.0	4512	1711	303
816	1332	14.74	47.0	3726	1466	154
748	1221	14.26	44.0	3623	1436	140
680	1110	13.90	43.0	3528	1402	127
612	999	13.46	41.0	3419	1366	113
476	777	12.42	37.0	3161	1280	85.72
340	555	10.61	28.0	2763	1175	58.43
204	333	8.81	22.0	2312	1025	32.30
136	222	7.33	17.0	1954	913	19.46
68	111	5.45	12.0	1472	735	7.64

**Figure 6.** Log-log plots of Y_n (lower trace), I_{pinch} (middle trace) and I_{peak} (top trace) versus E_0 for a high-performance bank up to 25 MJ; computed from numerical experiments.**Table III.** Numerical experiments on effect of increasing V_0 , at fixed C_0 of 777 μ F.

V_0 (kV)	E_0 (kJ)	b (cm)	a (cm)	z_0 (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_n (10^{10})
90	3147	39.92	27.65	25	7580	2483	1228
70	1904	31.14	22.40	30	5955	2091	631
50	971	23.44	16.86	35	4365	1652	246
35	476	16.69	12.01	37	3074	1286	88

An examination of the computed results shows that the computed effective beam energy [6,7,29] for 90 kV is already at the 330 keV level. Looking at data for the

**Figure 7.** Scaling of currents and Y_n as functions of operating voltage V_0 . Top curve: $\text{Log}(I_{\text{peak}})$, middle curve: $\text{Log}(I_{\text{pinch}})$ and bottom curve: $\text{Log}(Y_n)$.

D-D cross-section [31] as a function of beam energy, it is seen that above 300 keV, the rise in D-D fusion cross-section is very slow. Hence, we wish to highlight that there is little advantage operating above 90 kV. In fact, the situation is actually disadvantageous to increase operating voltage if one considers changing to D-T operation. The D-T fusion cross-section [31] has already peaked at 120 keV; and operating at 90 kV with the beam energy at 330 keV, the beam energy is already too high; the D-T cross-section having dropped by a factor around 3.6 from its peak. It seems then that from this point of view there is no advantage to operate a plasma focus at higher than 90 kV. For conversion to D-T operation, it would probably be better to operate at a lower voltage. It would then be necessary to increase C_0 until 10^{15} D-T neutrons is reached.

3.4. Investigating operation at 90 kV, varying E_0 by varying C_0 ; at 10 torr, $L_0 = 36$ nH, and $b/a = 1.39$; RESF = 0.12

We consider the effect of operating at 90 kV. We run the fourth series of numerical experiments at 90 kV with increasing E_0 (by increasing C_0) to obtain the energy required to reach $Y_n = 10^{13}$ D-D neutrons per shot. At each C_0 , z_0 is varied while adjusting 'a' for an end axial speed of 10 cm/ μ s. The optimum z_0 is thus found for each C_0 . Results are shown in Figure 8. Again at this higher voltage, no saturation is found for I_{peak} , I_{pinch} or Y_n . At 90 kV, the results show that the E_0 required for $Y_n = 10^{13}$ D-D fusion neutrons per shot is reduced further to 3 MJ, with $C_0 = 777$ μ F as shown in Figure 8. The values of I_{peak} and I_{pinch} are 7.6 and 2.5 MA, respectively. Furthermore, at 90 kV with the highest value of C_0 investigated as 39 960 μ F, the storage energy is 162 MJ. At that storage energy, optimized Y_n is 4.5×10^{14} D-D neutrons/shot with $I_{\text{peak}} = 17.3$ MA and $I_{\text{pinch}} = 5.7$ MA.

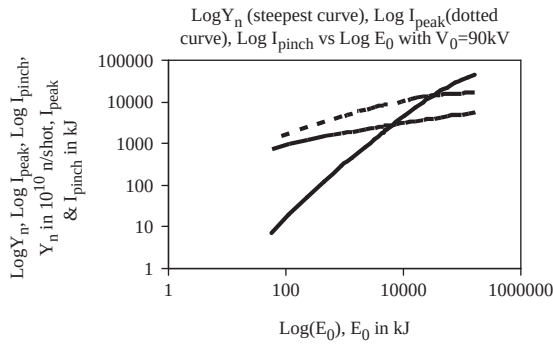


Figure 8. Numerical experiments at 90 kV, varying C_0 , to obtain scaling of I_{peak} , I_{pinch} and Y_n with E_0 . $\text{Log}(Y_n)$, steepest curve; $\text{Log}(I_{\text{peak}})$, dotted curve; $\text{Log}(I_{\text{pinch}})$, other curve. Y_n in units of 10^{10} D-D neutrons/shot; I_{peak} and I_{pinch} in kA.

4. CONCLUSIONS

This paper finds that it is possible to scale up the focus to useful fusion conditions. In the first series of numerical experiments, we have shown that as the bank energies of the plasma focus increase from 10 kJ to 25 MJ:

$$I_{\text{peak}} \sim E_0^x \text{ where } x = 4.7 \text{ at tens of kJ and } x = 0.22 \text{ at the highest energies (up to 25 MJ)}$$

$$I_{\text{pinch}} \sim E_0^x \text{ where } x = 4.1 \text{ at tens of kJ and } x = 0.22 \text{ at the highest energies (up to 25 MJ)}$$

$$Y_n \sim E_0^x, \text{ where } x = 2.0 \text{ at tens of kJ and } x = 0.84 \text{ at the highest energies (up to 25 MJ)}$$

Y_n does not saturate with increasing E_0 at the megajoule level and the resultant scaling laws for Y_n in relation to the total peak current, I_{peak} and the total pinch current I_{pinch} are

$$Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5} \text{ where } I_{\text{pinch}} (0.2 - 2.4) \text{ in MA}$$

$$Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8} \text{ where } I_{\text{peak}} (0.3 - 5.7) \text{ in MA}$$

To scale up from a PF1000-like capacitor bank requires close to 19 MJ to reach a target D-D neutron yield of 10^{13} per shot, this level of neutron yield being a desired landmark to achieve in a plasma focus device; from the point of view of possible exploitation as a powerful source of fusion neutrons for testing of prospective fusion reactor wall materials. The energy requirement can be reduced to 8 MJ using a modern capacitor bank with typical lower damping operating at typical voltage of 35 kV. By increasing the operational voltage to 90 kV, the energy requirement is further reduced to 3 MJ. Because of the high-effective beam energy already at 90 kV, there is little advantage to operate at voltages above 90 kV for D-D neutron yield, from the point of view of fusion cross-section.

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Review

Numerical Experiments Providing New Insights into Plasma Focus Fusion Devices

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Abstract: Recent extensive and systematic numerical experiments have uncovered new insights into plasma focus fusion devices including the following: (1) a plasma current limitation effect, as device static inductance is reduced towards very small values; (2) scaling laws of neutron yield and soft x-ray yield as functions of storage energies and currents; (3) a global scaling law for neutron yield as a function of storage energy combining experimental and numerical data showing that scaling deterioration has probably been interpreted as neutron ‘saturation’; and (4) a fundamental cause of neutron ‘saturation’. The ground-breaking insights thus gained may completely change the directions of plasma focus fusion research.

Keywords: plasma focus; numerical experiments; scaling laws; neutron saturation

1. Introduction

The Lee model code couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling a realistic simulation of all gross focus properties. The basic model, described in 1984 [1], was successfully used to assist several projects [2–4]. Radiation-coupled dynamics was included in the five-phase code, leading to numerical experiments on radiation cooling [5]. The vital role of a finite small disturbance speed discussed by Potter in a Z-pinch situation [6] was incorporated together with real gas thermodynamics and radiation-yield terms. This version of the code assisted other research projects [7–12] and was web published in 2000 [13] and 2005 [14]. Plasma

self-absorption was included in 2007 [13], improving the SXR yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [2,7,8,10,11,15–17], NX2 [9,12,18], and NX1 [18,19] and has been adapted for the Filippov-type plasma focus DENA [20]. A recent development is the inclusion of the neutron yield Y_n using a beam–target mechanism [21–25], incorporated in recent versions [26,27] of the code (versions later than RADPFV5.13), resulting in realistic Y_n scaling with I_{pinch} [21,22]. The versatility and utility of the model are demonstrated in its clear distinction of I_{pinch} from I_{peak} [28] and the recent uncovering of a plasma focus pinch current limitation effect [23,24,29] as static inductance is reduced towards zero. Extensive numerical experiments had been carried out systematically resulting in the uncovering of neutron [21–22,30–33] and SXR [30–37] scaling laws over a wider range of energies and currents than attempted before. The numerical experiments also gave insight into the nature and cause of ‘neutron saturation’ [31,33,38]. The description, theory, code, and a broad range of results of this “Universal Plasma Focus Laboratory Facility” are available for download from [26].

A brief description of the 5-phase model is given in the following.

2. The 5-Phase Lee Model Code

The five phases (*a-e*) are summarized [26,27,31,36,39] as follows:

a. Axial Phase (see Figure 1 left part): Described by a snowplow model with an equation of motion which is coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c . The mass swept-up factor f_m accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheet shock front structure, boundary layer effects, and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor f_c accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall, and current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.

b. Radial Inward Shock Phase (see Figure 1 right part, also Figure 2): Described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, (magnetic piston), allowing the current sheath to separate from the shock front by applying an adiabatic approximation [6]. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being particularly important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase using shock-jump equations. A communication delay between shock front and current sheath due to the finite small disturbance

speed [6,26,39] is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors f_{mr} and f_{cr} are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor f_{cr} accounts for the

fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

Figure 1. Schematic of the axial and radial phases. The left section depicts the axial phase, the right section the radial phase. In the left section, z is the effective position of the current sheath-shock front structure. In the right section r_s is the position of the inward moving shock front driven by the piston at position r_p . Between r_s and r_p is the radially imploding slug, elongating with a length z_f . The capacitor, static inductance and switch powering the plasma focus is shown for the axial phase schematic only.

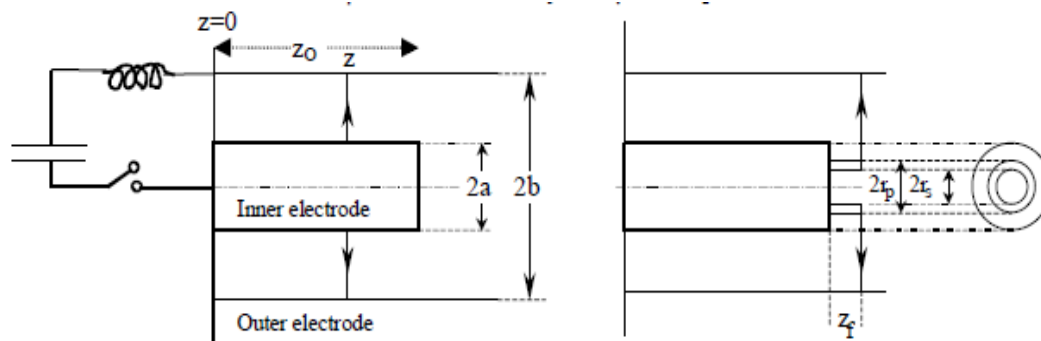
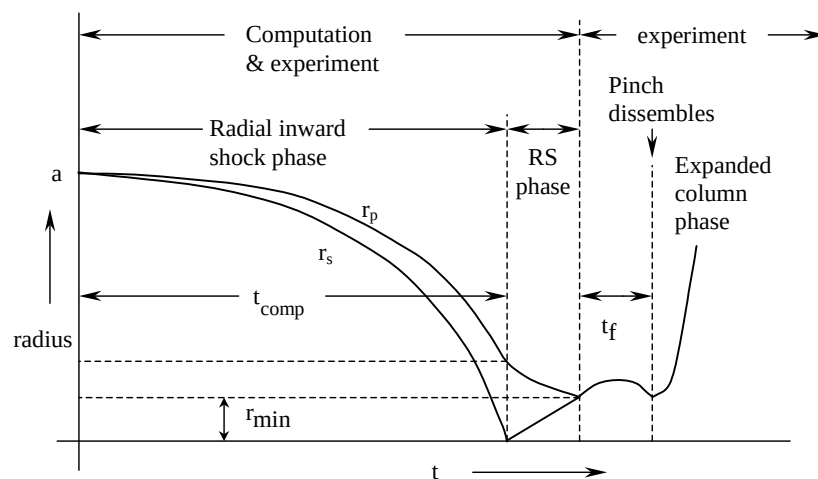


Figure 2. Schematic of radius *versus* time trajectories to illustrate the radial inward shock phase when r_s moves radially inwards, the reflected shock (RS) phase when the reflected shock moves radially outwards, until it hits the incoming piston r_p leading to the start of the pinch phase (t_f) and finally the expanded column phase.



c. Radial Reflected Shock (RS) Phase (See Figure 2): When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops which moves radially outwards, whilst the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the reflected shock moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the reflected shock undergoes a jump by a factor close to 2. Number densities are also computed using the reflected shock jump equations.

d. Slow Compression (Quiescent) or Pinch Phase (See Figure 2): When the out-going reflected shock hits the inward moving piston, the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.

e. Expanded Column Phase: To simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase the snowplow model is used, and two coupled equations are used similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

We note [36] that in radial phases *b*, *c* and *d*, axial acceleration and ejection of mass caused by necking curvatures of the pinching current sheath result in time-dependent strongly center-peaked density distributions. Moreover the transition from phase *d* to phase *e* is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These center-peaking density effects and localized regions are not modeled in the code, which consequently computes only an average uniform density and an average uniform temperature which are considerably lower than measured peak density and temperature. However, because the four model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modeled. Hence the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

2.1. From Measured Current Waveform to Modeling for Diagnostics

The Lee model code [26,27] is configured [21–24,26–39] to work as any plasma focus by inputting:

- Bank parameters, L_0 , C_0 and stray circuit resistance r_0 ;
- Tube parameters b , a and z_0 and
- Operational parameters V_0 and P_0 and the fill gas.

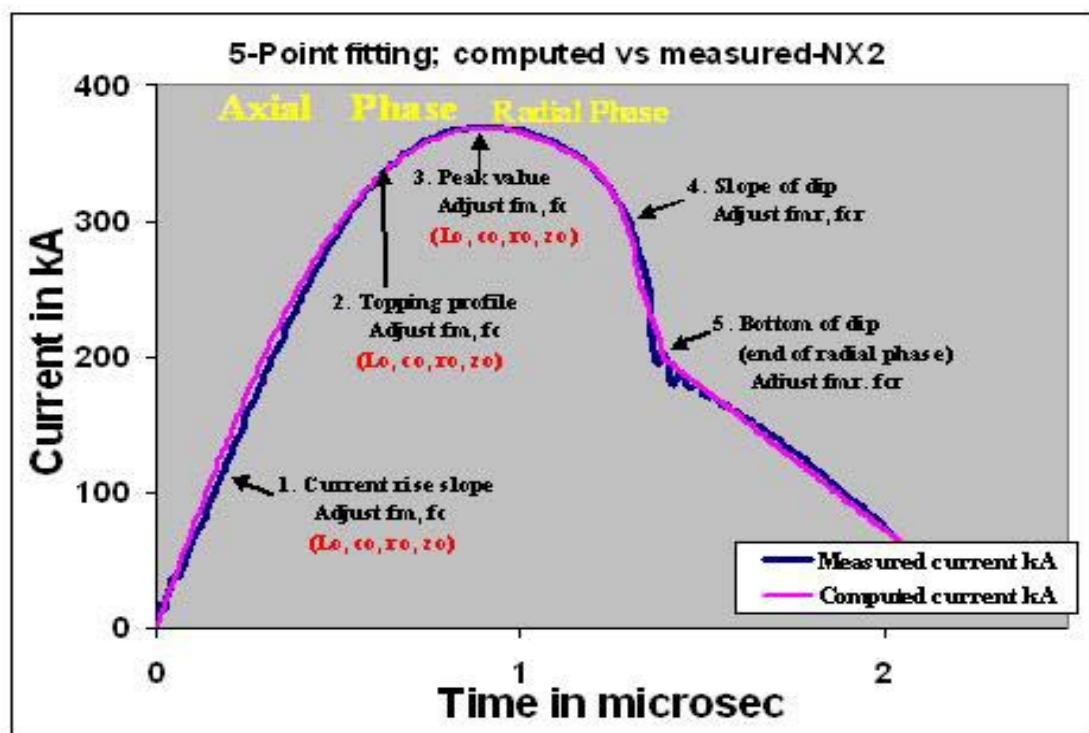
The computed total current waveform is fitted to the measured waveform by varying model parameters f_m , f_c , f_{mr} and f_{cr} one by one, until the computed waveform agrees with the measured waveform.

First, the axial model factors f_m , f_c are adjusted (fitted) until the features (1) computed rising slope of the total current trace and (2) the rounding off of the peak current as well as (3) the peak current itself are in reasonable (typically very good) fit with the measured total current trace (see Figure 3, measured trace fitted with computed trace).

Then we proceed to adjust (fit) the radial phase model factors f_{mr} and f_{cr} until features (4) the computed slope and (5) the depth of the dip agree with the measured. Note that the fitting of the computed trace with the measured current trace is done up to the end of the radial phase which is typically at the bottom of the current dip. Fitting of the computed and measured current traces beyond this point is not done. If there is significant divergence of the computed with the measured trace beyond the end of the radial phase, this divergence is not considered important.

In this case, after fitting the 5 features (1) to (5) above, the following fitted model parameters are obtained: $f_m = 0.1$, $f_c = 0.7$, $f_{mr} = 0.12$, $f_{cr} = 0.68$.

Figure 3. The 5-point fitting of computed current trace to the measured (or the reference) current trace. Point 1 is the current rise slope. Point 2 is the topping profile. Point 3 is the peak value of the current. Point 4 is the slope of the current dip. Point 5 is the bottom of the current dip. Fitting is done up to point 5 only. Further agreement or divergence of the computed trace with/from the measured trace is only incidental and not considered to be important.



From experience, it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace [26,27,31].

The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. There are many underlying mechanisms in the axial phase such as shock front and current sheet structure, porosity and inclination,

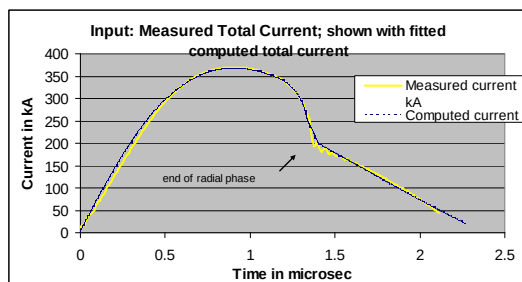
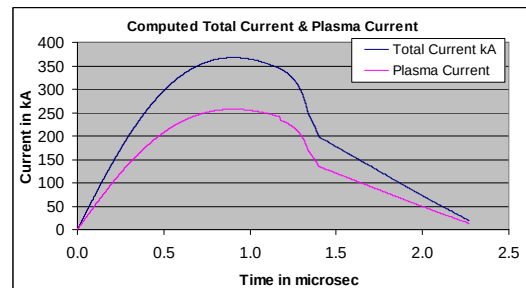
boundary layer effects and current shunting and fragmenting which are not simply modeled; likewise in the radial phase mechanisms such as current sheet curvatures and necking leading to axial acceleration and ejection of mass, and plasma/current disruptions. These effects may give rise to localized regions of high density and temperatures. The detailed profile of the discharge current is influenced by these effects and during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed total current trace to the measured total current trace in the procedure adopted by the Lee model code. Once matched, the fitted model parameters assure that the computation proceeds with all physical mechanisms accounted for, at least in the gross energy and mass balance sense.

2.2. Diagnostics-Time Histories of Dynamics, Energies and Plasma Properties Computed from the Measured Total Current Waveform by the Code

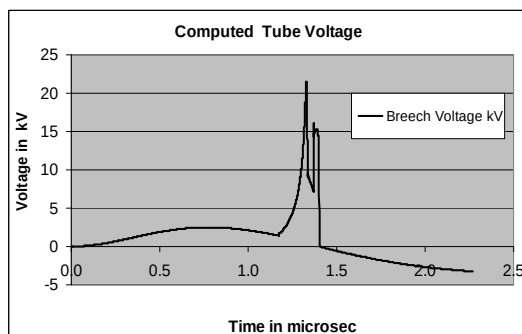
During every adjustment of each of the model parameters the code goes through the whole cycle of computation. In the last adjustment, when the computed total current trace is judged to be reasonably well fitted in all five waveform features, computed time histories are presented, in Figure 4a–o as an example, as follows: for the NX2 operated at 11 kV, 2.6 Torr neon [18,37].

2.3. Comments on Computed Quantities

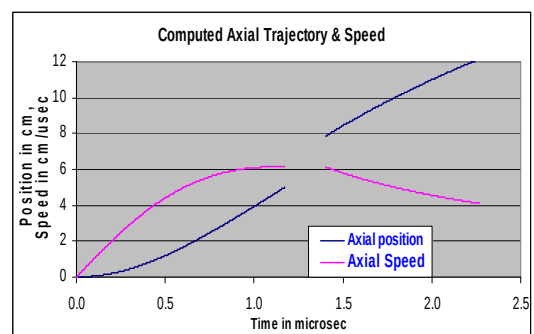
The computed total current trace typically agrees very well with the measured because of the fitting. The end of the radial phase is indicated in Figure 4a. Plasma currents are rarely measured. We had done a comparison of the computed plasma current with measured plasma current for the Stuttgart PF78 which shows good agreement of our computed to the measured plasma current [28]. The computed plasma current in this case of the NX2 is shown in Figure 4b. The computed tube voltage is difficult to compare with measured tube voltages in terms of peak values, typically because of poor response time of voltage dividers. However the computed waveform shape in Figure 4c. is generally as expected. The computed axial trajectory and speed, agree with experimentally obtained time histories. Moreover, the behavior with pressure, running the code again at different pressures, agrees well with experimental results. The radial trajectories and speeds are difficult to measure. The computed trajectories Figure 4e agree with the scant experimental data available. The length of the radial structure is shown in Figure 4f. Computed speeds radial shock front and piston speeds and speed of the elongation of the structure are shown in Figure 4g. The computed inductance (Figure 4h) shows a steady increase of inductance in the axial phase, followed by a sharp increase (rising by more than a factor of 2 in a radial phase time interval about 1/10 the duration of the axial phase for the NX2).

Figure 4. Computed plasma and electrodynamic properties of the NX2.(a) Fitted computed I_{total} .(b) Computed I_{total} & I_{plasma} .

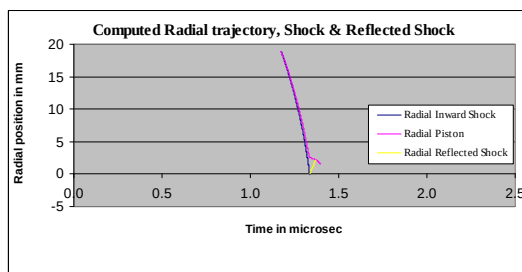
(c) Tube voltage.



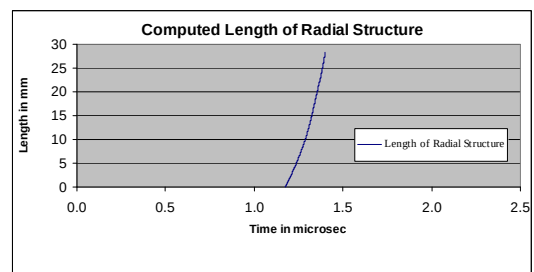
(d) Axial trajectory and speed.



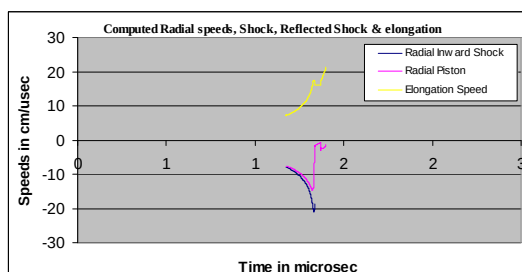
(e) Radial trajectories.



(f) Length of elongating structure.



(g) Speeds in radial phases.



(h) Tube inductance-axial & radial phases.

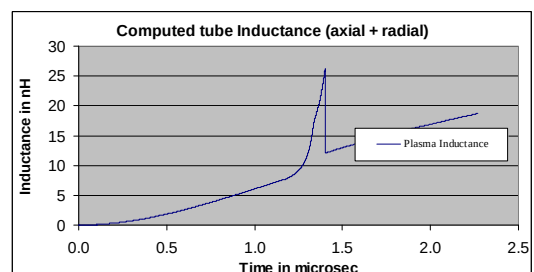
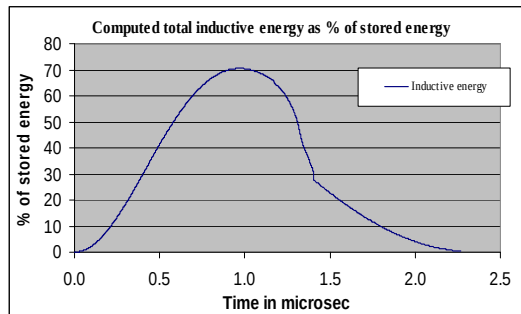
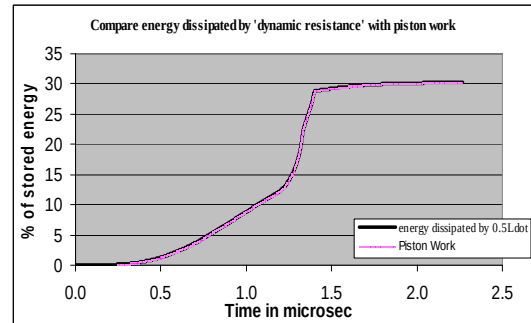


Figure 4. Cont.

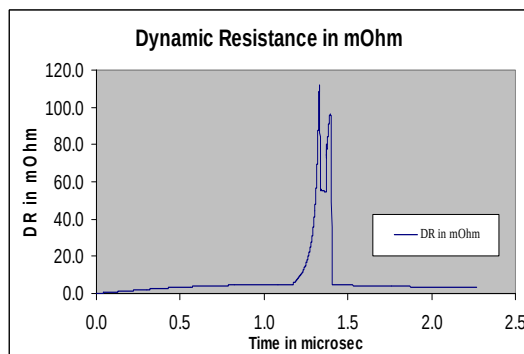
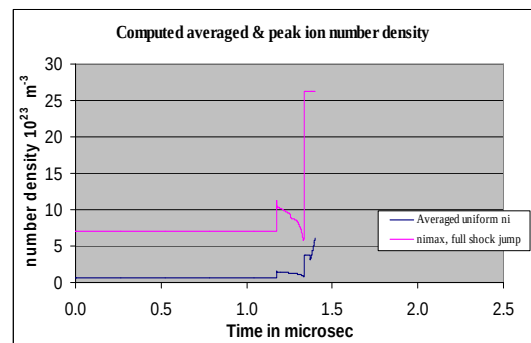
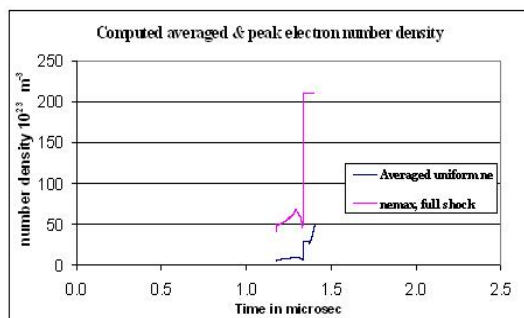
(i) Total inductive energy.



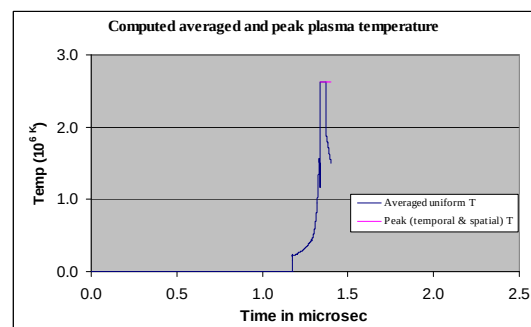
(j) Piston work and DR energy; both traces overlap.



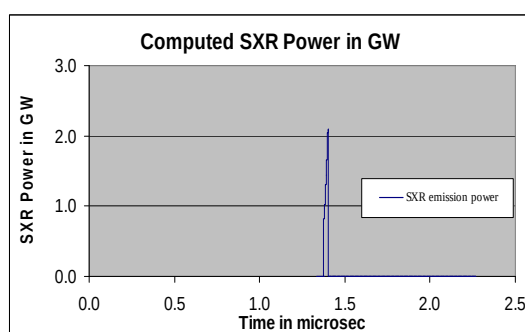
(k) DR axial and radial phases.

(l) Peak & averaged uniform n_i .(m) Peak & averaged uniform n_e .

(n) Peak and averaged uniform T.



(o) Neon Soft x-ray power.



The inductive energy ($0.5 LI^2$) peaks at 70% of initial stored energy, and then drops to 30% during the radial phase, as the sharp drop of current more than offsets the effect of sharply increased inductance (Figure 4i). In Figure 4j is shown the work done by the magnetic piston, computed using force integrated over distance method. Also shown is the work dissipated by the dynamic resistance, computed using dynamic resistance power integrated over time. We see that the two quantities and profiles agree exactly. This validates the concept of half $L\dot{I}$ as a dynamic resistance, DR (see Section 6.1). The piston work deposited in the plasma increases steadily to some 12% at the end of the axial phase and then rises sharply to just below 30% in the radial phase. Dynamic resistance is shown in Figure 4k. The values of the DR in the axial phase, together with the bank surge impedance, are the quantities that determine I_{peak} . The ion number density has a maximum value derived from shock-jump considerations, and an averaged uniform value derived from overall energy and mass balance considerations. The time profiles of these are shown in the Figure 4l. The electron number density (Figure 4m) has similar profiles to the ion density profile, but is modified by the effective charge numbers due to ionization stages reached by the ions. Plasma temperature too has a maximum value and an averaged uniform value derived in the same manner; are shown in Figure 4n. Computed neon soft x-ray power profile is shown in Figure 4o. The area of the curve is the soft x-ray yield in Joule. Pinch dimensions and lifetime may be estimated from Figure 4e,f. The model also computes the neutron yield, for operation in deuterium, using a phenomenological beam-target mechanism [25–27]. The model does not compute a time history of the neutron emission, only a yield number Y_n .

Thus as is demonstrated above, the model code when properly fitted is able to realistically model any plasma focus and act as a guide to diagnostics of plasma dynamics, trajectories, energy distribution and gross plasma properties.

Moreover, using such simulation, series of experiments have been systematically carried out to look for behavior patterns of the plasma focus. Insights uncovered by the series of experiments include: (i) pinch current limitation effect as static inductance is reduced; (ii) neutron and SXR scaling laws; (iii) a global scaling law for neutrons *versus* storage energy combining experimental and numerical experimental data; and (iv) insight into the nature and a fundamental cause of neutron saturation. These are significant achievements accomplished within a period of twenty months of intensive numerical experimentation.

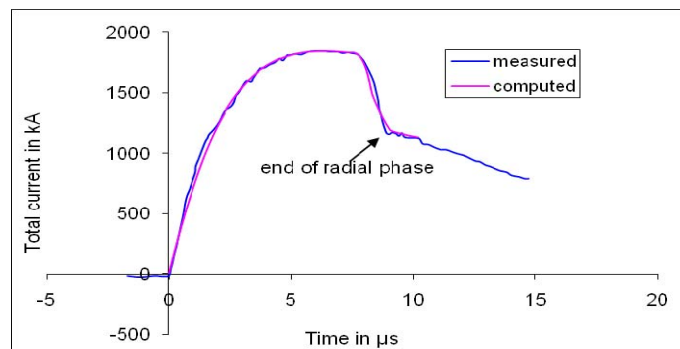
3. Insight 1-Pinch Current Limitation Effect as Static Inductance Is Reduced towards Zero

In a recent paper [25], there was expectation that the large MJ plasma focus PF1000 in Warsaw could increase its discharge current, and its pinch current, and consequently neutron yield by a reduction of its external or static inductance L_0 . To investigate this point, experiments were carried out using the Lee Model code. Unexpectedly, the results indicated that whilst I_{peak} indeed progressively increased with reduction in L_0 , no improvement may be achieved due to a pinch current limitation effect [23–24]. Given a fixed C_0 powering a plasma focus, there exists an optimum L_0 for maximum I_{pinch} . Reducing L_0 further will increase neither I_{pinch} nor Y_n . The numerical experiments leading to this unexpected result is described below.

A measured current trace of the PF1000 with $C_0 = 1332 \mu\text{F}$, operated at 27 kV, 3.5 torr deuterium, has been published [25], with cathode/anode radii $b = 16 \text{ cm}$, $a = 11.55 \text{ cm}$ and anode length

$z_0 = 60$ cm. In the numerical experiments we fitted external (or static) inductance $L_0 = 33.5$ nH and stray resistance $r_0 = 6.1$ m Ω (damping factor $RESF = r_0/(L_0/C_0)^{0.5} = 1.22$). The fitted model parameters are: $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The computed current trace [21,24,26] agrees very well with the measured trace through all the phases, axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase as shown in Figure 5.

Figure 5. Fitting computed current to measured current traces to obtain fitted parameters $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The measured current trace was for the PF1000 at 27 kV, storage capacity of 1332 μ F and fitted static inductance of 33.5 μ H.



We carried out numerical experiments for PF1000 using the machine and model parameters determined from Figure 5. Operating the PF1000 at 35 kV and 3.5 Torr, we varied the anode radius a with corresponding adjustment to b to maintain a constant $c = b/a = 1.39$ and in order to keep the peak axial speed at 10 cm/ μ s. The anode length z_0 was also adjusted to maximize I_{pinch} as L_0 was decreased from 100 nH progressively to 5 nH.

As expected, I_{peak} increased progressively from 1.66 to 4.4 MA. As L_0 was reduced from 100 to 35 nH, I_{pinch} also increased, from 0.96 to 1.05 MA. However, then unexpectedly, on further reduction from 35 to 5 nH, I_{pinch} stopped increasing, instead decreasing slightly to 1.03 MA at 20 nH, to 1.0 MA at 10 nH, and to 0.97 MA at 5 nH. Y_n also had a maximum value of 3.2×10^{11} at 35 nH.

To explain this unexpected result, we examine the energy distribution in the system at the end of the axial phase (see Figure 5) just before the current drops from peak value I_{peak} and then again near the bottom of the almost linear drop to the pinch phase indicated by the arrow pointing to ‘end of radial phase’. The energy equation describing this current drop is written as follows:

$$0.5I_{peak}^2(L_0 + L_a f_c^2) = 0.5I_{pinch}^2(L_0/f_c^2 + L_a + L_p) + \delta_{cap} + \delta_{plasma} \quad (1)$$

where L_a is the inductance of the tube at full axial length z_0 , δ_{plasma} is the energy imparted to the plasma as the current sheet moves to the pinch position and is the integral of $0.5(dL/dt)I^2$. We approximate this as $0.5L_p I_{pinch}^2$ which is an underestimate for this case. δ_{cap} is the energy flow into or out of the capacitor during this period of current drop. If the duration of the radial phase is short compared to the capacitor time constant, the capacitor is effectively decoupled and δ_{cap} may be put as zero. From this consideration we obtain

$$I_{pinch}^2 = I_{peak}^2(L_0 + 0.5L_a)/(2L_0 + L_a + 2L_p) \quad (2)$$

where we have taken $f_c = 0.7$ and approximated f_c^2 as 0.5.

Generally, as L_0 is reduced, I_{peak} increases; a is necessarily increased leading [7] to a longer pinch length z_p , hence a bigger L_p . Lowering L_0 also results in a shorter rise time, hence a necessary decrease in z_0 , reducing L_a . Thus, from Equation (2), lowering L_0 decreases the fraction I_{pinch} / I_{peak} . Secondly, this situation is compounded by another mechanism. As L_0 is reduced, the L - C interaction time of the capacitor bank reduces while the duration of the current drop increases (see Figure 6, discussed in the next section) due to an increasing a . This means that as L_0 is reduced, the capacitor bank is more and more coupled to the inductive energy transfer processes with the accompanying induced large voltages that arise from the radial compression. Looking again at the derivation of Equation (2) from Equation (1) a nonzero δ_{cap} , in this case, of positive value, will act to decrease I_{pinch} further. The lower the L_0 the more pronounced is this effect.

Summarizing this discussion, the pinch current limitation is not a simple effect, but is a combination of the two complex effects described above, namely, the interplay of the various inductances involved in the plasma focus processes abetted by the increasing coupling of C_0 to the inductive energetic processes, as L_0 is reduced.

3.1. Optimum L_0 for Maximum Pinch Current and Neutron Yield

From the pinch current limitation effect, it is clear that given a fixed C_0 powering a plasma focus, there exists an optimum L_0 for maximum I_{pinch} . Reducing L_0 further will increase neither I_{pinch} nor Y_n . The results of the numerical experiments carried out are presented in Figure 6 and Table 1.

With large $L_0 = 100$ nH it is seen (Figure 6) that the rising current profile is flattened from what its waveform would be if unloaded; and peaks at around $12 \mu s$ (before its unloaded rise time, not shown, of $18 \mu s$) as the current sheet goes into the radial phase. The current drop, less than 25% of peak value, is sharp compared with the current rise profile. At $L_0 = 30$ nH the rising current profile is less flattened, reaching a flat top at around $5 \mu s$, staying practically flat for some $2 \mu s$ before the radial phase current drop to 50% of its peak value in a time which is still short compared with the rise time. With L_0 of 5 nH, the rise time is now very short, there is hardly any flat top; as soon as the peak is reached, the current waveform droops significantly. There is a small kink on the current waveform of both the $L_0 = 5$ nH, $z_0 = 20$ cm and the $L_0 = 5$ nH, $z_0 = 40$ cm. This kink corresponds to the start of the radial phase which, because of the large anode radius, starts with a relatively low radial speed, causing a momentary reduction in dynamic loading. Looking at the three types of traces it is seen that for $L_0 = 100$ nH to 30 nH, there is a wide range of z_0 that may be chosen so that the radial phase may start at peak or near peak current, although the longer values of z_0 tend to give better energy transfers into the radial phase.

The optimized situation for each value of L_0 is shown in Table 1. The table shows that as L_0 is reduced, I_{peak} rises with each reduction in L_0 with no sign of any limitation. However, I_{pinch} reaches a broad maximum of 1.05 MA around 40–30 nH. Neutron yield Y_n also shows a similar broad maximum peaking at 3.2×10^{11} neutrons. Figure 7 shows a graphical representation of this I_{pinch} limitation effect. The curve going up to 4 MA at low L_0 is the I_{peak} curve. Thus I_{peak} shows no sign of limitation as L_0 is progressively reduced. However I_{pinch} reaches a broad maximum. From Figure 7 there is a stark and important message: one must distinguish clearly between I_{peak} and I_{pinch} . In general one cannot take I_{peak} to be representative of I_{pinch} .

Figure 6. PF1000 current waveforms computed at 35 kV, 3.5 Torr D₂ for a range of L_0 showing the changes in waveforms as L_0 varies.

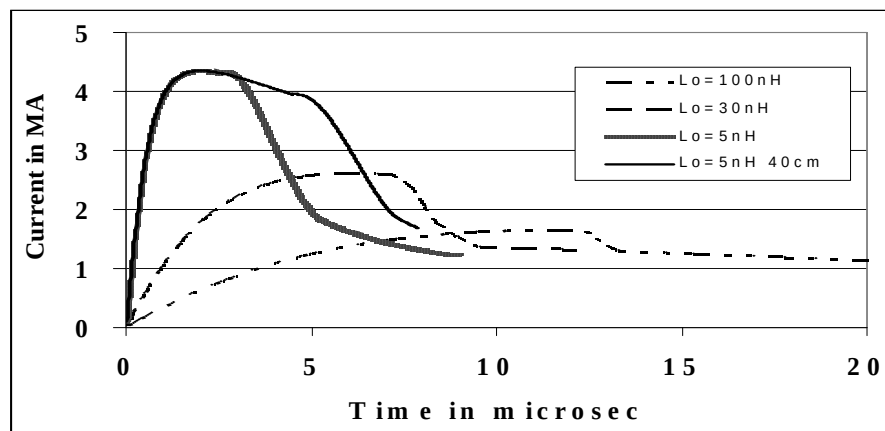
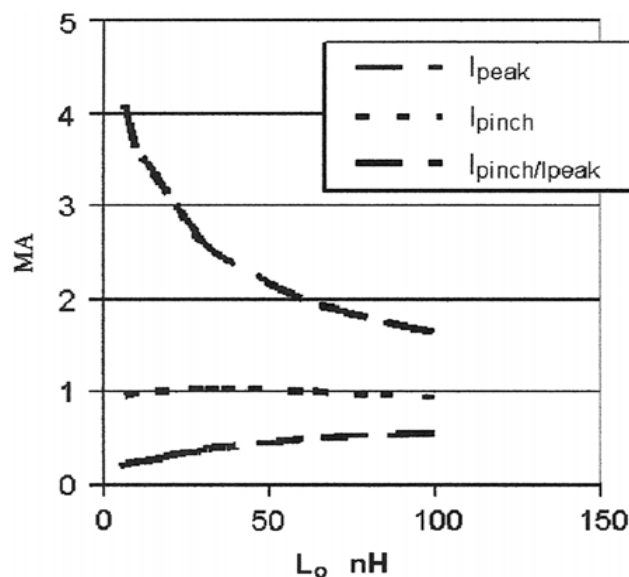


Table 1. Effect on currents and ratio of currents as L_0 is reduced-PF1000 at 35kV, 3.5 Torr Deuterium.

L_0 (nH)	b (cm)	a (cm)	z_0 (cm)	I_{peak} (MA)	I_{pinch} (M)	$Y_n(10^{11})$	I_{pinch}/I_{peak}
100	15.0	10.8	80	1.66	0.96	2.44	0.58
80	16.0	11.6	80	1.81	1.00	2.71	0.55
60	18.0	13.0	70	2.02	1.03	3.01	0.51
40	21.5	15.5	55	2.36	1.05	3.20	0.44
35	22.5	16.3	53	2.47	1.05	3.20	0.43
30	23.8	17.2	50	2.61	1.05	3.10	0.40
20	28.0	21.1	32	3.13	1.03	3.00	0.33
10	33.0	23.8	28	3.65	1.00	2.45	0.27
5	40.0	28.8	20	4.37	0.97	2.00	0.22

Figure 7. Effect on currents and current ratio (computed) as L_0 is reduced PF1000, 35 kV, 3.5 torr D₂.



We carried out several sets of experiments on the PF1000 for varying L_0 , each set with a different damping factor. In every case, an optimum inductance was found around 30–60 nH with I_{pinch} decreasing as L_0 was reduced below the optimum value. The results showed that for PF1000, reducing L_0 from its present 20–30 nH will increase neither the observed I_{pinch} nor the neutron yield, because of the pinch limitation effect. Indeed, the I_{pinch} decreases very slightly on further reduction to very small values. We would add that we have used a set of model parameters which in our experience is the most reasonable to be used in these numerical experiments. Variations of the model parameters could occur but we are confident that these variations are not likely to occur with such a pattern as to negate the pinch current limitation effect. Nevertheless these variations should be actively monitored and any patterns in the variations should be investigated.

4. Insight 2-Scaling Laws for Neutron

4.1. Computation of Neutron Yield-describing the Beam-target Mechanism

The neutron yield is computed using a phenomenological beam-target neutron generating mechanism described recently by Gribkov *et al.* [25] and adapted to yield the following equation. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. The beam-target yield is derived [21–24,26] as:

$$Y_{b-t} = C_n n_i I_{\text{pinch}}^2 z_p^2 (\ln(b/r_p)) \sigma / U^{0.5} \quad (3)$$

where n_i is the ion density, b is the cathode radius, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n- branch [40] and U , the beam energy. C_n is treated as a calibration constant combining various constants in the derivation process.

The D-D cross-section is sensitive to the beam energy in the range 15–150 keV; so it is necessary to use the appropriate range of beam energy to compute σ . The code computes induced voltages (due to current motion inductive effects) V_{max} of the order of only 15–50 kV. However it is known, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50–150 keV [25], and for smaller lower-voltage machines the relevant energy could be lower at 30–60 keV [16]. Thus in line with experimental observations the D-D cross section σ is reasonably obtained by using $U = 3V_{\text{max}}$. This fit was tested by using U equal to various multiples of V_{max} . A reasonably good fit of the computed neutron yields to the measured published neutron yields at energy levels from sub-kJ to near MJ was obtained when the multiple of 3 was used; with poor agreement for most of the data points when for example a multiple of 1 or 2 or 4 or 5 was used. The model uses a value of $C_n = 2.7 \times 10^7$ obtained by calibrating the yield [21–23,26], at an experimental point of 0.5 MA.

The thermonuclear component is also computed in every case and it is found that this component is negligible when compared with the beam-target component. It might be argued that an adjustment to the thermonuclear component could also be attempted in a similar way to the usage of the multiple to V_{max} . However, the usage of the multiple to V_{max} has some experimental basis due to ion energy measurements. Moreover the value of V_{max} in each numerical experiment is calculated from the slug model leading to the slow compression phase whilst it is known experimentally that after the slow compression phase, instability effects set in which will increase the electric fields operating within the

pinch. These are the basic arguments supporting the view that the operational beam energy has a value above V_{max} . For the thermonuclear component a feasible model to adjust the yield upwards has yet to be suggested.

4.2. Scaling Laws for Neutrons from Numerical Experiments over a Range of Energies from 10 kJ to 25 MJ

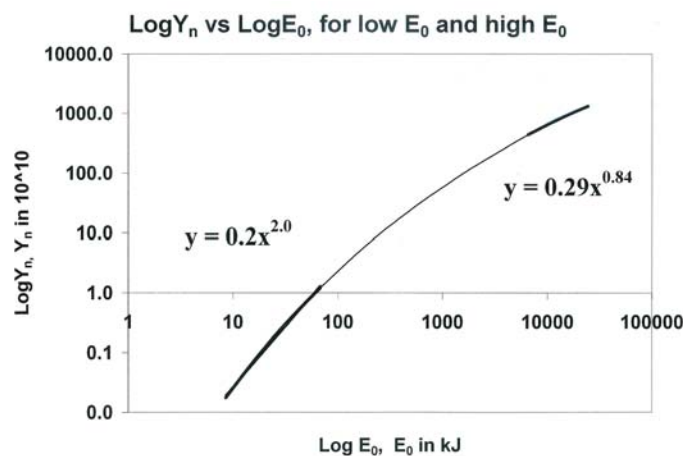
We apply the Lee model code to the MJ machine PF1000 over a range of C_0 to study the neutrons emitted by PF1000-like bank energies from 10 kJ to 25 MJ.

As shown earlier the PF1000 current trace has been used to fit the model parameters, with very good fitting achieved between the computed and measured current traces (Figure 5). Once the model parameters have been fitted to a machine for a given gas, these model parameters may be used with some degree of confidence when operating parameters such as the voltage are varied [26,27]. With no measured current waveforms available for the higher megajoule numerical experiments, it is reasonable to keep the model parameters that we have got from the PF1000 fitting.

The optimum pressure for this series of numerical experiments is 10 torr and the ratio $c = b/a$ is retained at 1.39. For each C_0 , anode length z_0 is varied to find the optimum. For each z_0 , anode radius a_0 is varied so that the end axial speed is 10 cm/ μ s. The numerical experiments were carried out for C_0 ranging from 14 μ F to 39,960 μ F corresponding to energies from 8.5 kJ to 24.5 MJ [22].

For this series of experiments we find that the Y_n scaling changes from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25 MJ) investigated in this series. This is shown in Figure 8.

Figure 8. Y_n plotted as a function of E_0 in log-log scale, showing Y_n scaling changes from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25 MJ).



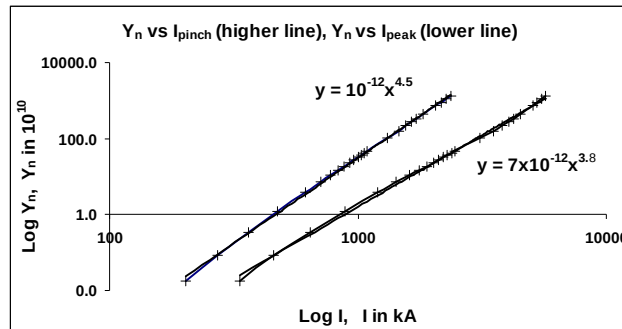
The scaling of Y_n with I_{peak} and I_{pinch} over the whole range of energies investigated up to 25 MJ (shown in Figure 9) are as follows:

$$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5} \text{ and}$$

$$Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}$$

where I_{peak} ranges from 0.3 MA to 5.7 MA and I_{pinch} ranges from 0.2 MA to 2.4 MA.

Figure 9. $\text{Log}(Y_n)$ scaling with $\text{Log}(I_{\text{peak}})$ and $\text{Log}(I_{\text{pinch}})$, for the range of energies investigated, up to 25 MJ.



This compares to an earlier study carried out on several machines with published current traces and Y_n yield measurements, operating conditions and machine parameters including the Chilean PF400J, the UNU/ICTP PFF, the NX2 and Poseidon providing a slightly higher scaling laws: $Y_n \sim I_{\text{pinch}}^{4.7}$ and $Y_n \sim I_{\text{peak}}^{3.9}$. The slightly higher value of the scaling is because those machines fitted are of mixed 'c' mixed bank parameters, mixed model parameters and currents generally below 1 MA and voltages generally below the 35 kV [21].

Summarising: Over wide ranges of energy, optimizing pressure, anode length and radius, the scaling laws for Y_n [15] are listed here:

$$Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5}; Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8}; I_{\text{peak}} (0.3 \text{ to } 5.7), I_{\text{pinch}} (0.2 \text{ to } 2.4) \text{ in MA.}$$

$Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at MJ level (up to 25MJ).

These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as optimized neutron sources.

5. Insight 3-Scaling Laws for Soft X-ray Yield

5.1. Computation of Neon SXR Yield- the Equations Used in the Computation

We note that the transition from Phase 4 to Phase 5 is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These localized regions are not modeled in the code, which consequently computes only an average uniform density, and an average uniform temperature which are considerably lower than measured peak density and temperature. However, because the 4 model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modeled. Hence the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

In the code [8,9,26–27], neon line radiation Q_L is calculated as follows:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} n_i^2 Z Z_n^4 (\pi r_p^2) z_f / T \quad (4)$$

where for the temperatures of interest in our experiments we take the SXR yield $Y_{sxr} = Q_L \cdot Z_n$ is the atomic number.

Hence the SXR energy generated within the plasma pinch depends on the properties: number density n_i , effective charge number Z , pinch radius r_p , pinch length z_f and temperature T . It also depends on the pinch duration since in our code Q_L is obtained by integrating over the pinch duration.

This generated energy is then reduced by the plasma self-absorption which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the SXR yield. These effects are included in the modeling by computing volumetric plasma self-absorption factor A derived from the photonic excitation number M which is a function of Z_n , n_i , Z and T . However, in our range of operation, the numerical experiments show that the self absorption is not significant. It was first pointed out by Liu Mahe [8,11,18] that a temperature around 300 eV is optimum for SXR production. Shan Bing's subsequent work [9] and our experience through numerical experiments suggest that around 2×10^6 K (below 200 eV) or even a little lower could be better. Hence unlike the case of neutron scaling, for SXR scaling there is an optimum small range of temperatures (T windows) to operate.

5.2. Scaling Laws for Neon Sxr from Numerical Experiments over A Range of Energies from 0.2 kJ to 1 MJ

We next use the Lee model code to carry out a series of numerical experiments over the energy range 0.2 kJ to 1 MJ [30,34]. In this case we apply it to a proposed modern fast plasma focus machine with optimized values for c the ratio of the outer to inner electrode radius and L_0 obtained from our numerical experiments.

The following parameters are kept constant : (i) the ratio $c = b/a$ (kept at 1.5, which is practically optimum according to our preliminary numerical trials; (ii) the operating voltage V_0 (kept at 20 kV); (iii) static inductance L_0 (kept at 30 nH, which is already low enough to reach the I_{pinch} limitation regime [23,24] over most of the range of E_0 we are covering) and; (iv) the ratio of stray resistance to surge impedance $RESF$ (kept at 0.1, representing a higher performance modern capacitor bank). The model parameters [26,27] f_m , f_c , f_{mr} , f_{cr} are also kept at fixed values 0.06, 0.7, 0.16 and 0.7. We choose the model parameters so they represent the average values from the range of machines that we have studied. A typical example of a current trace for these parameters is shown in Figure 10.

The storage energy E_0 is varied by changing the capacitance C_0 . Parameters that are varied are operating pressure P_0 , anode length z_0 and anode radius a . Parametric variation at each E_0 follows the order; P_0 , z_0 and a until all realistic combinations of P_0 , z_0 and a are investigated. At each E_0 , the optimum combination of P_0 , z_0 and a is found that produces the biggest Y_{sxr} . In other words at each E_0 , a P_0 is fixed, a z_0 is chosen and a is varied until the largest Y_{sxr} is found. Then keeping the same values of E_0 and P_0 , another z_0 is chosen and a is varied until the largest Y_{sxr} is found. This procedure is repeated until for that E_0 and P_0 , the optimum combination of z_0 and a is found. Then keeping the same value of E_0 , another P_0 is selected. The procedure for parametric variation of z_0 and a as described above is then carried out for this E_0 and new P_0 until the optimum combination of z_0 and a is

found. This procedure is repeated until for a fixed value of E_0 , the optimum combination of P_0 , z_0 and a is found.

The procedure is then repeated with a new value of E_0 . In this manner after systematically carrying out some 2000 runs, the optimized runs for various energies are tabulated in Table 2.

Figure 10. Computed total current versus time for $L_0 = 30$ nH and $V_0 = 20$ kV, $C_0 = 30$ μ F, $RESF = 0.1$, $c = 1.5$ and model parameters f_m , f_c , f_{mr} , f_{cr} are fixed at 0.06, 0.7, 0.16 and 0.7 for optimized $a = 2.285$ cm and $z_0 = 5.2$ cm.

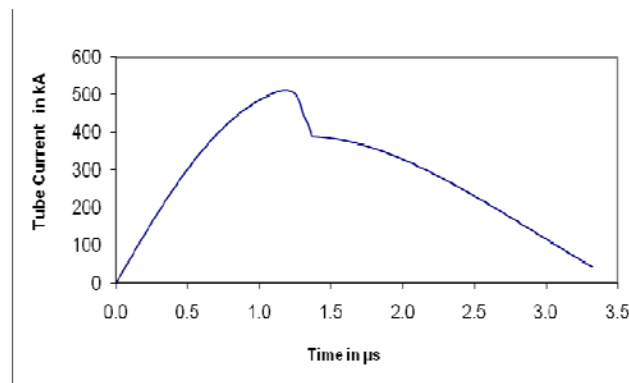


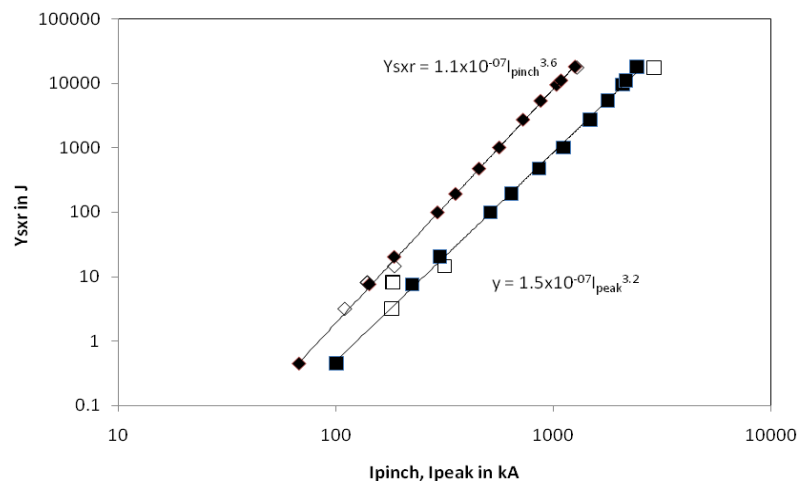
Table 2. Optimized configuration found for each E_0 . Optimization carried out with $RESF = 0.1$, $c = 1.5$, $L_0 = 30$ nH and $V_0 = 20$ kV and model parameters f_m , f_c , f_{mr} , f_{cr} are fixed at 0.06, 0.7, 0.16 and 0.7, respectively. The v_a , v_s and v_p are the peak axial, radial shock and radial piston speeds, respectively.

E_0 (kJ)	C_0 (μ F)	a (cm)	z_0 (cm)	P_0 (Torr)	I_{peak} (kA)	I_{pinch} (kA)	v_a (cm/ μ s)	v_s (cm/ μ s)	v_p (cm/ μ s)	Y_{srx} (J)
0.2	1	0.58	0.5	4.0	100	68	5.6	22.5	14.9	0.44
1	5	1.18	1.5	4.0	224	143	6.6	23.3	15.1	7.5
2	10	1.52	2.1	4.0	300	186	6.8	23.6	15.2	20
6	30	2.29	5.2	4.2	512	294	8.1	24.5	15.6	98
10	50	2.79	7.5	4.0	642	356	8.7	24.6	15.7	190
20	100	3.50	13	4.0	861	456	9.6	24.6	16.0	470
40	200	4.55	20	3.5	1109	565	10.3	24.7	16.2	1000
100	500	6.21	42	3.0	1477	727	11.2	24.8	16.4	2700
200	1000	7.42	63	3.0	1778	876	11.4	24.8	16.5	5300
400	2000	8.70	98	3.0	2079	1036	11.4	24.9	16.5	9400
500	2500	9.10	105	2.9	2157	1086	11.5	25.1	16.7	11,000
1000	5000	10.2	160	3.0	2428	1261	11.4	25.2	16.7	18,000

We then plot Y_{srx} against I_{peak} and I_{pinch} and obtain SXR yield scales as $Y_{srx} \sim I_{pinch}^{3.6}$ and $Y_{srx} \sim I_{peak}^{3.2}$. The I_{pinch} scaling has less scatter than the I_{peak} scaling. We next subject the scaling to further test when the fixed parameters $RESF$, c , L_0 and V_0 and model parameters f_m , f_c , f_{mr} , f_{cr} are varied. We add in the results of some numerical experiments using the parameters of several existing plasma focus devices including the UNU/ICTP PFF ($RESF = 0.2$, $c = 3.4$, $L_0 = 110$ nH and $V_0 = 14$ kV with fitted model parameters $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.2$, $f_{cr} = 0.8$) [8,26,27,36], the NX2 ($RESF = 0.1$,

$c = 2.2$, $L_0 = 20$ nH and $V_0 = 11$ kV with fitted model parameters $f_m = 0.10$, $f_c = 0.7$, $f_{mr} = 0.12$, $f_{cr} = 0.68$) [9,26,27,34,37] and PF1000 ($RESF = 0.1$, $c = 1.39$, $L_0 = 33$ nH and $V_0 = 27$ kV with fitted model parameters $f_m = 0.1$, $f_c = 0.7$, $f_{mr} = 0.15$, $f_{cr} = 0.7$) [26,27,23]. These new data points (white data points in Figure 11) contain wide ranges of c , V_0 , L_0 and model parameters. The resulting Y_{srx} versus I_{pinch} log-log curve remains a straight line, with the scaling index 3.6 unchanged and with no more scatter than before. However the resulting Y_{srx} versus I_{peak} curve now exhibits considerably larger scatter and the scaling index has changed.

Figure 11. Y_{srx} is plotted as a function of I_{pinch} and I_{peak} . The parameters kept constant for the black data points are: $RESF = 0.1$, $c = 1.5$, $L_0 = 30$ nH and $V_0 = 20$ kV and model parameters f_m , f_c , f_{mr} , f_{cr} at 0.06, 0.7, 0.16 and 0.7 respectively. The white data points are for specific machines which have different values for the parameters c , L_0 and V_0 .



We would like to highlight that the consistent behavior of I_{pinch} in maintaining the scaling of $Y_{srx} \sim I_{pinch}^{3.6}$ with less scatter than the $Y_{srx} \sim I_{peak}^{3.2}$ scaling particularly when mixed-parameters cases are included, strongly support the conclusion that I_{pinch} scaling is the more universal and robust one. Similarly conclusions on the importance of I_{pinch} in plasma focus performance and scaling laws have been reported [21–24,26–28].

It may also be worthy to note that our comprehensively surveyed numerical experiments for Mather configurations in the range of energies 0.2 kJ to 1 MJ produce an I_{pinch} scaling rule for Y_{srx} not compatible with Gates' rule [41]. However it is remarkable that our I_{pinch} scaling index of 3.6, obtained through a set of comprehensive numerical experiments over a range of 0.2 kJ to 1 MJ, on Mather-type devices, is within the range of 3.5 to 4 postulated on the basis of sparse experimental data, (basically just two machines one at 5 kJ and the other at 0.9 MJ), by Filippov [42], for Filippov configurations in the range of energies 5 kJ to 1 MJ.

It must be pointed out that the results represent scaling for comparison with baseline plasma focus devices that have been optimized in terms of electrode dimensions. It must also be emphasized that the scaling with I_{pinch} works well even when there are some variations in the actual device from $L_0 = 30$ nH, $V_0 = 20$ kV and $c = 1.5$.

Summarising: over wide ranges of energy, optimizing pressure, anode length and radius, the scaling laws for neon SXR are found to be:

$$Y_{sxr} = 8.3 \times 10^3 \times I_{pinch}^{3.6}$$

$$Y_{sxr} = 600 \times I_{peak}^{3.2}$$

where I_{peak} (0.1 to 2.4), I_{pinch} (0.07 to 1.3) in MA, $Y_{sxr} \sim E_0^{1.6}$ (kJ range) to $Y_{sxr} \sim E_0^{0.8}$ (towards MJ).

These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as neon SXR sources.

6. Insight 4-Neutron Saturation

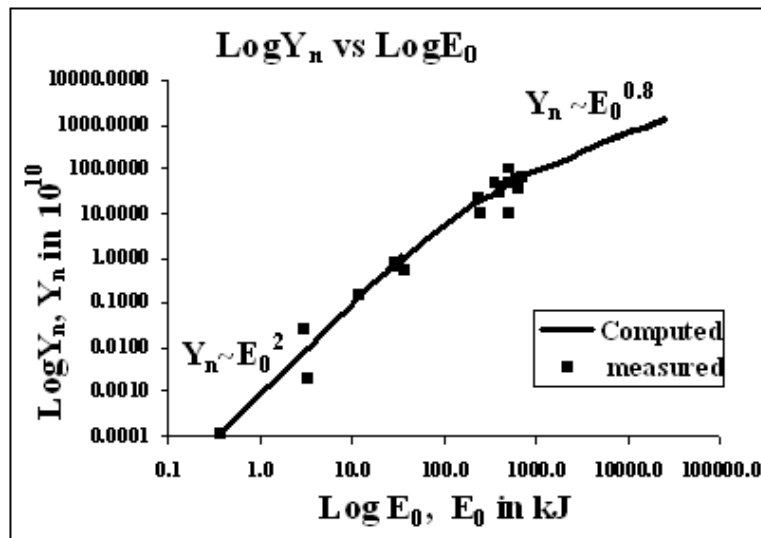
Besides being accurately descriptive and related to wide-ranging experimental reality, desirable characteristics of a model include predictive and extrapolative scaling. Moreover a useful model should be accessible, usable and user-friendly and should be capable of providing insights. Insight however cannot be a characteristic of the model in isolation, but is the interactive result of the model with the modeler or model user.

It was observed early in plasma focus research [43] that neutron yield $Y_n \sim E_0^2$ where E_0 is the capacitor storage energy. Such scaling gave hopes of possible development as a fusion energy source. Devices were scaled up to higher E_0 . It was then observed that the scaling deteriorated, with Y_n not increasing as much as suggested by the E_0^2 scaling. In fact some experiments were interpreted as evidence of a neutron saturation effect [44,45] as E_0 approached several hundreds of kJ. As recently as 2006 Krauz [46] and November 2007, Scholz [47] have questioned whether the neutron saturation was due to a fundamental cause or to avoidable machine effects such as incorrect formation of plasma current sheath arising from impurities or sheath instabilities [45]. We should note here that the region of discussion (several hundreds of kJ approaching the MJ region) is in contrast to the much higher energy region discussed by Schmidt at which there might be expected to be a decrease in the role of beam target fusion processes [45,48].

6.1. The Global Neutron Scaling Law

Recent extensive numerical experiments [21,22] also showed that whereas at energies up to tens of kJ the $Y_n \sim E_0^2$ scaling held, deterioration of this scaling became apparent above the low hundreds of kJ. This deteriorating trend worsened and tended towards $Y_n \sim E_0^{0.8}$ at tens of MJ. The results of these numerical experiments are summarized in Figure 1 with the solid line representing results from numerical experiments. Experimental results from 0.4 kJ to MJ, compiled from several available published sources [43,45–47,49–52], are also included as squares in the same figure. The combined experimental and numerical experimental results [31–33,38] (Figure 12) appear to have general agreement particularly with regards to the $Y_n \sim E_0^2$ at energies up to 100 kJ, and the deterioration of the scaling from low hundreds of kJ to the 1 MJ level. The global data of Figure 12 suggests that the apparently observed neutron saturation effect is overall not in significant variance with the deterioration of the scaling shown by the numerical experiments.

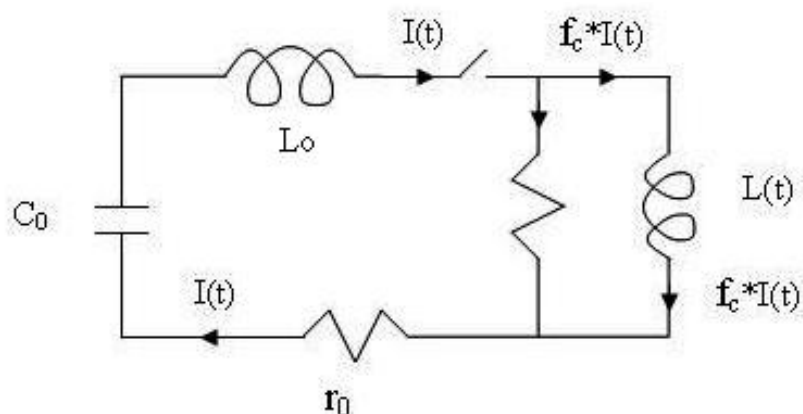
Figure 12. The global scaling law, combining experimental and numerical data. The global data illustrates Y_n scaling deterioration observed in numerical experiments from 0.4 kJ to 25 MJ (solid line) using the Lee model code, compared to measurements compiled from publications (squares) of various machines from 0.4 kJ to 1 MJ.



6.2. The Cause of Neutron ‘Saturation’ is the Dynamic Resistance

A simple yet compelling analysis of the cause of this neutron saturation has been published [38]. In Figure 1 left side is shown a schematic of the plasma dynamics in the axial phase of the Mather-type plasma focus. In that work the simplest representation was used, in which the current sheet is shown to go from the anode to the cathode perpendicularly. Observation shows that there is actually a canting of the current sheet [53] and also that only a fraction (typically 0.7) of the total current participates in driving the current sheet. These points are accounted for in the modeling by model parameters f_m and f_c . We now represent the plasma focus circuit as shown in Figure 13.

Figure 13. Plasma focus circuit schematic. The capacitor bank with static inductance L_0 and stray resistance r_0 is switched into the plasma focus tube where a fraction f_c of the circuit current $I(t)$ effectively drives the plasma creating a time-varying inductance $L(t)$ in the focus tube.



We consider only the axial phase. By surveying published results of all Mather-type experiments we find that all deuterium plasma focus devices operate at practically the same speeds [7] and are characterized by a constancy of energy density (per unit mass) over the whole range of devices from the smallest sub-kJ to the largest MJ devices. The time varying tube inductance is $L = (\mu/2\pi)\ln(c) z$, where $c = b/a$ and μ is the permeability of free space. The rate of change of inductance is $dL/dt = 2 \times 10^{-7}\ln(c) dz/dt$ in SI units. Typically on switching, as the capacitor discharges, the current rises towards its peak value, the current sheet is accelerated, quickly reaching nearly its peak speed and continues accelerating slightly towards its peak speed at the end of the axial phase. Thus for most of its axial distance the current sheet is travelling at a speed close to the end-axial speed. In deuterium the end-axial speed is observed to be about 10 cm/ μ s over the whole range of devices. This fixes the rate of change of inductance dL/dt as 1.4×10^{-2} H/s for all the devices, if we take the radius ratio $c = b/a = 2$. This value of dL/dt changes by at most a factor of 2, taking into account the variation of c from low values of 1.4 (generally for larger machines) to 4 (generally for smaller machines). This typical dL/dt may also be expressed as 14 m Ω .

We need now to inquire into the nature of the change in the inductance $L(t)$.

Consider instantaneous power P delivered to $L(t)$ by a change in $L(t)$

Induced voltage:

$$V = d(LI)/dt = I(dL/dt) + L(dI/dt) \quad (5)$$

Hence instantaneous power into $L(t)$:

$$P = VI = I^2(dL/dt) + LI(dI/dt) \quad (6)$$

Next, consider instantaneous power associated with the inductive energy ($\frac{1}{2} LI^2$):

$$P_L = d(\frac{1}{2} LI^2)/dt = \frac{1}{2} I^2(dL/dt) + LI(dI/dt) \quad (7)$$

We note that P_L of Equation (7) is not the same as P of Equation (6).

The difference $= P - P_L = (\frac{1}{2})(dL/dt)I^2$ is not associated with the inductive energy stored in L . We conclude that whenever $L(t)$ changes with time, the instantaneous power delivered to $L(t)$ has a component that is not inductive. Hence this component of power $(\frac{1}{2})(dL/dt)I^2$ must be resistive in nature; and the quantity $(\frac{1}{2})(dL/dt)$ also denoted as half $L\dot{}$ is identified as a resistance, due to the motion associated with dL/dt ; which we call the dynamic resistance DR [27,31,33,38]. Note that this is a general result and is independent of the actual processes involved. In the case of the plasma focus axial phase, the motion of the current sheet imparts power to the shock wave structure with consequential shock heating, Joule heating, ionization, radiation *etc.* The total power imparted at any instant is just the amount $(\frac{1}{2})(dL/dt)I^2$, with this amount powering all consequential processes. We denote the dynamic resistance of the axial phase as DR_0 .

We have thus identified for the axial phase of the plasma focus a typical dynamic resistance of 7 m Ω due to the motion of the current sheet at 10 cm/ μ s. It should be noted here that similar ideas of the role of dL/dt as a resistance were discussed by Bernard *et al.* [45]. In that work the effect of dL/dt was discussed only for the radial phase. In our opinion the more important phase for the purpose of neutron saturation is actually the axial phase for the Mather-type plasma focus.

We now resolve the problem into its most basic form as follows. We have a generator (the capacitor charged to 30 kV), with an impedance of $Z_0 = (L_0/C_0)^{0.5}$ driving a load with a near constant resistance

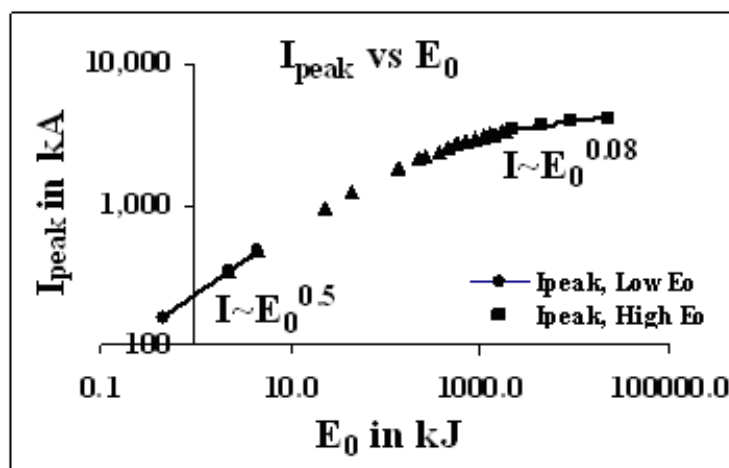
of 7 mΩ. We also assign a value for stray resistance of $0.1Z_0$. This situation may be shown in Table 3 where L_0 is given a typical value of 30 nH. We also include in the last column the results from a circuit (L-C-R) computation, discharging the capacitor with initial voltage 30 kV into a fixed resistance load of 7 mΩ, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

Table 3. Discharge characteristics of equivalent PF circuit, illustrating the ‘saturation’ of I_{peak} with increase of E_0 to very large values. The last column presents results using circuit (L-C-R) computation, with a fixed resistance load of 7 mΩ, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

E_0 (kJ)	C_0 (μF)	Z_0 (mΩ)	DR_0 (mΩ)	Z_{total} (mΩ)	$I_{peak} = V_0/Z_{total}$ (kA)	I_{peak} , L-C-R (kA)
0.45	1	173	7	197	152	156
4.5	10	55	7	67	447	464
45	100	17	7	26	1156	1234
135	300	10	7	18	1676	1819
450	1000	5.5	7	12.9	2321	2554
1080	2400	3.5	7	10.8	2781	3070
4500	10,000	1.7	7	8.8	3407	3722
45,000	100,000	0.55	7	7.6	4209	4250

Plotting the peak current as a function of E_0 we obtain Figure 14, which shows the tendency of the peak current towards saturation as E_0 reaches large values; the deterioration of the curve becoming apparent at the several hundred kJ level. This is the case for $I_{peak} = V_0/Z_{total}$ and also for the L-C-R discharge with simulated value of the DR_0 . In both cases it is seen clearly that a capacitor bank of voltage V_0 discharging into a constant resistance such as DR_0 will have a peak current I_{peak} approaching an asymptotic value of $I_{peak} = V_0/DR_0$ when the bank capacitance C_0 is increased to such large values that the value of $Z_0 = (L_0/C_0)^{0.5} \ll DR_0$. Thus DR_0 causes current ‘saturation’.

Figure 14. I_{peak} versus E_0 on log-log scale, illustrating I_{peak} ‘saturation’ at large E_0 .



In Section 4.2 we had shown the following relationships between Y_n and I_{peak} and I_{pinch} as follows:

$$Y_n \sim I_{pinch}^{4.5} \quad (8)$$

$$Y_n \sim I_{peak}^{3.8} \quad (9)$$

Hence saturation of I_{peak} will lead to saturation of Y_n .

At this point we note that if we consider that only 0.7 of the total current takes part in driving the current sheet, as typically agreed upon from experimental observations, then there is a correction factor which reduces the axial dynamic resistance by some 40%. That would raise the asymptotic value of the current by some 40%; nevertheless there would still be ‘saturation’.

Thus we have shown that current ‘saturation’ is inevitable as E_0 is increased to very large values by an increase in C_0 , simply due to the dominance of the axial phase dynamic resistance. This makes the total circuit impedance tend towards an asymptotic value which approaches the dynamic resistance at infinite values of E_0 . The ‘saturation’ of current inevitably leads to a ‘saturation’ of neutron yield. Thus the apparently observed neutron ‘saturation’ which is more accurately represented as a neutron scaling deterioration is inevitable because of the dynamic resistance. In line with current plasma focus terminology we will continue to refer to this scaling deterioration as ‘saturation’. The above analysis applies to the Mather-type plasma focus. The Filippov-type plasma focus does not have a clearly defined axial phase. Instead it has a lift-off phase and an extended pre-pinch radial phase which determine the value of I_{peak} . During these phases the inductance of the Filippov discharge is changing, and the changing $L(t)$ will develop a dynamic resistance which will also have the same current ‘saturation’ effect as the Filippov bank capacitance becomes big enough.

6.3. Beyond Presently Observed Neutron Saturation Regimes

Moreover the ‘saturation’ as observed in presently available data is due also to the fact that all tabulated machines operate in a narrow range of voltages of 15–50 kV. Only the SPEED machines, most notably SPEED II [51,54] operated at low hundreds of kV. No extensive data have been published from the SPEED machines. Moreover SPEED II, using Marx technology, has a large bank surge impedance of 50 mΩ, which itself would limit the current. If we operate a range of such high voltage machines at a fixed high voltage, say 300 kV, with ever larger E_0 until the surge impedance becomes negligible due to the very large value of C_0 , then the ‘saturation’ effect would still be there, but the level of ‘saturation’ would be proportional to the voltage. In this way we can go far above presently observed levels of neutron ‘saturation’; moving the research, as it were into presently beyond-saturation regimes.

Could the technology be extended to 1 MV? That would raise I_{peak} to beyond 15 MA and I_{pinch} to over 6 MA. Also multiple Blumleins at 1 MV, in parallel, could provide driver impedance of 100 mΩ, matching the radial phase dynamic resistance and provide fast rise currents peaking at 10 MA with I_{pinch} value of perhaps 5 MA. Bank energy would be several MJ. The push to higher currents may be combined with proven neutron yield enhancing methods such as doping deuterium with low % of krypton [55]. Further increase in pinch current might be by fast current injection near the start of the radial phase. This could be achieved with charged particle beams or by circuit manipulation

such as current-stepping [31,33,56–57]. The Lee model is ideally suited for testing circuit manipulation schemes.

7. Conclusions

This paper has reviewed the extensive and systematic numerical experiments which have been used to uncover new insights into plasma focus fusion devices including the following. A plasma current limitation effect was unexpectedly found, as the static inductance of any focus device is reduced towards very small values. Scaling laws of neutron yield and soft X-ray yield as functions of storage energies, circuit peak current as well as plasma pinch current, were developed over wider range of parameters than attempted previously. A global scaling law for neutron yield as a function of storage energy was uncovered combining experimental and extensive numerical data, showing that scaling deterioration has been wrongly interpreted as neutron ‘saturation’. However in keeping with conventional terminology, the effect of scaling deterioration will continue to be referred to as neutron ‘saturation’. The cause of neutron ‘saturation’ as device storage energy is increased was found to be the axial phase ‘dynamic resistance’. With the fundamental cause discovered, it is suggested that beyond ‘present saturation’ regimes may be reached by going to higher voltages, and using plasma current enhancement techniques such as current-steps. It is expected that numerical experiments will continue to play a major role complementing and even guiding laboratory measurements and practices. The ground-breaking insights thus gained will completely open up the directions of plasma focus fusion research.

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Scaling Laws for Plasma Focus Machines from Numerical Experiments

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Abstract: Numerical experiments carried out systematically using the Lee Model code unveil insightful and practical wide-ranging scaling laws for plasma focus machines for nuclear fusion energy as well as other applications. An essential feature of the numerical experiments is the fitting of a measured current waveform to the computed waveform to calibrate the model for the particular machine, thus providing a reliable and rigorous determination of the all-important pinch current. The thermodynamics and radiation properties of the resulting plasma are then reliably determined. This paper provides an overview of the recently published scaling laws for neutron (Y_n) and neon soft x-ray, SXR (Y_{sxr}) yields:

$$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5}; Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}; I_{peak} (0.3 \text{ to } 5.7), I_{pinch} (0.2 \text{ to } 2.4) \text{ in MA.}$$

$$Y_n \sim E_0^{2.0} \text{ at tens of kJ to } Y_n \sim E_0^{0.84} \text{ at MJ level (up to 25 MJ) and}$$

$$Y_{sxr} = 8.3 \times 10^3 I_{pinch}^{3.6}; Y_{sxr} = 6 \times 10^2 I_{peak}^{3.2}; I_{peak} (0.1 \text{ to } 2.4), I_{pinch} (0.07 \text{ to } 1.3) \text{ in MA.}$$

$$Y_{sxr} \sim E_0^{1.6} \text{ (kJ range) to } Y_{sxr} \sim E_0^{0.8} \text{ (towards MJ).}$$

Keywords: dense plasma focus, plasma focus scaling laws, neutron scaling laws, soft x-ray scaling laws, plasma focus modeling, Lee model code

1. Introduction

Plasma focus machines of various energies are increasingly being studied as sources of neutrons and soft x-rays. The most exciting prospect is for scaling the plasma focus up to regimes relevant for fusion energy studies. However, even a simple machine such as the UNU/ICTP PFF 3 kJ machine consistently produces 10^8 neutrons when operated in deuterium [1]. Plasma focus machines operated in neon have also been studied as intense sources of soft x-rays with potential applications [2–4]. Whilst many recent experiments have concentrated efforts on low energy devices [2–4] with a view of operating these as repetitive pulsed sources, other experiments have looked at x-ray pulses from larger plasma focus devices [5,6] extending to the MJ regime. Numerical experiments simulating x-ray pulses from plasma focus devices are also gaining more interest in the public domain. For example, the Institute of Plasma Focus Studies [7] conducted a recent international Internet Workshop on Plasma Focus Numerical Experiments [8], at which it was demonstrated that the Lee model code [9] not only computes realistic focus pinch parameters, but also absolute values of neutron yield Y_n and soft x-ray yield Y_{sxr} which are consistent with those measured experimentally. A com-

parison was made for the case of the NX2 machine [4], showing good agreement between computed and measured Y_{sxr} as a function of P_0 [8,10]. This gives confidence that the Lee model code gives realistic results in the computation of Y_n and Y_{sxr} .

In this paper, we show the comprehensive range of numerical experiments conducted to derive scaling laws on neutron yield Y_n [11,12] and neon Y_{sxr} in terms of storage energy E_0 , peak discharge current I_{peak} and peak focus pinch current I_{pinch} obtained from studies carried out over E_0 varying from 0.2 kJ to 25 MJ for optimised machine parameters and operating parameters. It is worth mentioning that the scaling laws in terms of I_{pinch} and I_{peak} have also been obtained for numerical experiments using the Lee model code fitted with the actual machine parameters and operating parameters and the difference from that obtained for the optimised conditions are within the order of 0.1 in the scaling laws power factor for neutrons and no change for neon SXR yield with I_{pinch} .

We also wish to point out that the distinction of I_{pinch} from I_{peak} is of basic importance [13–15]. The scaling with I_{pinch} is the more fundamental and robust one; since obviously there are situations (no pinching or poor pinching however optimized) where I_{peak} may be large but Y_n is zero or small; whereas the scaling with I_{pinch} is

certainly more consistent with all situations. In these works the primary importance of I_{pinch} for scaling plasma focus properties including neutron yield Y_n , has been firmly established [11–15].

2. The Lee Model Code

The Lee model code couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation, enabling realistic simulation of all gross focus properties. The basic model, described in 1984 [16] was successfully used to assist several projects [17–19]. Radiation-coupled dynamics was included in the five-phase code leading to numerical experiments on radiation cooling [20]. The vital role of a finite small disturbance speed discussed by Potter in a Z-pinch situation [21] was incorporated together with real gas thermodynamics and radiation-yield terms. Before this ‘communication delay effect’ was incorporated, the model consistently overestimated the radial speeds. This is serious from the point of view of neutron yields. A factor of two in shock speeds gives a factor of four in temperatures leading to a difference in fusion cross-sections of ~1000 at the range of temperatures we are dealing with. This version of the code assisted other research projects [22–27] and was web-published in 2000 [28] and 2005 [29]. Plasma self-absorption was included in 2007 [27] improving SXR yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [1, 17, 22, 23, 25–27, 30, 31], NX2 [24, 27, 32], NX1 [3, 32] and adapted for the Filippov-type plasma focus DENA [33]. A recent development is the inclusion of the neutron yield Y_n using a beam–target mechanism [11, 12, 14, 15, 34], incorporated in recent versions [9] of the code (versions later than RADPFV5.13), resulting in realistic Y_n scaling with I_{pinch} [11, 12]. The versatility and utility of the model are demonstrated in its clear distinction of I_{pinch} from I_{peak} [13] and the recent uncovering of a plasma focus pinch current limitation effect [14, 15]. The description, theory, code and a broad range of results of this ‘Universal Plasma Focus Laboratory Facility’ are available for download from [9].

A brief description of the code is given below. The five phases are summarised as follows:

1) Axial Phase: Described by a snowplow model with an equation of motion coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c respectively. The mass swept-up factor f_m accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheet-shock front structure and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure during the axial phase. The current factor f_c accounts for the fraction of current effectively flowing in the moving structure (due to all effects

such as current shedding at or near the back-wall and current sheet inclination). This defines the fraction of current effectively driving the structure during the axial phase.

2) Radial Inward Shock Phase: Described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, also called the magnetic piston, allowing the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock front and current sheath due to the finite small disturbance speed is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors f_{mr} and f_{cr} respectively are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug during the radial phase. The current factor f_{cr} accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

3) Radial Reflected Shock (RS) Phase: When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops which moves radially outwards, whilst the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the reflected shock moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the RS undergoes a jump by a factor of approximately two.

4) Slow Compression (Quiescent) or Pinch Phase: When the out-going reflected shock hits the in-coming piston the compression enters a radiative phase, in which for gases such as neon radiation emission may actually enhance the compression, where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects are incorporated into this phase. The duration of this slow compression phase is set as the time

of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.

4) Expanded Column Phase: To simulate the current trace beyond this point, we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase the snowplow model is used, and two coupled equations are used; similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

2.1 Computation of Neutron Yield

The neutron yield is computed using a phenomenological beam-target neutron generating mechanism described recently by Gribkov *et al* [34] and adapted to yield the following equation. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. The beam-target yield is derived [11,12,14,28] as:

$$Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 (\ln(b/r_p) \sigma / U)^{0.5} \quad (1)$$

where n_i is the ion density, b is the cathode radius, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n- branch [35] and U , the beam energy. C_n is treated as a calibration constant combining various constants in the derivation process.

The D-D cross-section is sensitive to the beam energy in the range 15–150 kV; so it is necessary to use the appropriate range of beam energy to compute σ . The code computes induced voltages (due to current motion inductive effects) V_{max} of the order of only 15–50 kV. However it is known, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50–150 keV [34], and for smaller lower-voltage machines the relevant energy could be lower at 30–60 keV [31]. Thus in line with experimental observations the D-D cross section σ is reasonably obtained by using $U=3V_{max}$. This fit was tested by using U equal to various multiples of V_{max} . A reasonably good fit of the computed neutron yields to the measured published neutron yields at energy levels from sub-kJ to near MJ was obtained when the multiple of 3 was used; with poor agreement for most of the data points when for example a multiple of 1 or 2 or 4 or 5 was used. The model uses a value of $C_n=2.7 \times 10^7$ obtained by calibrating the yield [9,13,14] at an experimental point of 0.5 MA.

The thermonuclear component is also computed in every case and it is found that this component is negligible when compared with the beam-target component.

2.2 Computation of Neon SXR Yield

We note that the transition from Phase 4 to Phase 5 is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These localized regions are not modelled in the code, which consequently computes only an average uniform density, and an average uniform temperature which are considerably lower than measured peak density and temperature. However, because the 4 model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modelled. Hence the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

In the code [9], neon line radiation Q_L is calculated as follows:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} n_i^2 Z Z_n^4 (\pi r_p^2) z_f / T \quad (2)$$

where for the temperatures of interest in our experiments we take the SXR yield $Y_{sxr} = Q_L$. Z_n is the atomic number. Hence the SXR energy generated within the plasma pinch depends on the properties: number density n_i , effective charge number Z , pinch radius r_p , pinch length z_f and temperature T . It also depends on the pinch duration since in our code the Q_L is obtained by integrating over the pinch duration.

This generated energy is then reduced by the plasma self-absorption which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the SXR yield. These effects are included in the modelling by computing volumetric plasma self-absorption factor A derived from the photonic excitation number M which is a function of Z_n , n_i , Z and T . However, in our range of operation, the numerical experiments show that the self absorption is not significant. It was first pointed out by Liu Mahe [23] that a temperature around 300 eV is optimum for SXR production. Shan Bing's subsequent work [24] and our experience through numerical experiments suggest that around 2×10^6 K (below 200 eV) or even a little lower could be better. Hence unlike the case of neutron scaling, for SXR scaling there is an optimum small range of temperatures (T windows) to operate.

3. Numerical Experiments

The Lee code is configured to work as any plasma focus by inputting the bank parameters, L_0 , C_0 and stray circuit resistance r_0 ; the tube parameters b , a and z_0 and operational parameters V_0 and P_0 and the fill gas. The standard

practice is to fit the computed total current waveform to an experimentally measured total current waveform [11,13–15,28,29] using the four model parameters representing the mass swept-up factor f_m , the plasma current factor f_c for the axial phase and factors f_{mr} and f_{cr} for the radial phases.

From experience it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace.

The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The detailed profile of the discharge current during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed current trace to the measured current trace in the procedure adopted by the Lee model code.

3.1 Scaling Laws for Neutrons from Numerical Experiments over a Range of Energies from 10kJ to 25 MJ

We apply the Lee model code to the MJ machine PF1000 over a range of C_0 to study the neutrons emitted by PF1000-like bank energies from 10kJ to 25 MJ.

First, we fitted a measured current trace to obtain the model parameters. A measured current trace of the PF1000 with $C_0 = 1332 \mu\text{F}$, operated at 27 kV, 3.5 torr deuterium, has been published [34], with cathode/anode radii $b=16 \text{ cm}$, $a=11.55 \text{ cm}$ and anode length $z_0=60 \text{ cm}$. In the numerical experiments we fitted external (or static) inductance $L_0 = 33.5 \text{ nH}$ and stray resistance $r_0 = 6.1 \text{ m}\Omega$ (damping factor $RESF = r_0/(L_0/C_0)^{0.5} = 1.22$). The fitted model parameters are: $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The computed current trace [11], [15] agrees very well with the measured trace through all the phases; axial and radial, right down to the bottom of the current dip

indicating the end of the pinch phase as shown in Figure 1.

This agreement confirms the model parameters for the PF1000. Once the model parameters have been fitted to a machine for a given gas, these model parameters may be used with some degree of confidence when operating parameters such as the voltage are varied [9]. With no measured current waveforms available for the higher megajoule numerical experiments, it is reasonable to keep the model parameters that we have got from the PF1000 fitting.

This series of numerical experiments is carried out at 35 kV, 10 torr deuterium, inductance $L_0 = 33.5 \text{ nH}$, stray resistance $r_0 = 6.1 \text{ m}\Omega$ (damping factor $RESF = r_0/(L_0/C_0)^{0.5} = 1.22$). The ratio $c=b/a$ is retained at 1.39. The numerical experiments were carried out for C_0 ranging from $14 \mu\text{F}$ to $39960 \mu\text{F}$ corresponding to energies from 8.5 kJ to 24 MJ [12]. For each C_0 , anode length z_0 is varied to find the optimum. For each z_0 , anode radius a_0 is varied so that the end axial speed is $10 \text{ cm}/\mu\text{s}$.

For this series of experiments we find that the Y_n scaling changes from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ) investigated in this series. This is shown in Figure 2.

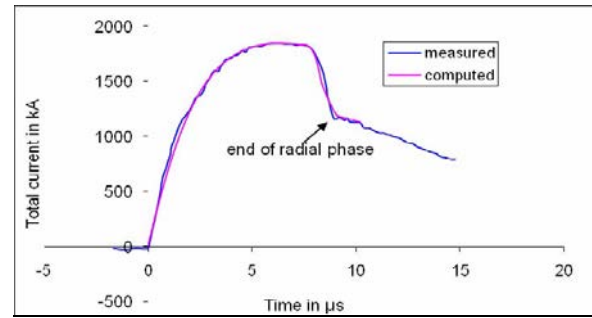


Figure 1. Current fitting of computed current to measured current traces to obtain fitted parameters $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$

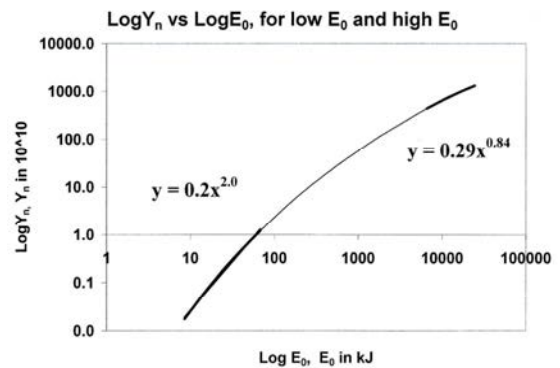


Figure 2. Y_n plotted as a function of E_0 in log-log scale, showing Y_n scaling changes from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ). The scaling deterioration observed in this Figure is discussed in the Conclusion section

The scaling of Y_n with I_{peak} and I_{pinch} over the whole range of energies investigated up to 25 MJ (Figure 3) is as follows:

$$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5} \text{ and}$$

$$Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}$$

where I_{peak} ranges from 0.3 to 5.7 MA and I_{pinch} ranges from 0.2 to 2.4 MA.

This compares to an earlier study carried out on several machines with published current traces with Y_n yield measurements, operating conditions and machine parameters including the PF400, UNU/ICTP PFF, the NX2 and Poseidon providing a slightly higher scaling laws: $Y_n \sim I_{pinch}^{4.7}$ and $Y_n \sim I_{peak}^{3.9}$. The slightly higher value of the scaling is because those machines fitted are of mixed 'c' mixed bank parameters, mixed model parameters and currents generally below 1MA and voltages generally below the 35 kV [11].

3.2 Scaling Laws for Neon SXR from Numerical Experiments over a Range of Energies from 0.2 kJ to 1 MJ

We next use the Lee model code to carry out a series of numerical experiments to obtain the soft x-ray yield in neon for bank energies from 0.2 kJ to 1 MJ [36]. In this case we apply it to a proposed modern fast plasma focus machine with optimised values for c the ratio of the outer to inner electrode radius and L_0 obtained from our numerical experiments.

The following parameters are kept constant: 1) the ratio $c=b/a$ (kept at 1.5, which is practically optimum according to our preliminary numerical trials; 2) the operating voltage V_0 (kept at 20 kV); 3) static inductance L_0 (kept at 30 nH, which is already low enough to reach the I_{pinch} limitation regime [13,14] over most of the range of E_0 we are covering) and; 4) the ratio of stray resistance to surge impedance $RESF$ (kept at 0.1, representing a higher performance modern capacitor bank). The model parameters [8-14] f_m, f_c, f_{mr}, f_{cr} are also kept at fixed values 0.06, 0.7, 0.16 and 0.7. We choose the model parameters as they represent the average values from the range of machines that we have studied. A typical current waveform is shown in Figure 4.

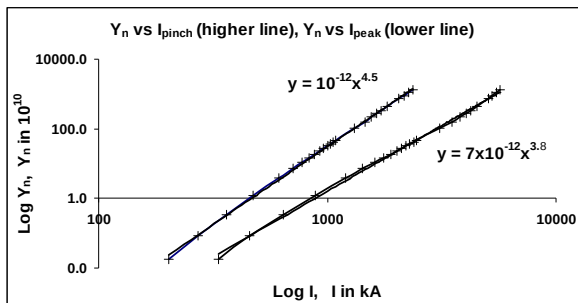


Figure 3. $\text{Log}(Y_n)$ scaling with $\text{Log}(I_{peak})$ and $\text{Log}(I_{pinch})$, for the range of energies investigated, up to 25 MJ

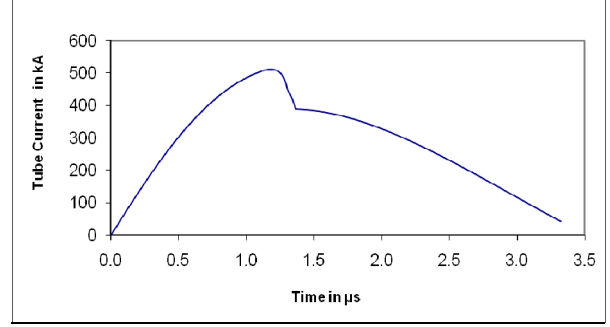


Figure 4. Computed total current versus time for $L_0=30$ nH and $V_0=20$ kV, $C_0=30$ uF, $RESF=0.1$, $c=1.5$ and model parameters f_m, f_c, f_{mr}, f_{cr} are fixed at 0.06, 0.7, 0.16 and 0.7 for optimised $a=2.285$ cm and $z_0=5.2$ cm

The storage energy E_0 is varied by changing the capacitance C_0 . Parameters that are varied are operating pressure P_0 , anode length z_0 and anode radius ' a '. Parametric variation at each E_0 follows the order; P_0, z_0 and a until all realistic combinations of P_0, z_0 and a are investigated. At each E_0 , the optimum combination of P_0, z_0 and a is found that produces the biggest Y_{srx} . In other words at each E_0 , a P_0 is fixed, a z_0 is chosen and a is varied until the largest Y_{srx} is found. Then keeping the same values of E_0 and P_0 , another z_0 is chosen and a is varied until the largest Y_{srx} is found. This procedure is repeated until for that E_0 and P_0 , the optimum combination of z_0 and a is found. Then keeping the same value of E_0 , another P_0 is selected. The procedure for parametric variation of z_0 and a as described above is then carried out for this E_0 and new P_0 until the optimum combination of z_0 and a is found. This procedure is repeated until for a fixed value of E_0 , the optimum combination of P_0, z_0 and a is found.

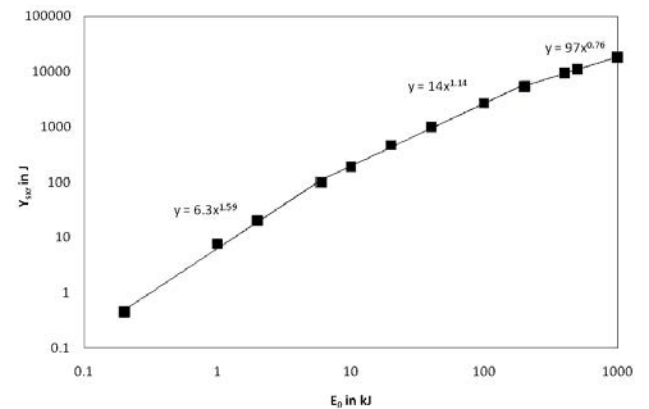


Figure 5. Y_{srx} vs E_0 . The parameters kept constants are: $RESF=0.1$, $c=1.5$, $L_0=30$ nH and $V_0=20$ kV and model parameters f_m, f_c, f_{mr}, f_{cr} at 0.06, 0.7, 0.16 and 0.7 respectively. The scaling deterioration observed in this Figure is discussed in the Conclusion section

The procedure is then repeated with a new value of E_0 . In this manner after systematically carrying out some 2000 runs, the optimized runs for various energies are obtained. A plot Y_{srx} against E_0 is shown in Figure 5.

We then plot Y_{srx} against I_{peak} and I_{pinch} and obtain SXR yield scales as $Y_{srx} \sim I_{pinch}^{3.6}$ and $Y_{srx} \sim I_{peak}^{3.2}$. The I_{pinch} scaling has less scatter than the I_{peak} scaling. We next subject the scaling to further test when the fixed parameters $RESF$, c , L_0 and V_0 and model parameters f_m , f_c , f_{mr} , f_{cr} are varied. We add in the results of some numerical experiments using the parameters of several existing plasma focus devices including the UNU/ICTP PFF ($RESF = 0.2$, $c = 3.4$, $L_0 = 110$ nH and $V_0 = 14$ kV with fitted model parameters $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.2$, $f_{cr} = 0.8$) [7-9], [23], the NX2 ($RESF = 0.1$, $c = 2.2$, $L_0 = 20$ nH and $V_0 = 11$ kV with fitted model parameters $f_m = 0.06$, $f_c = 0.7$, $f_{mr} = 0.16$, $f_{cr} = 0.7$) [7-10,24] and PF1000 ($RESF = 0.1$, $c = 1.39$, $L_0 = 33$ nH and $V_0 = 27$ kV with fitted model parameters $f_m = 0.1$, $f_c = 0.7$, $f_{mr} = 0.15$, $f_{cr} = 0.7$) [7-9,14]. These new data points (unblackened data points in Figure 6) contain wide ranges of c , V_0 , L_0 and model parameters. The resulting Y_{srx} versus I_{pinch} log-log curve remains a straight line, with the scaling index 3.6 unchanged and with no more scatter than before. However the resulting Y_{srx} versus I_{peak} curve now exhibits considerably larger scatter and the scaling index has changed.

We would like to highlight that the consistent behaviour of I_{pinch} in maintaining the scaling of $Y_{srx} \sim I_{pinch}^{3.6}$ with less scatter than the $Y_{srx} \sim I_{peak}^{3.2}$ scaling particularly when mixed-parameters cases are included, strongly support the conclusion that I_{pinch} scaling is the more universal and robust one. Similarly conclusions on the importance of I_{pinch} in plasma focus performance and scaling laws have been reported [11-15].

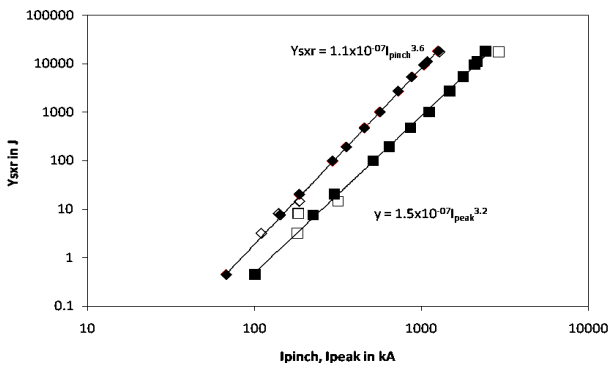


Figure 6. Y_{srx} is plotted as a function of I_{pinch} and I_{peak} . The parameters kept constant for the black data points are: $RESF = 0.1$, $c = 1.5$, $L_0 = 30$ nH and $V_0 = 20$ kV and model parameters f_m , f_c , f_{mr} , f_{cr} at 0.06, 0.7, 0.16 and 0.7 respectively. The unblackened data points are for specific machines which have different values for the parameters c , L_0 , V_0 and $RESF$

It may also be worthy of note that our comprehensively surveyed numerical experiments for Mather configurations in the range of energies 0.2 kJ to 1 MJ produce an I_{pinch} scaling rule for Y_{srx} not compatible with Gates' rule [37]. However it is remarkable that our I_{pinch} scaling index of 3.6, obtained through a set of comprehensive numerical experiments over a range of energies 0.2 kJ to 1 MJ, on Mather-type devices is within the range of 3.5 to 4 postulated on the basis of sparse experimental data, (basically just two machines one at 5 kJ and the other at 0.9 MJ), by Filippov [6], for Filippov configurations in the range of energies 5 kJ to 1 MJ.

It must be pointed out that the results represent scaling for comparison with baseline plasma focus devices that have been optimized in terms of electrode dimensions. It must also be emphasized that the scaling with I_{pinch} works well even when there are some variations in the actual device from $L_0 = 30$ nH, $V_0 = 20$ kV and $c = 1.5$. However there may be many other parameters which can change and could lead to a further enhancement of x-ray yield.

4. Conclusions

Numerical experiments carried out using the universal plasma focus laboratory facility based on the Lee model code gives reliable scaling laws for neutrons production and neon SXR yields for plasma focus machines. The scaling laws obtained:

For neutron yield:

$$Y_n = 3.2 \times 10^{11} I_{pinch}^{4.5}; Y_n = 1.8 \times 10^{10} I_{peak}^{3.8}; I_{peak} (0.3 \text{ to } 5.7), I_{pinch} (0.2 \text{ to } 2.4) \text{ in MA.}$$

$$Y_n \sim E_0^{2.0} \text{ at tens of kJ to } Y_n \sim E_0^{0.84} \text{ at MJ level (up to 25 MJ).}$$

For neon soft x-rays:

$$Y_{srx} = 8.3 \times 10^3 I_{pinch}^{3.6}; Y_{srx} = 6 \times 10^2 I_{peak}^{3.2}; I_{peak} (0.1 \text{ to } 2.4), I_{pinch} (0.07 \text{ to } 1.3) \text{ in MA.}$$

$$Y_{srx} \sim E_0^{1.6} \text{ (kJ range) to } Y_{srx} \sim E_0^{0.8} \text{ (towards MJ).}$$

These laws provide useful references and facilitate the understanding of present plasma focus machines. More importantly, these scaling laws are also useful for design considerations of new plasma focus machines particularly if they are intended to operate as optimized neutron or neon SXR sources. More recently, the scaling of Y_n versus E_0 as shown above has been placed in the context of a global scaling law [38] with the inclusion of available experimental data. From that analysis, the cause of scaling deterioration for neutron yield versus energy as shown in Figure 2 (which has also been given the misnomer 'neutron saturation') has been uncovered as due to a current scaling deterioration caused by an almost constant axial phase 'dynamic resistance' interacting with a reducing bank impedance as energy storage is increased at essentially constant voltage. Solutions suggested include the use

of ultra-high voltages and circuit enhancement techniques such as current-steps [39,40]. It is suggested here that the deterioration of soft x-ray yield with storage energy as shown in Figure 5 could also be ascribed to the same axial phase 'dynamic resistance' effect as described in that reference [38].

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***In situ* determination of the static inductance and resistance of a plasma focus capacitor bank**

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The static (unloaded) electrical parameters of a capacitor bank are of utmost importance for the purpose of modeling the system as a whole when the capacitor bank is discharged into its dynamic electromagnetic load. Using a physical short circuit across the electromagnetic load is usually technically difficult and is unnecessary. The discharge can be operated at the highest pressure permissible in order to minimize current sheet motion, thus simulating zero dynamic load, to enable bank parameters, static inductance L_0 , and resistance r_0 to be obtained using lightly damped sinusoid equations given the bank capacitance C_0 . However, for a plasma focus, even at the highest permissible pressure it is found that there is significant residual motion, so that the assumption of a zero dynamic load introduces unacceptable errors into the determination of the circuit parameters. To overcome this problem, the Lee model code is used to fit the computed current trace to the measured current waveform. Hence the dynamics is incorporated into the solution and the capacitor bank parameters are computed using the Lee model code, and more accurate static bank parameters are obtained. © 2010 American Institute of Physics. [doi:10.1063/1.3429207]

I. INTRODUCTION

To model any electromagnetic device, among the input parameters are the static bank parameters. For example for any modeling of a plasma focus discharge (see Fig. 1), among the input parameters are the static inductance L_0 and resistance r_0 , given the bank capacitance C_0 . The word static is used here in preference to the phrase “short circuit” as the former conveys the more accurate meaning that these parameters are the relevant parameters when there is no dynamic electromagnetic load. Moreover static also implies *in situ*; that everything remains the same except that there is no motion attributed to the current distribution. The term short circuit is not sufficiently accurate in this context, as the position and design of the short circuit are subject to sometimes arbitrary definition which would give unreliable results.

A simple approximate method is to operate the plasma focus at high pressures, say 20 Torr neon; so that there is little current sheet motion. Then the discharge current approximates a lightly damped L_0 - C_0 - r_0 oscillation where C_0 is the bank capacitance and L_0 and r_0 are the static inductance and resistance of the bank, respectively. Analysis of two or three cycles of the current waveform then yield approximate values of L_0 and r_0 , given that C_0 is known.

However it turns out that even at 20 Torr neon, there is usually significant motion of the current sheet which affects the current waveform, as this motion introduces an additional time-varying inductance into the circuit; even though at such high pressures the axial motion is not fast enough to reach

the end of the anode at the end of the first half cycle drive. So there is no focus (radial) phase (see Fig. 1). Nevertheless, because of the axial motion, the estimated static inductance will be too high. Therefore an analysis of the current waveform including the small but significant current sheet dynamics is necessary in order to obtain a true measure of L_0 . In this paper we discuss the estimate of capacitor parameters using a high pressure discharge first using damped L_0 - C_0 - r_0 analysis, assuming no current sheet dynamics. Then we show that actually there is significant current sheet motion. Finally we use a plasma focus code, with included dynamics to analyze the measured current waveform, thus separating the L_0 and the additional amount of inductance due to motion.

II. EXPERIMENTAL PROCEDURES AND RESULTS

A. Estimate of capacitor bank static parameters from a high pressure discharge

An example is given here, using the INTI-PF, which is one of the plasma focus machines developed from the UNU/ICTP PFF.^{1,2} The capacitor bank has a capacitance of $C_0 = 30 \mu\text{F}$. The plasma focus is filled with neon at 20 Torr. The capacitor bank is charged up to 10 kV and discharged. The current trace is monitored with a current transformer. The discharge current waveform is as shown in Fig. 2.

Assuming that the discharge current waveform is that from a lightly damped L_0 - C_0 - r_0 circuit, the waveform may be treated as sinusoid with period T ; the following approximate equations³ hold:

$$L_0 = T^2/(4\pi^2 C_0), \quad (1)$$

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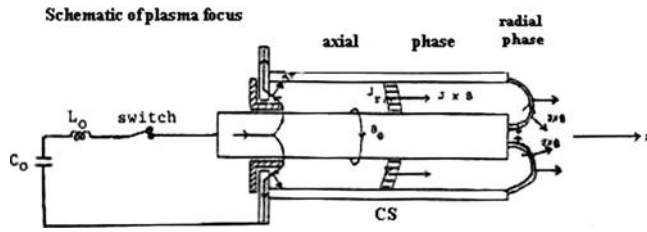


FIG. 1. Showing the plasma focus with representative current sheet positions in the axial phase and the radial phase. The plasma focus acts in the following way: A capacitor bank C_0 discharges a large current into the coaxial tube. The current flows in a current sheath CS which is driven by the $J \times B$ force, axially down the tube. At the end of the axial phase the CS implodes radially, forming an elongating pinch. The static resistance r_0 of the discharge circuit is not shown.

$$r_0 = -(2/\pi)[\ln(f)](L_0/C_0)^{0.5}, \quad (2)$$

$$I_0 = \pi C_0 V_0 (1 + f)/T, \quad (3)$$

where f is the reversal ratio obtained from the successive current peaks I_1, I_2, I_3, I_4 , and I_5 with $f_1 = I_2/I_1$, $f_2 = I_3/I_2$, $f_3 = I_4/I_3$, $f_4 = I_5/I_4$, and $f = (1/4)(f_1 + f_2 + f_3 + f_4)$; and I_0 the highest peak current is written here as I_1 , the peak current of the first half cycle.

From Fig. 2, we estimate the following:

$$3T = 36.6 \mu s,$$

giving $T = 12.2 \mu s$ and

$$L_0 = 126 \text{ nH}.$$

$f_1 = 0.737$, $f_2 = 0.612$, $f_3 = 0.760$, and $f_4 = 0.524$, giving $f = 0.658$; and

$$r_0 = 17.1 \text{ m}\Omega$$

and peak current $I_0 = 128 \text{ kA}$.

The coil gives a peak first half cycle output of 24 V. Thus additionally the coil sensitivity is obtained as $128/24 = 5.3 \text{ kA/V}$.

B. Correction required if the current sheet had moved

If the current sheet had moved, then the movement will add to the circuit inductance; and the value of the inductance measured will have a part which is due to inductance increment as a result of motion. To estimate whether there is significant motion at this pressure we fire a shot at a low enough pressure to obtain a strong focus in order to obtain the axial speed at the lower pressure. This is shown in Fig. 3.

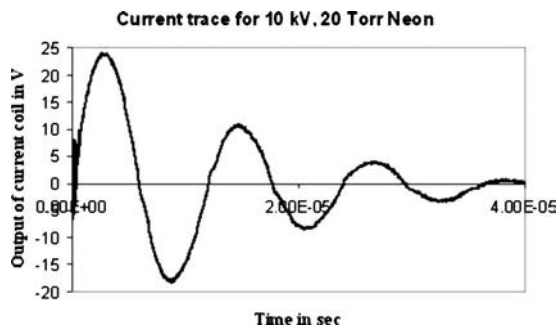


FIG. 2. Measured discharge current waveform at 10 kV, 20 Torr neon.

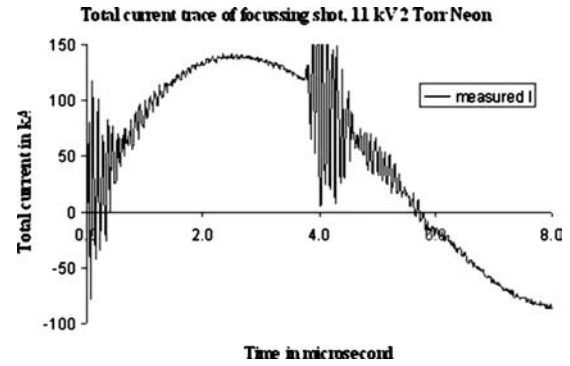


FIG. 3. Discharge current waveform at 11 kV, 2 Torr neon.

From this shot we deduce the following. The current dip starts at the point which is approximately the end of the axial phase and the start of the radial phase.

So it takes $3.9 \mu s$ for the current sheet to travel the 16 cm which is the distance traveled by the current sheet over the full length of the axial phase. The average speed of the current sheet down the tube in this axial phase is $4.1 \text{ cm}/\mu s$ for this shot at 11 kV, 2 Torr neon.

From these data we can estimate the average speed of the high pressure shot at 10 kV 20 Torr neon; using the scaling relation⁴ for electromagnetic drive

$$\text{speed} \sim (I/a)/\rho^{0.5}.$$

The anode radius a is constant, and the current is approximately proportional to the charging voltage. Here ρ is the density which is proportional to the pressure for a fixed gas. Hence the average axial speed for the 10 kV, 20 Torr neon shot is estimated from the relationship to be $(10/11)(2/20)^{0.5}(4.1) \sim 1.2 \text{ cm}/\mu s$.

So in the time of the first half cycle of $6.1 \mu s$, the current sheet would have moved 7 cm. The coaxial tube has an inductance of $\sim 2.4 \text{ nH/cm}$. So the current sheet movement could have added a maximum amount of 17 nH to the measured inductance of the high pressure shot. This is not a negligible amount. Hence the motion should be taken into account for the measurement of L_0 .

C. Fitting the computed current trace to the measured current trace to determine the static inductance

The Lee model code⁵ couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling realistic simulation of all gross focus properties.⁶⁻¹⁰ For the high pressure shot involving only the axial phase, the code is very precise in its computation of the discharge current, including the interaction of the circuit with the current sheet dynamics. The Lee model code is used in order to generate a computed current trace for fitting to the measured current trace. In this fitting, adjustments are normally made to four model parameters, f_m , f_c for the axial phase, and f_{mr} and f_{cr} for the radial phase. In this case we are fitting the 20 Torr neon shot, which has no radial phase, since the current sheet does not move fast enough to reach the end of the anode during the drive time of the current pulse.

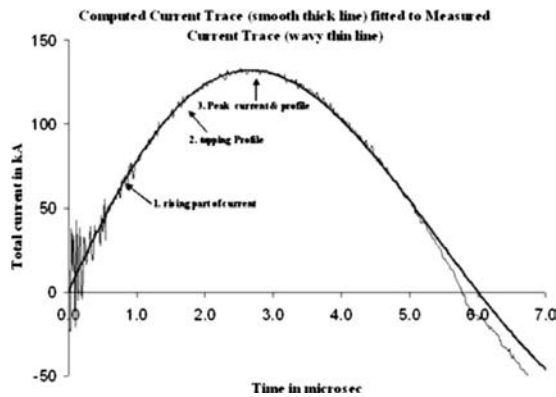


FIG. 4. Fitting the computed current trace to the measured current trace by varying model parameters f_m and f_c .

Our experience has shown us that the three features to be fitted for the axial phase, i.e., the rising part of the current trace, the topping profile, and the peak current have to be fitted by adjusting the value of f_m and f_c (see Fig. 4).

The method is also sensitive enough that the static inductance and circuit stray resistance also need to be correctly fitted; otherwise no good fit would be obtained. Further, for some low voltage shots, it may also be necessary to move the whole measured current trace a small amount in time; otherwise no good fit may be obtained. This is due to the nonperfect switching characteristics of the spark gap which typically takes a little time to go from nonconducting to conducting, whereas the code assumes instantaneous switching at time zero. This is apparent when the early rising slope of the high pressure shot is examined. The rising section of the measured current does not rise as fast as the computed current trace no matter how the model parameters f_m and f_c are adjusted and no matter how L_0 and r_0 are adjusted. The correct way is to shift either current trace relative to the other by the addition of a small time delay.

Figure 4 shows the results of the fitting of the computed current trace to the measured trace according to the procedures summarized in the earlier paragraph, and discussed in greater detail below. We start the fitting process by using the static inductance and resistance estimated earlier assuming no plasma dynamics; i.e., $L_0=126$ nH and $r_0=17.1$ m Ω , with capacitance $C_0=30$ μ F. Since for this high pressure shot the current sheet does not move beyond the end of the anode, we only need the first two of the model parameters, which we initialize by taking $f_m=0.08$ and $f_c=0.7$.

By comparing the computed with the measured current waveforms, it was clear that the first part of the current trace was not going to fit except by shifting the whole measured current trace. This was done, and the required shift was 0.2 μ s to make the measured current trace come earlier in time relative to the computed. Next it was found that no suitable fit could be found unless we change the value of L_0 to $L_0=114$ nH, $r_0=15$ m Ω . Finally in small incremental steps the following values of model parameters (axial phase) were found to be $f_m=0.05$ and $f_c=0.71$. The resultant fit is shown in Fig. 4.

III. DISCUSSION

The results of this method of analysis for this 10 kV, 20 Torr neon shot are the following: bank parameters: $L_0=114$ nH, $C_0=30$ μ F, and $r_0=15$ m Ω ; model parameters (axial phase only): $f_m=0.05$ and $f_c=0.71$; tube parameters: $b=3.2$ cm, $a=0.95$ cm and $z_0=16$ cm. Here b and a are the outer and inner radii of the coaxial tube and z_0 is the length of the anode. At the same time the Rogowski coil (used as a current transformer) was calibrated with a sensitivity of 5.4 kA/V.

In obtaining the best fit as shown in Fig. 4, it was noted that for $t > 5.0$ μ s, there is a feature in the measured current trace which is not present in the computed current trace. The computed current trace assumes that the current sheet continues to move axially down the tube. However it is known that in electromagnetic shock tubes, which the axial part of the plasma focus is, the voltage across the tube reverses just before the current changes direction in the external circuit. At that point of time when the voltage changes direction, the anode to cathode current starts to close upon itself to form a closed loop.¹¹ With this loop decoupled electrically from the capacitor, the inductance of the capacitor discharge circuit is reduced. This causes the external current to drop quicker to zero and the modeled current trace, which does not have this closing loop feature, is no longer able to be fitted to the measured current trace. We include this discussion here to show how sensitive is the method of fitting modeled current to measured current.

IV. CONCLUSION

In this paper, a two-step method is discussed to determine the capacitor bank static parameters of a plasma focus. In the first step of the estimate, the assumption is made that there is no current sheet movement for the high pressure discharge. Hence the discharge current may be analyzed by equations that assume a lightly damped sinusoid generated from an L_0 - C_0 - r_0 discharge circuit, where C_0 is known, and L_0 and r_0 are constant in time and are thus easily determined from Eqs. (1) and (2). The second step takes into account the current sheet motion. This step involves fitting the current trace computed with the Lee model code to the measured current trace, using the estimated values of L_0 and r_0 obtained from the first step. For this fitting, it is found that the static inductance L_0 has to be adjusted before the fitting is possible, with the adjustment of r_0 also playing a significant role. Finally the model parameters have to be adjusted for the best fit.

This two-step process enables the values of L_0 and r_0 to be correctly measured. At the same time the current measuring device is also more accurately calibrated.

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Radiation Emission Correlated with the Evolution of Current Sheath from a Deuterium Plasma Focus

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Abstract The time resolved emission of neutrons and X-rays (both soft and hard) is correlated with the current sheath evolution during the radial phase of a 3.2 kJ Mather-type plasma focus device operated in deuterium at an optimised pressure of 4 mbar. A three-frame computer-controlled laser shadowgraphy system was incorporated in the experiment to investigate the time evolution of the radial phase of the plasma focus. The dynamics of the sheath was then correlated with the time resolved X-rays and neutron emission. The time-resolved neutron and hard X-ray emission was detected by a Scintillator-photo-multiplier system while the time resolved soft X-rays were detected employing filtered PIN photo diodes. The observations were recorded with a temporal accuracy of a few ns. For the reference, the total neutron yield was also monitored by an Indium Foil activation detector. The correlation with the High Voltage Probe signal of the discharge, together with the X-ray and neutron emission regimes enabled to identify the important periods of the sheath evolution i.e. the radial compression (pre focus), minimum pinch radius (focus) and the post focus phenomena. During the initial stage of the radial phase, velocities of 10–23 cm/ μ s, while at the later stage of the radial phase (up till the compression), velocities up to 32–42 cm/ μ s were measured in our experiment. For the

discharges with the lower neutron yield (lower than the average value $\sim 1 \times 10^8$ n/discharge), the current sheath appears to be disturbed and neutron and hard X-ray signal profiles do not carry much information whereas the soft X-ray emission is significant. For the discharges with high neutron yield (higher than the average value), the current sheath has a smooth structure until the maximum compression occurs. Hard X-ray emission is maximum for the discharges with high neutron yield, especially whenever there is development of $m = 0$ instability compressing the column to very high densities. The neutrons are emitted long after the maximum compression supporting the beam target fusion. For the discharges with High neutron yield, the soft X-ray production is less as compared to the discharges with low neutron yield.

Keywords Plasma focus device · Hard X-rays · Soft X-rays · Neutrons

Introduction

Plasma Focus (PF) device is capable of producing a short lived, high density plasma. The pinched plasma leads to the emission of radiations such as neutrons (operated in deuterium) [1] X-rays (both soft and hard) [2–6] and ions [7]. In the DPF device a current sheath is formed in the vicinity of insulator sleeve (breakdown phase). An azimuthal magnetic field is established as a result leading towards the formation of a Lorentz force. Due to this force the sheath moves in the axial direction along the anode (axial acceleration phase). The next sequential phase is the radial phase in which the current sheath moves in a radial direction towards the centre of the anode tip until it reaches the centre and collapses (the pinch). The imaging of the

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plasma sheath structure and its dynamics during the radial phase is of vital interest in knowing the actual behaviour and variety of phenomena taking place in the plasma. Different studies were made in the past to investigate the current sheath dynamics and related phenomena. The structure of the pinched column for small plasma focus devices (≤ 10 kJ) was investigated by several authors [8–12]. It was found that in small scale experiments, the primary filamentation of the current sheath is destroyed during the collapse phase, when a plasma column of a few millimetres in diameter is formed [9]. Yamamoto et al. [13] studied the neutron production mechanism through the time correlation between the dynamic behaviour of the focused plasma and the burst of the deuteron beams, X-rays and neutrons. Toshikazu et al. [14] suggested that the generation of the ion and the electrons beam occur in an interval of 20 ns around the peak of the voltage spike and the plasma inductance increases monotonically, but rather slowly during the process of the $m = 0$ instability growth. Comisar [15] suggested that the lifetime of the resulting fountain like pinch column appear to be governed by the formation of sausage instabilities. Nevertheless, the build up time of the $m = 0$ instabilities are at least 5–10 times shorter than the observed pinch lifetime (~ 50 ns).

The structure and the time dependent evolution of the sheath is responsible to set the emission parameters of the radiations like neutrons and X-rays.

Mohammadi et al [16] correlated the time resolved soft X-ray emission with the dynamics of the sheath by increasing the length of the conventional anode.

In this paper, the current sheath structure, dynamics and its influence on the radiation emission during the radial phase of a 3.2 kJ Mather-type plasma focus is presented. The dynamics during the radial phase of the sheath is correlated with the time resolved emission of radiation (neutron and X-rays) along with high voltage probe signal profiles.

Experimental Set-up

The experiments were carried out on a 3.2 kJ Mather type plasma Focus device. The device was operated with a 33 μ F/14 kV capacitor giving a discharge current of about 180 kA (a typical Rogowski Coil signal). The main tube of the device consists of a cylindrical copper anode of 1.9 cm in diameter with 16 cm length surrounded by 6 equally spaced copper cathode rods making a diameter of 6.4 cm concentric with the anode. The insulation between the cathode and anode is provided by a glass insulator sleeve.

The device was operated in deuterium at an optimised pressure of 4 mbar.

In order to investigate the radial phase of the plasma focus evolution three shadowgraphic images of a single discharge of the device were captured. A CCD based three-frame computer-controlled laser shadowgraphy system was employed in the experiment. The first part of the shadowgraphy system was a three locally fabricated nitrogen dye lasers (used as probe beams) synchronized with a few nano second accuracy with an appropriate delay (a few ns) between each pulse to capture three images of a single shot of plasma focus device [17]. The second part was a telescopic arrangement for image resolution to fit to the field of view of the CCD. The third part was the video capture system comprising of a data acquisition and transfer facility. The triggering of the CCD was done internally by transferring the image to the disk when there was a significant increase in the light level. The timing between the probe laser and the discharge was adjusted using the laser pulse and the high voltage signal. A schematic of the shadowgraphy system is shown in Fig. 1a. A detailed description of the system can be found in [17].

The images obtained thus were then correlated with the time resolved neutron, hard X-ray and soft X-ray emission. A scintillator-photomultiplier system was used to explore the time resolved behaviour of neutrons and hard X-rays. The detector consisted of a plastic scintillator (NE 102A of 50 mm diameter and 40 mm thickness) and a photomultiplier (EMI 9813B) tube. The plastic scintillator was wrapped in an Aluminum foil, with the exception of the side facing towards the window of the photomultiplier tube. For proper optical coupling, Dow Corning 200 silicon grease was used between the plastic scintillator and the tube.

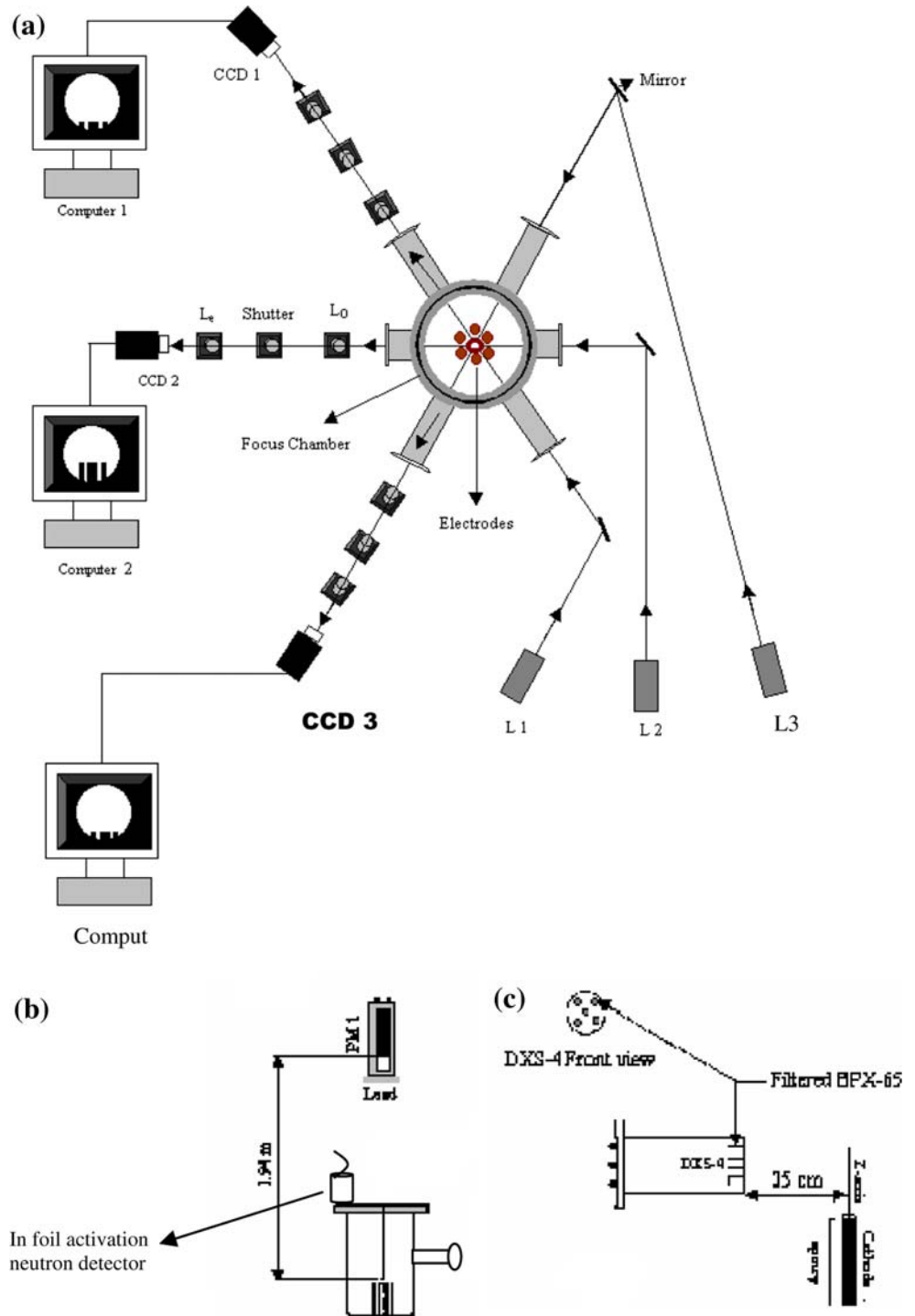
The detector, referred to as PM1, was placed at a distance of 1.94 m in the axial direction at 0° with respect to the pinch (Fig. 1b). To cut off most of the hard X-rays or to avoid the saturation of the tube, lead sheets (3 mm thick) were placed in front of PM1. The neutron time of flight from the pinch to the PMT was also compensated to synchronize with the Hard X-ray timings. The total neutron yield was measured with the help of an Indium foil activation detector (BICON). The calibration factor for the detector was found to be 1.5×10^5 neutrons/count [17].

For the time resolved soft X-ray measurements two of a five-channel soft X-ray spectrometer (DXS-4) was used the details of which can be found in [17, 18]. The arrangement is shown in Fig. 1c. The data was acquired on a ten channel HP 250 MHz Oscilloscope.

Results and Discussion

In order to investigate various phenomena involved in the focus dynamics when operated in deuterium at an optimised

Fig. 1 **a** A set up for the three frame shadowgraphy system (*top view*). **b** A schematic of a Scintillator-photomultiplier system for the time-resolved neutron and hard X-rays measurements along with the In foil activation detector for total neutron yield measurements. **c** A schematic of the arrangement for time resolved soft X-ray measurements



pressure of 4 mbar and to identify their role in the radiation emission, the radial phase is classified as: (1) Pre-focus Phase, (2) Focus Phase and (3) Post Focus phase. This classification is based on the time correlation between the high voltage probe signal and the probing laser beam signals. Time is taken $t = 0$ at the time of maximum compression indicated by the high voltage

pulse and is observed by the shadowgraphs. Time is assigned a negative value before focusing (maximum compression) and is positive after the maximum compression. For each phase the results are discussed with reference to the total neutron yield. The average neutron yield (taken for 5000 discharges) is found to be 1×10^8 n/discharge.

Pre-Focus Phase

This is the phase in which the current sheath starts from the open end (tip) of the anode and accelerates towards the centre. This phase ends when the current sheath meets the axis at the centre of the anode ($t = 0$).

Figure 2 presents a three-frame sequence of a single shot during pre-focus phase within a time span of -10 ns to -5 ns. The time resolved hard X-rays (HX), neutrons (dY_N/dt)_{PM1}, soft X-rays (dY_{sx}/dt)₃, (dY_{sx}/dt)₄ and high voltage probe (V) signal profiles of are also shown. The neutron yield for this shot is 4.5×10^7 lower than the average value. The voltage signal profile (V) shows multiple spikes indicating that the discharge belongs to the multiple-compression regime. This multiple compression might be due to the uneven transfer of energy from the capacitor bank. The experimentally measured radial velocity is 26 cm/ μ s which is in agreement with the work done by Lee [19] and Zhang [20]. In the last frame ($t = -5$ ns), A slight breakage of the sheath appears (encircled) which might effect the focusing at the later stages. No significant information could be extracted from these images. The time resolved hard X-ray and neutron signal profiles do not show significant peaks. However, soft X-ray signal profiles (dY_{sx}/dt)₃ and (dY_{sx}/dt)₄ show that there is emission of soft X-rays but not at the first focus peak (V at $t = 0$). The emission starts 150 ns after the main focus and lasts for about 400 ns. This emission corresponds bremsstrahlung due to the presence of hot electron.

Figure 3 shows another discharge with total neutron yield of 3.6×10^7 (lower than the average value) during the pre-focus phase. It is again a multi-compression discharge as indicated by the High voltage probe signal (V). Almost after 150 ns, there is an onset of another

focussing event which lasts for about 150 ns. This multiple focus discharges are attributed to the weak setting up of the break down phase near the insulator sleeve surface. The current sheath seems to have a lower density and is much wider than the “normal” discharge. At the instant 1 ns before the maximum compression, the column is compressed at one point relatively far from the tip of the anode, while, near the tip of the anode the sheath is yet to reach the centre. The axial elongation of the plasma as well as the short radius of curvature of the sheath is observed. The low neutron yield could then be explained by a significant loss of the plasma in the axial direction (towards and away from the anode), which reduces the number of particles to be confined, thus, the final density and temperature might have low values. Experimentally measured average radial velocity for this discharge is 30 cm/ μ s.

The time resolved hard X-ray (HX) and neutron (dY_N/dt)_{PM1} signals are flat. The reason could be attributed to the less dense sheath as it appears in the shadowgraph. The density at the maximum compression might not have reached the sufficient value to create hot electrons and strong ion beams. The soft X-ray production (dY_{sx}/dt)₃ and (dY_{sx}/dt)₄ are emitted with the first focussing of the sheath. 150 ns after the first focus there is another burst of the soft X-rays which corresponds to the second focussing event of the sheath. A very weak third emission is observed at 600 ns after the first focus which might be due to the sputtering of the copper anode.

A comparison of the two discharges with low neutron yield shows that the implosion speed alone is not a critical parameter for neutron production. The compression may be fast, but if the geometrical profile of the sheath favours significant losses, the confinement efficiency decreases.

Fig. 2 A three frame sequence of a discharge during the pre-focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{sx}/dt and V. The neutron yield for this shot is 4.5×10^7 lower than the average value

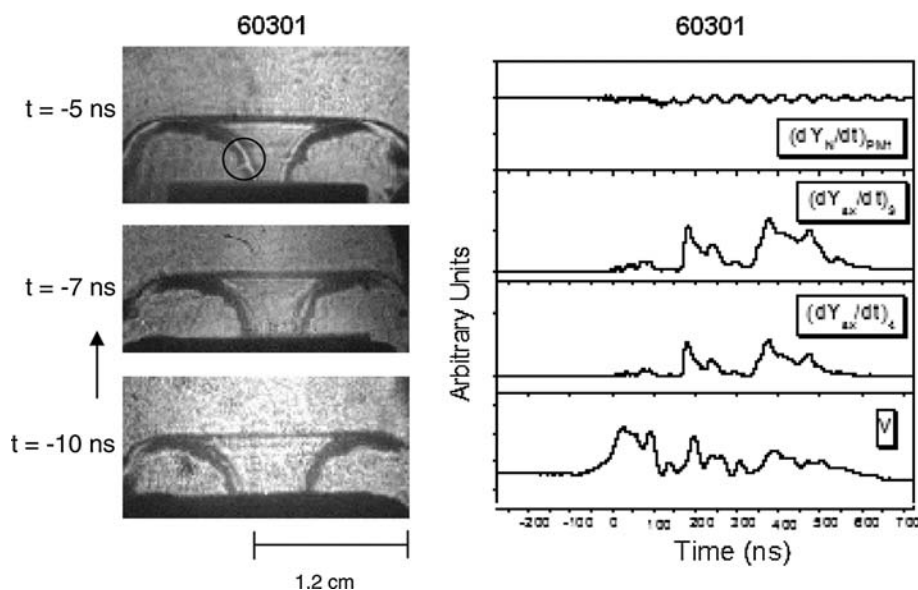


Fig. 3 A three frame sequence of a discharge during the pre-focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{sx}/dt and V . The neutron yield for this discharge is 3.6×10^7 (lower than the average value)

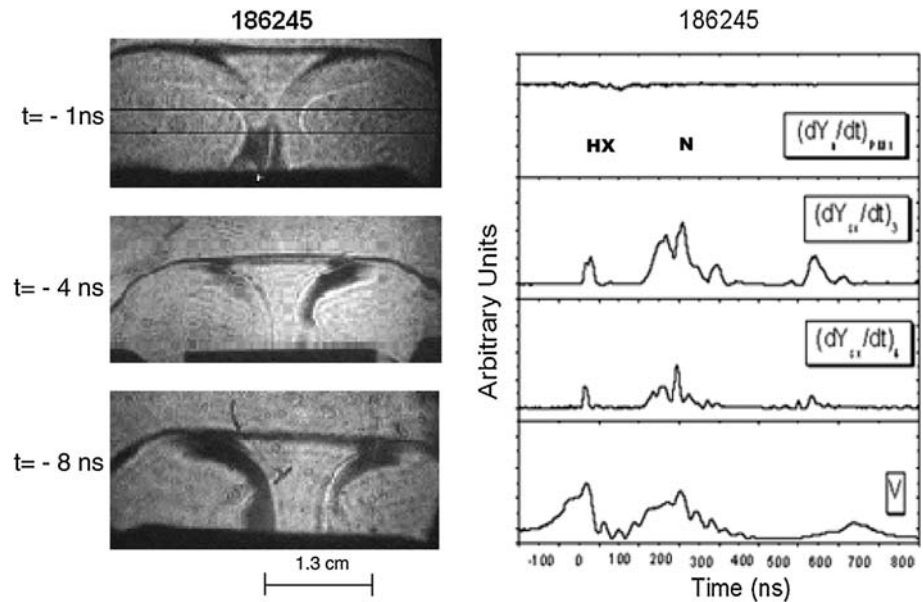


Fig. 4 A three frame sequence of a discharge during the pre-focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{sx}/dt and V . The neutron yield is 1.3×10^8 higher than the average value

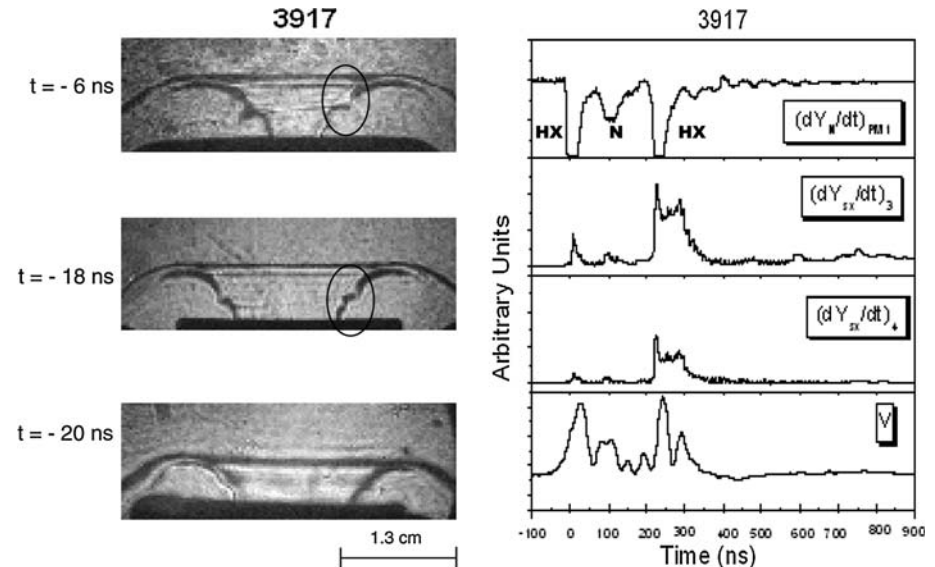


Figure 4 shows a three-frame sequence of a discharge exhibiting neutron yield 1.3×10^8 higher than the average value along with the time resolved signal profiles of hard X-rays (HX) and neutrons dY_N/dt , soft X-rays $(dY_{sx}/dt)_3$ and $(dY_{sx}/dt)_4$ and high voltage probe (V).

In this particular discharge, the development of RT instability (encircled) is seen on both the wings of the sheath. Normally, the growth of instability is associated with a high neutron yield and intense hard X-ray bursts. The average radial velocity for this discharge is 21 cm/ μ s.

The hard X-ray burst (HX) appears immediately at the first focus point ($t = 0$), the signal is saturated which means that intense burst of X-ray is emitted at this instant. Which might be attributed to the hot electrons which are generated immediately after the sheath is tightly focussed

and a very high dense compression is formed. There is emission of neutrons 100 ns after the maximum compression which corresponds to the first focussing event. The neutron signal lasts for about 70–80 ns. The reason for the delayed emission is due to the setting up of a self generated electric fields which causes the separation of the two major species of the plasma. Thus, an ion beam is formed emitted away from the anode tip. The ions undergo beam target fusion resulting in the emission of the neutrons.

Another burst of the hard X-rays observed 22 ns after the first peak which is due to the second focussing event as clearly seen from the voltage signal.

The soft X-rays are also emitted at the first focus point but the intensity is much lower especially of the signal from the 4th detector $(dY_{sx}/dt)_4$. The reason is quite

obvious that at the first focus, the whole of the electron contributed towards the emission of the hard X-rays but a little emission is in the soft X-ray range. However, at the second focus point about 200 ns after the first focus, the soft X-ray emission is quite significant. The reason could be related to the formation of the electron beam after the setting up of the self generated electric field in the plasma causing more collisions and generating more soft X-ray.

Focus Phase

The results presented in this section describe the shadow-graphs and signal profiles taken during and near the maximum compression of the current sheath.

Figure 5 shows a sequence of a three-frame shadow-graph, along with the corresponding time-resolved neutron, X-ray (soft and hard), current and voltage signal profiles. The discharge belongs to multiple-compression regime, as indicated by the voltage signal profile. Neutron yield for this discharge is 4.3×10^7 which is lower than the average value. The development of RT instabilities is seen on both the wings of the funnel shaped current sheath. The sheath seems to have less density. Hard X-ray (HX) and neutrons emission signals (N) are not identified which might be due to the less dense current sheath forming a wider plasma column at the focus point.

An intense burst of soft X-rays is observed. The strong soft X-ray emission is related to the emission of strong relativistic electron beams long after the maximum compression.

Figure 6 shows a sequence of a three-frame shadow-graph, and the corresponding time-resolved neutron, X-ray (soft and hard), current and voltage signal profiles with low

neutron yield (2.4×10^7) in which the development of $m = 0$ instability is clearly seen at the later stage (20 ns) after the pinch (top frame). At $t = 0$, there is the onset of the instability. It appears that the focussed column has not attained the optimum length after the maximum compression. That is why the shot does not have such a high neutron and hard X-ray emission even though there is appearance of the instability long after the maximum compression.

In Fig. 7, two unusual discharge are shown. The neutron yield is 7.4×10^7 quite near to the average. The sheath dynamics (Fig. 7a) seems to be normal during the initial stage (bottom frame), but, 1 ns before the compression the right wing of the sheath appears to move faster than the left one and the compression occurs in a very peculiar manner. It looks like that the column is formed somewhere inside the hollow anode. An RT instability disturbing the left wing can be seen in the middle and the top frames. In Fig. 7b (bottom and middle frames), the column seems to behave like a “tornado”. The column looks twisted and highly compressed at the tip of the anode (bottom frame). This kind of discharges is not commonly observed (only 5% of the recorded discharges). This might be due the onset of the $m = 1$ instability but the B field might not me so strong to let the $m = 1$ instability fully develop.

For the discharge with a very high neutron yield (shown in Fig. 8), the high voltage signal exhibits a single peak indicating a single compression. The neutron yield for this discharge is 1.4×10^8 higher than the average value. The plasma column is broken at three places due to the development of the $m = 0$ instability. The time resolved hard X-ray signal is saturated and it appears immediately after the maximum compression ($t = 0$). About 100 ns after the compression, there is a neutron signal and lasts for about

Fig. 5 A three frame sequence of a discharge during the pre-focus and focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{SX}/dt and V . The neutron yield for this discharge is 4.3×10^7 lower than the average value

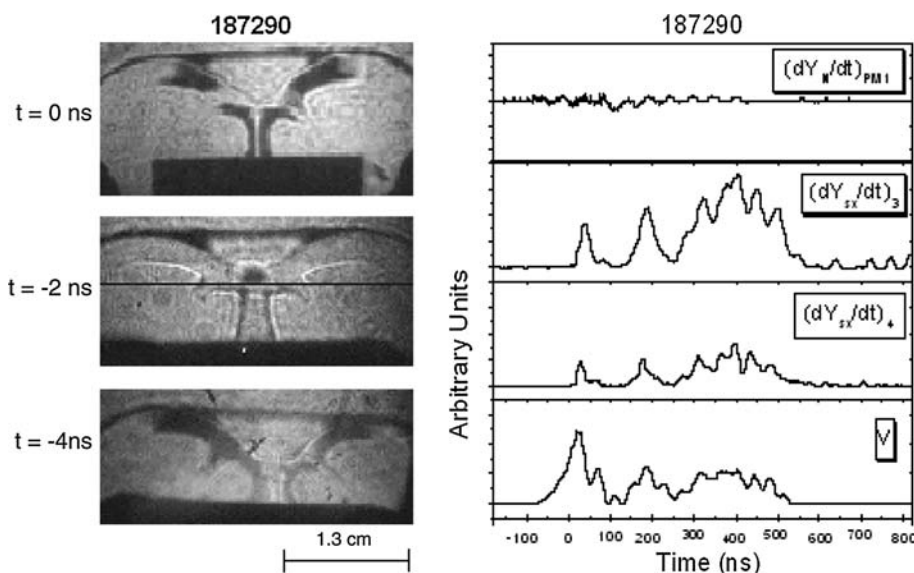


Fig. 6 A three frame sequence of a discharge low neutron yield (2.4×10^7) during the pre-focus and focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{SX}/dt and V

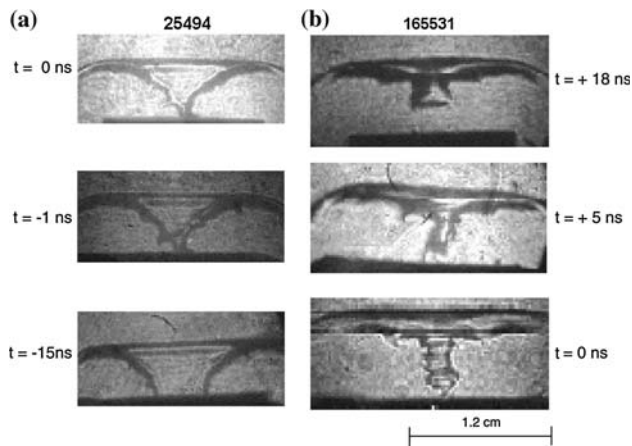
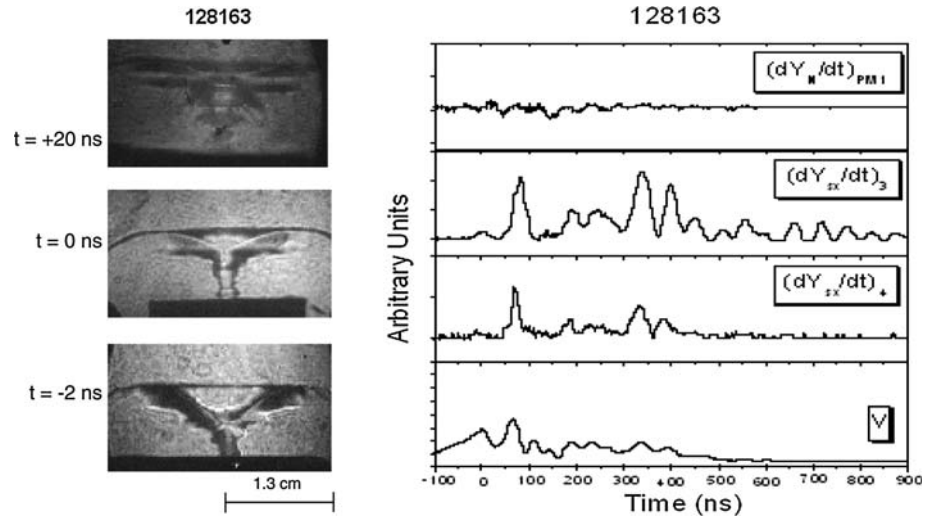
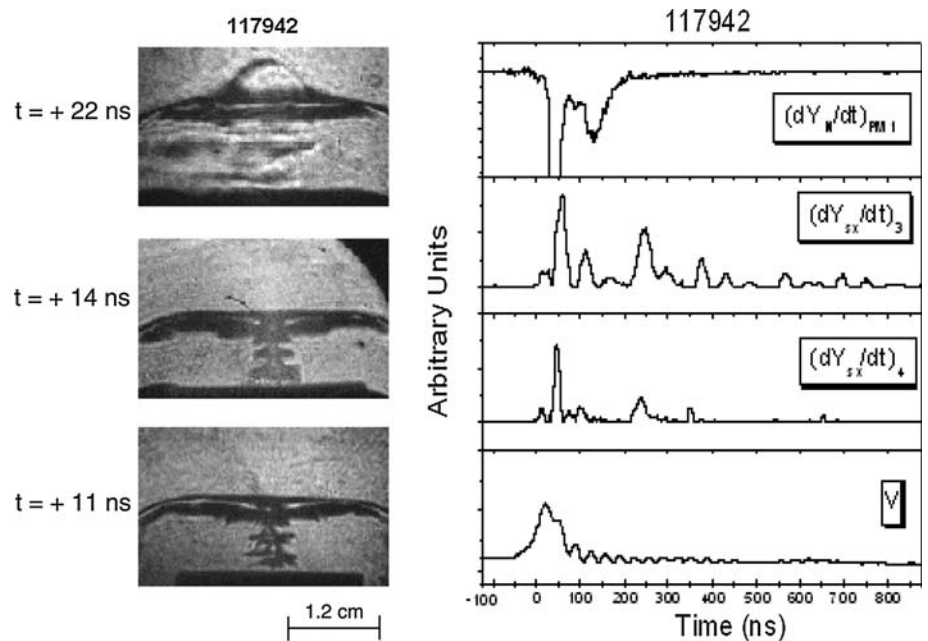


Fig. 7 Three frame sequence of two discharges with average neutron yield during the pre-focus, focus and post focus phases (-15 to $+18$ ns)

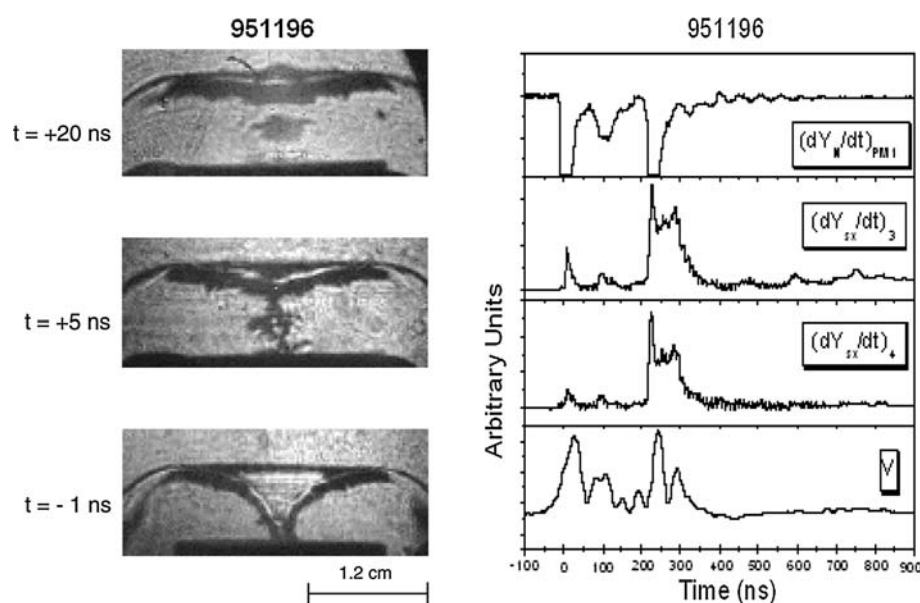
Fig. 8 A three frame sequence of a discharge during the focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{SX}/dt and V . The neutron yield for this discharge is 1.4×10^8 higher than the average value



60 ns. This signal is indicative of a set up of a strong deuteron beams under the influence of a self generated electric fields to undergo a beam target fusion.

An other three frame shadowgraph of a discharge with exceptionally high neutron yield (1.9×10^8) along with the time resolved signal profiles of hard X-rays (HX), neutrons (N), soft X-rays and high voltage is shown in Fig. 9. In the bottom frame, the left wing of the sheath seems to reach earlier than the right one. The column is broken in two places but the column seems to be compressed with an intense magnetic field. The instant at which the $m = 0$ instability starts to destroy the column, the neutron and X-rays emission begins. The breaking up of the column is characterised also by the development of micro instabilities.

Fig. 9 A three frame sequence of a discharge with exceptionally high neutron yield (1.9×10^8) during the focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{SX}/dt and V



The breaking up may also correspond to the density fluctuation connected with the micro turbulences. When the filament breaks up, the spectral analysis [21] of the scattered light indicates that the ion velocity distribution is no longer Maxwellian. Simultaneously, its intensity increases beyond the thermal level and this fact can be used to study the development of the micro instabilities in the plasma. These instabilities are attributed to the current which flows on the axis. The current creates a drift between the electrons and ions. The instabilities are related to the drift velocity as $V_D = 1/n_e e$. This means that with the increase of the drift velocity, the instability growth increases. In other words, the greater the drift velocity between the ion and electron, the greater the number of break-ups and higher the radiation output. Our data agrees with this assumption, because more than 80% of the recorded discharges with the high neutron yield show 2 and sometimes 3 break-ups.

The impedance of the plasma sheath consists of the inductance $L(t)$ and the resistance $R(t)$. Both vary during the propagation and the radial compression. The voltage between the two electrodes across the insulator is $V = d(LI)/dt + RI$. From the values given by magneto-hydrodynamics computation for the electron temperature and the filament diameter, the resistance term is negligible compared to the inductance term. It is known, however, that the instabilities can increase the resistivity well above the Spitzer's expression for the plasma in equilibrium [21]. The dissipation process caused in the thermal plasma by collisions of the current driven electrons can be enhanced when instabilities occur. The anomalous resistance is caused by the collisions of the electrons with the waves when they reach high intensity level. A typical problem in

the plasma focus studies is the role played by the anomalous resistivity in heating the plasma. This is important in heating the plasma in the connective phase of the discharge, following the break ups. This plasma appears to act as a source of deuterons, which are accelerated axially into the plasma formed further away from the anode, and produce neutrons.

Post-Focus Phase

After the compression, the plasma column decays. The decay time depends on the development of the instabilities during the pinch phase.

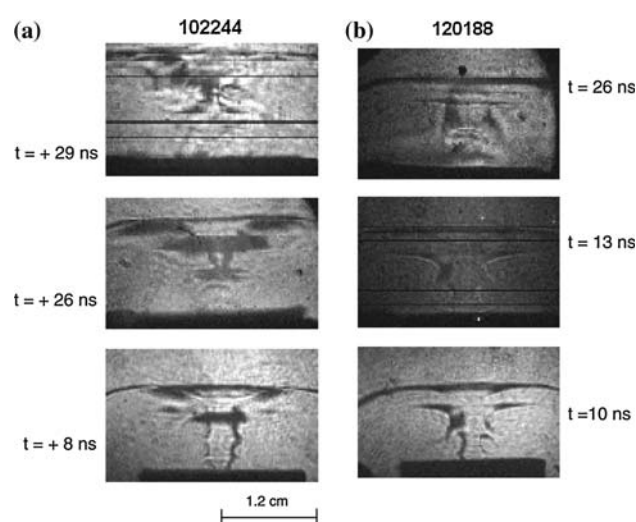


Fig. 10 Three frame sequence of two discharges during the post-focus phase. Both discharges have low neutron yield (a) 3.6×10^7 and (b) 2.8×10^7

Figure 10a, b shows shadowgraphs of two discharges with low neutron yield (3.6×10^7 and 2.8×10^7 respectively). There are distinct ripples formed along the length of the plasma column (Fig. 10a). These kind of ripples are well known and were reported by Herold et al. from schlieren pictures [22]. These ripples are caused by flute instabilities during the compression or at the later stage. The kinks are seen on both sides of the surface of the column. This kind of ripples could also be caused by the $m = 0$ instability, because in the middle and the top frame it seems that the column breaks up in the same manner as it does for the $m = 0$ instability. These ripples might have

been developed from primary current sheath ripples maintained during the maximum compression.

In Fig. 10b, the column has a broken surface on both the sides (bottom frame). It seems that this kind of ripple is a continuity of a RT instability developed during the initial stage of the radial phase. The signs of the development of the $m = 0$ instability is also seen in the same frame. At the later stage of the pinch phase (middle and top frame), the column is broken due to the $m = 0$ instability. It appears that this kind of discharges are neither suitable for the neutron yield, nor for the X-ray production. However, the percentage of this kind of discharges is very low.

Fig. 11 A three frame sequence of a discharge during the post-focus phase along with signal profiles of dY_N/dt , dY_{HX}/dt , dY_{sx}/dt and V . The neutron yield for this discharge is 1.63×10^8 higher than the average value

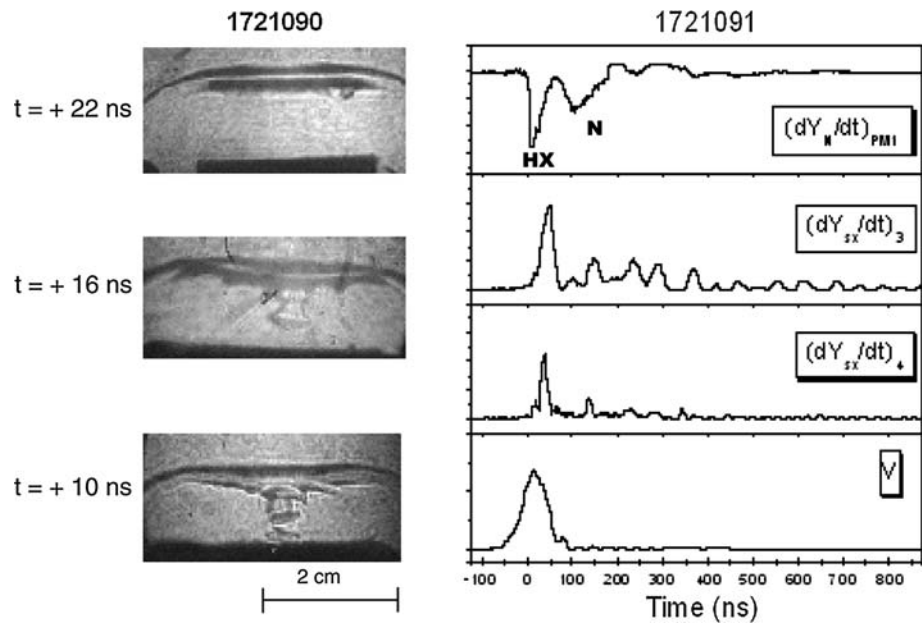


Fig. 12 A three frame sequence of a discharge 500 ns after the first compression. This discharge exhibits a second focusing event

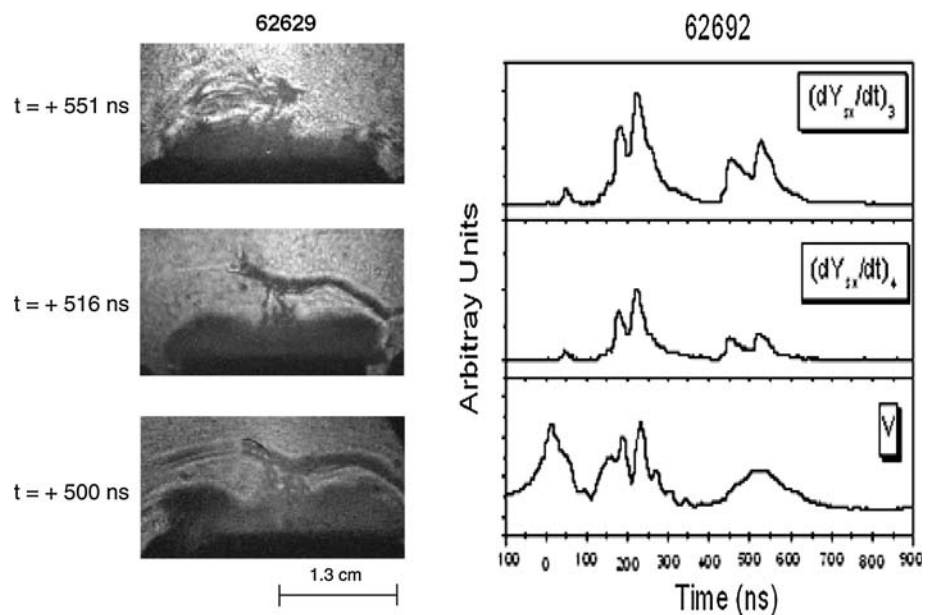


Figure 11 shows a sequence taken during the post-focus phase along with the time resolved signal profiles of hard and soft X-rays, neutrons, high voltage probe and Rogowski coil. The neutron yield for this discharge is 1.63×10^8 higher than the average value. This discharge corresponds to a single compression regime. Time resolved hard X-rays appear exactly at the time of the compression indicating the generation of energetic electrons responsible for the production of hard X-rays. The neutron signals appears 60 ns after the compression again explaining the generation of strong deuteron beams and producing these neutron via beam target mechanism. As far as the soft X-rays are concerned, a single burst is seen a few ns after the compression which is attributed to the low energy remnant electrons after producing hard X-ray burst.

The plasma column in this figure is broken only on one side (bottom frame). This might be the combination of the $m = 0$ and $m = 1$ instabilities occurring simultaneously. The magnetic field might be acting in such a manner that the column is compressed only on one side.

Figure 12 shows an interesting sequence taken 500 ns after the maximum compression. In this discharge, a second focused column, along with the emission of the copper vapours is seen. The voltage probe signal shows a hump at the instant at which the shadowgraph is taken (+500 ns). This clearly indicates the existence of the second compression at this instant. The column is diffused and the current sheath is asymmetrical. There is also a soft X-ray emission during this time. This emission is partly due to the copper vapours from the anode material.

Conclusions

The correlation of shadowgraphic images of the focus with the high voltage probe signal, together with the X-ray and neutron emission characteristics, enabled us to identify the important regimes of the focus. The focus operates in two regimes namely; the single compression regime and the multiple compression regime. The multiple compression regime is indicative of the low radiation output, especially the neutron and hard X-ray and is supportive to the soft X-rays whereas the single compression regime is best suited for the high neutron and hard X-ray output.

For the discharges with low neutron yield the sheath is not smooth and the radial compression does not favour the high dense column formation with no disturbances due to instabilities. Whereas, for the discharges with high neutron

yield, there is multiple break ups of the plasma column due to on set of $m = 0$ instability which favours the high density plasma and in turn the high neutron and hard X-ray output. However, the soft X-ray out put is relatively higher for the discharges with low neutron yield or vice versa. Therefore, the conditions for the plasma focus to operate in the neutron-optimized mode are different from that for the soft X-ray optimized mode.

During the initial stage of the radial phase the sheath moves with velocities ranging from 10–23 cm/ μ s while at later stages (up till the maximum compression), the velocities reach 32–42 cm/ μ s.

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Pinch Current and Soft X-Ray Yield Limitations by Numerical Experiments on Nitrogen Plasma Focus

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Abstract The modified version of the Lee model code RADPF5-15a is used to run numerical experiments with nitrogen gas, for optimizing the nitrogen soft X-ray yield on PF-SY1. The static inductance L_0 of the capacitor bank is progressively reduced to assess the effect on pinch current I_{pinch} . The experiments confirm the I_{pinch} limitation effect in plasma focus, where there is an optimum L_0 below which although the peak total current, I_{peak} , continues to increase progressively with progressively reduced inductance L_0 , the I_{pinch} and consequently the soft X-ray yield, Y_{sxr} , of that plasma focus would not increase, but instead decreases. For the PF-SY1 with capacitance of 25 μF , the optimum $L_0 = 5$ nH, at which $I_{\text{pinch}} = 254$ kA, $Y_{\text{sxr}} = 5$ J; reducing L_0 further increases neither I_{pinch} nor nitrogen Y_{sxr} . The obtained results indicate that reducing the present L_0 of the PF-SY1 device will increase the nitrogen soft X-ray yield.

Keywords Plasma focus SY1 · Pinch current limitation · Soft X-ray · Nitrogen gas · Lee model RADPF5.15a

Introduction

The plasma focus is well known as a source of fusion neutrons and X-rays. Besides being a ready source of hot dense plasma and fusion neutrons, the focus also emits plentiful amounts of soft X-rays, especially when operated with high Z gases rather than deuterium. Because of its simple construction, cost-effectiveness and easy maintenance, the plasma focus appears to be a promising device for X-ray generation, with enhanced efficiency. The nitrogen plasma focus is used as an emitter of the X-ray radiation [1–3].

The total current I_{total} waveform, which is a “fingerprint” of the plasma focus discharge, is easily measured using a Rogowski coil, and from experience, it is known that the current trace of the focus is one of the best indicators of gross performance [4–9]. The focus pinch current I_{pinch} , which is defined as the value of the plasma sheath current at the start of pinch, is difficult to measure and this is the reason that the total current I_{peak} is experimentally used instead of I_{pinch} , despite the fact that yields should more consistently be scaled to the focus pinch current I_{pinch} , since it is I_{pinch} which directly powers the emission processes. The numerical method to consistently deduce I_{pinch} from any measured trace of I_{total} was developed in numerical experiments using the Lee Model [4–9].

For enhancing of the neutron and X-ray yields from plasma focus devices, many experiments have been investigated by some modifications on the bank, tube and operating parameters of the devices; for example, the two plasma focus devices UNU/ICTP PFF and the NX2 both have capacitance of about 30 μF and maximum operating voltage V_0 of 15 kV. The UNU/ICTP PFF has L_0 of 110 nH whilst the NX2 was designed for much higher performance with $L_0 = 20$ nH. As a result of the much

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Numerical Experiments on Oxygen Soft X-Ray Emissions from Low Energy Plasma Focus Using Lee Model

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Abstract The X-ray emission properties of oxygen plasmas are numerically investigated using corona plasma equilibrium model. The Lee model is here modified to include oxygen in addition to other gases. It is then applied to characterize the Rico Plasma Focus (1 kJ), finding a oxygen soft X-ray yield (Y_{sxr}) of 0.04 mJ in its typical operation. Keeping the bank parameters and operational voltage unchanged but systematically changing other parameters, numerical experiments were performed finding the optimum combination of pressure = 3 Torr, anode length = 1.5 cm and anode radius = 1.29 cm. The optimum Y_{sxr} was 43 mJ. Thus we expect to increase the oxygen Y_{sxr} of PF-1 kJ thousand-fold from its present typical operation; without changing the capacitor bank, merely by changing the electrode configuration and operating pressure. The modified version of the Lee model code is also used to run numerical experiments with oxygen gas, for optimizing the oxygen soft X-ray yield on the new plasma focus device PF-SY2 (2.8 kJ). The static inductance L_0 of the capacitor bank is progressively reduced to assess the effect on pinch current I_{pinch} . The experiments

confirm the I_{pinch} , limitation effect in plasma focus, where there is an optimum L_0 below which although the peak total current, I_{peak} , continues to increase progressively with progressively reduced inductance L_0 , the I_{pinch} and consequently the soft X-ray yield, Y_{sxr} , of that plasma focus would not increase, but instead decreases. The obtained results indicate that reducing the present L_0 of the PF-SY2 device will increase the oxygen soft X-ray yield till the maximum value after that the Y_{sxr} will decrease with I_{pinch} decreasing.

Keywords Low energy plasma focus · Soft X-ray · Oxygen gas · Lee Model RADPF5.15 K

Introduction

The dynamics of plasma focus discharges is complicated; for this purpose, to investigate the plasma focus phenomena, the Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation, enabling realistic simulation of all gross focus properties.

In the radial phases, axial acceleration and ejection of mass are caused by necking curvatures of the pinching current sheath result in time-dependent strongly center-peaked density distributions. Moreover laboratory measurements show that rapid plasma/current disruptions result in localized regions of high densities and temperatures particularly in the heavy gases like xenon. We need to point out that these center-peaking density effects and localized regions are not modeled in the code, which consequently computes only an average uniform density and an average uniform temperature which are considerably lower than measured peak density and temperature. However, because the 4-model parameters are obtained by

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fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modeled. Hence the computed gross features such as speeds and trajectories and integrated soft X-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

Thus the code provides a useful tool to conduct scoping studies, as it is not purely a theoretical code, but offers means to conduct phenomenological scaling studies for any plasma focus device from low energy to high energy machines.

The model in its two-phase form was described in 1984 [1]. It was successfully used to assist in the design and interpretation of several experiments [2–6]. Radiation-coupled dynamics was included in the five-phase code leading to numerical experiments on radiation cooling [7]. The vital role of a finite small disturbance speed discussed by Potter [8] in a Z-pinch situation was incorporated together with real gas thermodynamics and radiation-yield terms. Before this ‘communication delay effect’ was incorporated, the model consistently over-estimated the radial speeds by a factor of ~ 2 and shock temperatures by a factor ~ 4 . This version using the ‘signal-delay slug’ assisted other research projects [9–11] and was web-published in 2000 [12] and 2005 [13]. All subsequent versions of the Lee model code incorporate the ‘signal-delay slug’ as a must-have feature. Plasma self-absorption was included in 2007 [12] improving soft X-ray yield simulation in neon, argon and xenon among other gases. The model has been used extensively as a complementary facility in several machines, for example, UNU/ICTP PFF [2, 5, 9, 10, 14–17], NX1 [10, 18] NX2 [10, 11] and DENA [19]. It has also been used in other machines for design and interpretation including sub-kJ plasma focus machines [20]. Information obtained from the model includes axial and radial velocities and dynamics [19], dimensions and duration of the focus pinch, gross information of temperatures and densities within the pinch, soft X-ray emission characteristics and yield [10, 11, 21], design and optimization [20, 21], of machines, and adaptation to other machine types such as the Filippov-type DENA [19]. The versatility and utility of the improved model is demonstrated in the clear distinction of pinch current from the peak current [22] and the recent uncovering of a plasma focus pinch current limitation effect [23–26]. The detailed description, theory, latest code and a broad range of results of this ‘Universal Plasma Focus Laboratory Facility’ are available for download from ref. [27].

Oxygen has been used in plasma focus devices as a rich ion source for material science applications [28, 29].

To characterize the plasma focus device operated in oxygen, in this work the Lee model code was modified to include oxygen gas. For this purpose, the oxygen ionization energy data was extracted from NIST [30], and using the corona-model sub-routines available on the IPFS website [27], thermodynamic data such as ionization curves, effective charge numbers and specific heat ratios for oxygen were calculated. Next, these data were fed into the latest Lee code RADPF5.15 K using the 6-polynomials method for the oxygen thermodynamic data.

This work has progressed the Lee model code to the version RADPF5.15 K, which enables to run numerical experiments with the following gases: hydrogen, deuterium, deuterium-tritium, helium, neon, argon, xenon, krypton, nitrogen and oxygen.

In this paper, the Lee Model RADPF5.15 K was used in numerical experiments on two low energy plasma focus devices: Rico Plasma focus (1 kJ) [28, 29] and PF-SY2 (2.8 kJ) operating with oxygen gas.

Calculations of Oxygen Plasma Parameters Using Corona Model

The X-ray radiation properties of plasma are dependent on the plasma temperature, ionization states and density. Plasma equilibrium model can be used to calculate the ion fraction α , the effective ionic charge number Z_{eff} , the effective specific heat ratio γ and X-ray emission of the plasma at different temperatures.

The corona model [11, 27, 31–33] has been used as an approximation for computing the thermodynamic data of the oxygen plasma in the plasma focus. The data of ionization potentials and X-ray emission spectrum of highly ionized oxygen plasma are taken from NIST [30]. Based on the corona model, the ion fraction, effective ionic charge number and effective specific heat ratio for oxygen plasma have been calculated at different temperatures, for more details see [11, 32]. The obtained results for the ion fraction and effective ionic charge number are shown in Figs. 1, 2.

Looking at the results displayed in Fig. 1, the suitable temperature range for generating H-like 1s-2p, O₂: 18.97 Å ($h\nu_1 = 653.68$ eV), 1s-3p, O₂: 16 Å ($h\nu_2 = 774.634$ eV) and He-like 1s²-1s2p, O₂: 21.6 Å ($h\nu_3 = 573.947$ eV), 1s²-1s3p, O₂: 18.62 Å ($h\nu_4 = 665.615$) ions in oxygen plasma (therefore soft X-ray emissions) is between 119–260 eV (1.38×10^6 – 3×10^6 K). Also the important feature can be seen from Fig. 2, that the temperatures range 49.44–80.53 eV corresponds to the 1s² close shell for the oxygen ions. Therefore, the yield from X-ray line emissions is low in this temperature range from oxygen. And it can be noticed that the oxygen atoms become fully ionized around 2,000–3,000 eV.

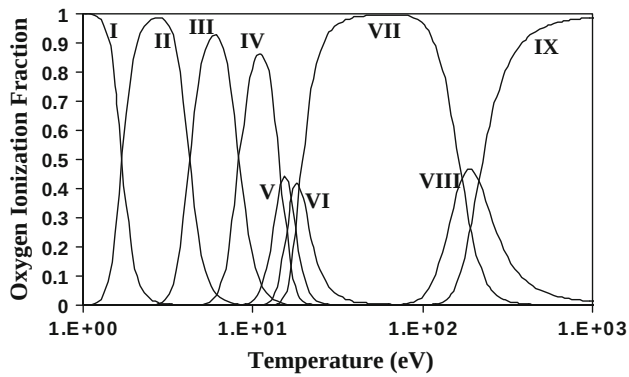


Fig. 1 Oxygen ion fractions at different temperature, where IX indicates the ion O^{+8}

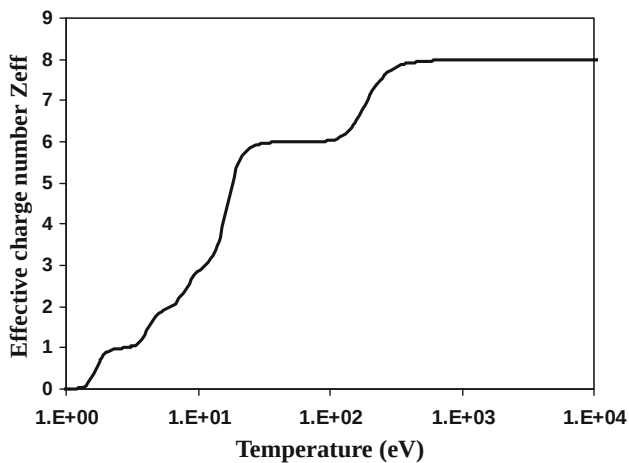


Fig. 2 Effective ionization number Z_{eff} of oxygen (calculated by corona model)

The power of line emissions can also be calculated [11, 32], and the intensities of Lyman—alpha (Ly_{α}), helium—alpha (He_{α}) lines are proportional to the H-like and He-like ion densities, respectively. Using the following equations the radiation power of Ly_{α} , Ly_{β} and He_{α} , He_{β} lines for oxygen as functions of photon energy $h\nu$ are [11]:

$$P_{Ly\alpha(1s-2p)} = k_1 \cdot N_i^2 \cdot \alpha_7 \cdot Z_{\text{eff}} \cdot e^{\left(\frac{-h\nu_1}{T_{\text{ev}}}\right)} / \sqrt{T_{\text{ev}}} \quad (1)$$

$$P_{Ly\beta(1s-3p)} = k_2 \cdot N_i^2 \cdot \alpha_7 \cdot Z_{\text{eff}} \cdot e^{\left(\frac{-h\nu_2}{T_{\text{ev}}}\right)} / \sqrt{T_{\text{ev}}} \quad (2)$$

$$P_{He\alpha(1s^2-1s2p)} = k_3 \cdot N_i^2 \cdot \alpha_6 \cdot Z_{\text{eff}} \cdot e^{\left(\frac{-h\nu_3}{T_{\text{ev}}}\right)} / \sqrt{T_{\text{ev}}} \quad (3)$$

$$P_{He\beta(1s^2-1s3p)} = k_4 \cdot N_i^2 \cdot \alpha_6 \cdot Z_{\text{eff}} \cdot e^{\left(\frac{-h\nu_4}{T_{\text{ev}}}\right)} / \sqrt{T_{\text{ev}}} \quad (4)$$

where: $k_1 = 2 \times 10^{-31}$, $k_2 = 4 \times 10^{-32}$, $k_3 = 1.3 \times 10^{-31}$, $k_4 = 3 \times 10^{-32}$. From these equations it can be seen that the radiation power is proportional to the density squared, ion fraction and Z_{eff} . Then the normalized emission intensity can be calculated by putting $N_i = 1$. The calculated Ly and He emission intensities from one oxygen ion at unit density are given in Fig. 3. The related ion

fractions are also plotted in this figure, which clearly shows the relationship between the line emission intensity with corresponding ions. The peaks of the Ly and He lines are located at the higher temperature side of the H-like and He-like ion distributions. It means the ions need to be heated to excited states to give X-ray emissions. The locations of the peaks give us a rough knowledge of the optimum temperatures for generating X-rays from oxygen plasma, i.e. 225 eV. Choice of optimum temperature is made in comparison with the optimum temperatures chosen for nitrogen [33], neon and argon in Shan Bing's work [11]. The comparison is shown in Table 1. It can be seen that much more energy is required to heat the argon to its X-ray optimum temperature. Hence the usual method for increasing the plasma temperature (by increase the energy density) is to decrease the filling gas pressure. However, the X-ray yield is also related to the total number of X-ray emitters which is proportional to the gas pressure. Therefore, the optimum oxygen X-ray yield temperature may be lower than 260 eV in a plasma focus.

X-Ray Emissions in Plasma Focus and its Incorporation in Model Code

The focused plasma, with electron temperature of a few hundreds eV to about keV and high enough electron density, is a copious source of X-rays. The plasma focus emits both soft (thermal) as well as hard (non-thermal) X-rays but for the scope of this paper we will concentrate only on soft thermal X-rays. The plasma focus emits soft thermal X-rays by three processes [34, 35], namely: Bremsstrahlung (free-free transition) from the coulomb interactions between electrons and ions; recombination radiation (free-bound transition) emitted by an initially free electron as it

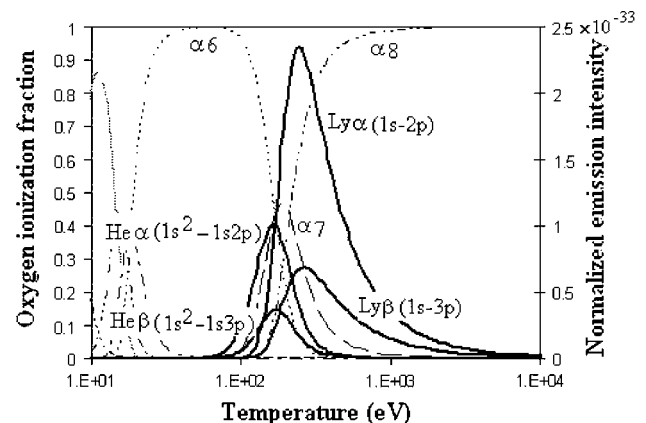


Fig. 3 Calculated X-ray line emission intensities from oxygen Ly and He lines vs temperature. The dot lines are the ions fractions (calculated by corona model)

Table 1 Optimization conditions for X-ray radiative plasma of Oxygen, Nitrogen, Neon and Argon

Gas	Temperature	Z_{eff}	$E_{\text{ion}} = E_i + (3/2)(1 + Z_{\text{eff}})kT$
Oxygen	~ 225 eV	7.38	$1.59 + 2.82 \text{ keV}$
Nitrogen	~ 160 eV	6.48	$1.16 + 1.79 \text{ keV}$ [33]
Neon	~ 420 eV	9.38	$2.69 + 4.36 \text{ keV}$ [11]
Argon	~ 3 keV	17.00	$11.03 + 54 \text{ keV}$ [11]

loses energy on recombination with an ion; and de-excitation radiation (bound-bound transition) when a bound electron loses energy by falling to a lower ionic energy state. The first two processes give rise to the continuum of the X-ray spectrum while the third process produces the characteristic line radiation of the plasma. The relative strengths of the continuum and line emissions depend on how the plasma was formed; typically, for plasma formed from a high-Z material continuum emission dominates, while for a low-Z material line emission can be stronger. The calculation of the power emitted by processes within the plasma depends on assumptions made about the state of the plasma. In the code [27, 36–38] in pinch phase, line radiation Q_L is calculated using the relation

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z_{\text{eff}}^4 Z_n^4 (\pi a_{\text{min}}^2) Z_{\text{max}} / T \quad (5)$$

after being integrated over the pinch duration. Hence the soft X-ray energy generated within the plasma pinch depends on the properties: number density N_i , effective charge number Z_{eff} , atomic number of gas Z_n , pinch radius a_{min} , pinch length Z_{max} , plasma temperature T and the pinch duration. This generated energy is then reduced by the plasma self-absorption which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the soft X-ray yield.

Based on the corona model, in the code we take the oxygen soft X-ray yield (generation H-like and He-like ions) to be equivalent to line radiation yield i.e. $Y_{\text{sxr}} = Q_L$ at the following temperature range 119–260 eV.

Procedures for Numerical Experiments Using RADPF5.15 K

The Lee code is configured to work as any plasma focus by inputting the bank parameters, the tube parameters, operational parameters and the fill gas. The standard practice is to fit the computed total current waveform to an experimentally measured total current waveform using four model parameters representing the mass swept-up factor f_m , the plasma current factor f_c for the axial phase and factors f_{mr} and f_{cr} for the radial phase.

The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace. The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of the mass swept up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The detailed profile of the discharge current during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus, the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed current trace to the measured current trace in the procedure adopted by the Lee model code [12, 13, 22–24, 39, 40].

The numerical experiments for soft X-ray optimization from oxygen plasma were investigated on two low energy plasma focus devices: Rico Plasma focus (1 kJ) and PF-SY2.

PF-1 kJ:-The Numerical Experiments

The Rico Plasma focus (1 kJ) [28, 29] operated with oxygen filling gas at the following bank and tube parameters:

Bank parameters: $L_0 = 65 \text{ nH}$, $C_0 = 3.86 \text{ } \mu\text{F}$, $r_0 = 22 \text{ m}\Omega$,

Tube parameters: $a = 1.75 \text{ cm}$, $b = 4.9 \text{ cm}$, $z_0 = 6.75 \text{ cm}$,

Operating parameters: $V_0 = 14.9 \text{ kV}$, $p_0 = 0.2 \text{ Torr}$, oxygen gas,

where L_0 is the static inductance (nominal), C_0 the storage capacitance (nominal), b the tube outer radius, a the inner radius, z_0 the anode length, V_0 the operating voltage and p_0 the operating initial pressure. The measured current derivative waveform at the above conditions is shown in Fig. 4.

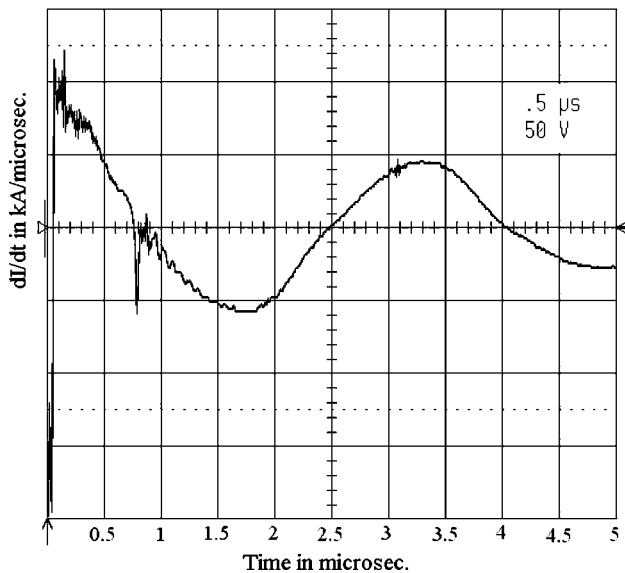


Fig. 4 The temporal evolution of derivative current of the oxygen discharge during the plasma focus formation in PF-1 kJ. $V_0 = 14.9$ kV, $p_0 = 0.2$ Torr, $C_0 = 3.86$ μ F, $L_0 = 65$ nH, $r_0 = 22$ m Ω

We first digitize the measured current derivative waveform using an open access source digitizing program, Engauge [41] and then integrate the data with time to obtain the current waveform. Then we fit the computed current waveform to the measured waveform as follows:

We configure the Lee model code (version RADPF5.15 K) to operate as the Rico Plasma focus (1 kJ) starting with the above bank and tube parameters.

To obtain a reasonably good fit the following parameters are used:

Bank parameters: $L_0 = 69$ nH, $C_0 = 3.86$ μ F, $r_0 = 20$ m Ω ,

Tube parameters: $b = 4.9$ cm, $a = 1.75$ cm, $z_0 = 6.75$ cm,

Operating parameters: $V_0 = 14.9$ kV, $p_0 = 0.2$ Torr, oxygen gas,

together with the following fitted model parameters:

$f_m = 0.004$, $f_c = 0.7$, $f_{mr} = 0.015$ and $f_{cr} = 0.45$.

It can be seen that the computed discharge current waveform agrees well with the measured current waveform up to and slightly beyond the bottom of the current dip (Fig. 5). This means that the agreement covers all the regions of interest from axial to radial phases up to the end of the pinch phase; all five plasma focus phases of interest to us.

The numerical experiments using RADPF5.15 K at the bank and tube parameters last mentioned above and using the fitted model parameters give then the following results: the end axial speed to be $V_a = 17.6$ cm/ μ s, the final plasma column is 0.19 cm in radius, and 2.1 cm in length.

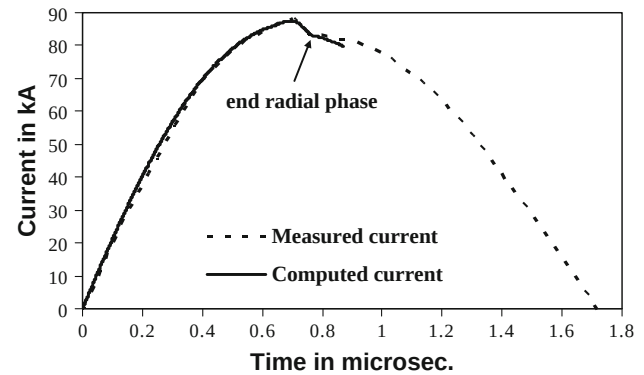


Fig. 5 Comparison of the computed current trace (solid smooth line) with the experimental one (dotted line) of the PF-1 kJ at 14.9 kV, 0.2 Torr at oxygen filling gas

Also the Ysxr emitted from the oxygen plasma is calculated at the above conditions, to be 0.04 mJ (see Table 2). From Table 2 at $p_0 = 0.2$ Torr, it can be found, that the axial speed is very high, and consequently the temperature will be higher than suitable for soft X-ray generation. This low soft X-ray yield from Rico Plasma focus (1 kJ) at experimental conditions is already expected, since Rico Plasma focus (1 kJ) was modified to use as ion beam source for material science applications, and not to generate soft X-ray or line radiation.

As the first step, the code RADPF5.15 K was run to optimize X-ray yield from PF-1 kJ with oxygen gas as function of only pressure; fixing all the mentioned above parameters. The pressure was varied from 0.2 to 0.9 Torr.

As is well known, when the operating pressure is increased, the plasma speeds decrease; hence, the duration of the axial phase increases. From Table 2 it is seen that the Ysxr increases with increasing pressure until it reaches the maximum value about 6.8 mJ at $p_0 = 0.745$ Torr, after which it decreases with higher pressures. As expected as p_0 is increased, the end axial speed, the inward shock speed and the radial piston speed all reduced. The decrease in speeds lead to lowering of plasma temperatures below that needed for soft X-ray production. From Table 2 we note that a shift of operating pressure to 0.745 Torr would increase the computed Ysxr to 6.8 mJ.

To optimize the soft X-ray yield from Rico Plasma focus (1 kJ) with oxygen gas, more numerical experiments were carried out with the above model parameters; but varying p_0 , z_0 and 'a' keeping $c = b/a$ constant at value $c = 2.8$. The pressure p_0 was varied from 1 to 15 Torr.

The following procedure was used [33]:

- At each p_0 , the anode length z_0 was fixed at a certain value,
- Then the anode radius 'a' was smoothly varied, till the maximum X-ray yield (Ysxr) was obtained for this certain value of z_0 .

Table 2 Variation PF-1 kJ parameters with pressure at: $L_0 = 69$ nH, $C_0 = 3.86$ μ F, $r_0 = 20$ m Ω , $V_0 = 14.9$ kV, RESF = 0.150, $c = b/a = 2.8$, $f_m = 0.004$, $f_c = 0.7$, $f_{mr} = 0.015$, $f_{cr} = 0.45$, oxygen gas

p_0 (Torr)	I_{peak} (kA)	I_{pinch} (kA)	V_a (cm/ μ s)	V_s (cm/ μ s)	V_p (cm/ μ s)	Pinch dur. (ns)	T_{pinch} (10^6 K)	Y_{srx} (mJ)
0.2	87	38	17.6	35.3	24.4	12.7	8.39	0.04
0.3	89	38	15.2	29.8	20.9	14.8	5.62	0.16
0.5	91	35	12.4	22.8	16.3	19.2	2.94	1.2
0.7	92	30	10.6	17.9	13.1	24.9	1.60	5.1
0.72	92	30	10.5	17.5	12.8	25.6	1.50	5.7
0.73	92	29	10.4	17.2	12.7	25.9	1.45	6.1
0.74	92	29	10.4	17.0	12.5	26.2	1.41	6.5
0.745	92	29	10.3	16.9	12.5	26.5	1.38	6.8
0.75	92	29	10.3	16.8	12.4	26.7	1.36	6
0.77	93	28	10.2	16.4	12.1	27.6	1.27	2
0.8	93	27	10.0	15.8	11.7	28.9	1.14	0
0.9	93	24	9.4	13.8	10.4	34.5	0.78	0

- After that, we chose another value of z_0 , varying the value of 'a' looking for the maximum of Y_{srx} , until we found the optimum combination of z_0 and 'a' for the best X-ray yield at the fixed p_0 .
- Then we changed p_0 and repeated the above procedure to find the optimum combination of z_0 and 'a' corresponding to this new value of p_0 . We proceed until we had obtained the optimum combination of p_0 , z_0 and 'a' for the maximum soft X-ray yield.

For optimum oxygen Y_{srx} , as mentioned earlier, there is an optimum temperature. This implies that there is an optimum speed factor [6] $S = (I_{peak}/a)/p_0^{0.5}$. As p_0 was increased in order to maintain the optimum S , (I_{peak}/a) had to be correspondingly increased, by a reduction of 'a'. The numerical experiments also showed that z_0 needed to be increased to optimize the Y_{srx} (see Table 3). Thus whilst external inductance L_0 is fixed at a constant value and an axial section inductance L_a is increased due to increasing the anode length, the pinch inductance L_p is reduced due to decreasing the pinch length [6, 23].

The optimized results for each value of p_0 are shown in table 3. The table shows that as p_0 is increased, anode length z_0 rises and inner radius 'a' decreases with each increase in p_0 , while the soft X-ray yield slightly increases

with increasing p_0 until it reaches a maximum value of 43 mJ at $p_0 = 3$ Torr; then the Y_{srx} decreases with further pressure increase.

Nevertheless the numerical experiments have shown that with the present capacitor bank, Rico Plasma focus (1 kJ) can be improved from its present computed Y_{srx} of 0.04 mJ corresponding to the yield with its present geometry and usual operating pressure. The optimum geometry requires making the anode length and the anode radius shorter, at the same time increasing its operational pressure.

PF-SY2 (2.8 kJ)-The Numerical Experiments

The numerical experiments were investigated using the parameters of the low energy plasma focus PF-SY2 and optimizing for a X-ray source. The bank parameters were $L_0 = 200$ nH, $C_0 = 25$ μ F and $r_0 = 14$ m Ω . The tube parameters were the outer radius $b = 3.2$ cm, the inner radius $a = 0.95$ cm, and the anode length $z_0 = 16$ cm. The operating parameters were $V_0 = 15$ kV, and $p_0 = 10$ Torr, filling oxygen gas. The above mentioned parameters were put into the code RADPF5.15 K.

Table 3 X-ray yield optimization from PF-1 kJ for each value of p_0 varying z_0 and 'a' at filling oxygen gas

p_0 (Torr)	z_0 (cm)	a (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_{srx} (mJ)	V_a (cm/ μ s)	a_{min} (cm)	z_{max} (cm)
1.0	1.0	2.18	92	40	41	4.8	0.20	2.7
2.0	1.3	1.57	92	41	42	5.3	0.14	1.9
3.0	1.5	1.29	92	41	43	5.6	0.12	1.6
5.0	1.8	1.01	93	41	42	5.9	0.09	1.2
10.0	2.8	0.71	95	42	37	6.7	0.07	0.9
15.0	3.0	0.58	95	42	33	6.8	0.05	0.7

$L_0 = 69$ nH, $C_0 = 3.86$ μ F, $r_0 = 20$ m Ω , $V_0 = 14.9$ kV, RESF = 0.150, $c = b/a = 2.8$, $f_m = 0.004$, $f_c = 0.7$, $f_{mr} = 0.015$, $f_{cr} = 0.45$

In this work we would like to present the Mather-type plasma focus device PF-SY2 as a X-ray source with oxygen filling gas using Lee model RADPF5.15 K.

The numerical experiments were conducted using a constant value of a factor $RESF = 0.157$ ($RESF$ = stray resistance/surge impedance), where at each L_0 the corresponding resistance value was found. Also at each L_0 the ratio ($c = b/a$) was kept constant at value $c = 3.368$.

To optimize the soft X-ray yield from PF-SY2 with oxygen gas, varying L_0 , z_0 and 'a' keeping 'c' and $RESF$ constant. The external inductance L_0 was varied from 200 to 1 nH.

As we haven't any oxygen measured current trace from PF-SY2, the numerical experiments for optimization soft X-ray from oxygen plasma were carried out with the two different model parameters:

The first model parameters: $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.1$, and $f_{cr} = 0.7$,

The second model parameters: $f_m = 0.03$, $f_c = 0.7$, $f_{mr} = 0.08$, and $f_{cr} = 0.7$.

The following procedures were used:

At each L_0 , the pressure was fixed at constant value (in our case $p_0 = 10$ Torr) and also the anode length was fixed at a certain value:

- Then the inner radius 'a' was varied, whilst keeping $c = 3.368$, until the maximum X-ray yield was obtained for this certain value of z_0 .
- After that we chose another value of z_0 , varying 'a' until maximum X-ray yield and so on, until we have obtained the combination of z_0 and 'a' for the best maximum X-ray yield at a fixed L_0 (Y_{srx} vs z_0 and 'a' at fixed L_0 and p_0).
- We repeated the above procedure for progressively smaller L_0 until $L_0 = 1$ nH.

At each L_0 , after z_0 was varied, the inner radius 'a' was adjusted to obtain the optimum X-ray yield, which we find to correspond closely to the largest I_{pinch} .

The soft X-ray optimization for each value of L_0 , varying z_0 and 'a' is shown in Tables 4, 5. The tables show that as L_0 is reduced, I_{peak} increases with each reduction in L_0 with no sign of any limitation as function of L_0 . However, I_{pinch} reaches a maximum value at $L_0 = 5$ nH, then it decreases with each reduction in L_0 , but the ratio I_{pinch}/I_{peak} drops progressively as L_0 decreases. Thus I_{peak} does not show any limitation as L_0 is progressively reduced. However, I_{pinch} has a maximum value. This pinch current limitation effect is not a simple, but it is a combination of the two complex effects: the interplay of the various inductances involved in the plasma focus processes abetted by the increasing coupling of C_0 to the inductive energetic processes, as L_0 is reduced [23, 25].

From Tables 4, 5 it can be seen, that as L_0 is decreased, the soft X-ray yield increases until it reaches a maximum value of 10 J at $L_0 = 5$ nH (where I_{pinch} also has maximum); beyond which the soft X-ray yield does not increase with reducing L_0 . Thus with decreasing L_0 the pinch current I_{pinch} and the soft X-ray yield show limitation. The obtained results confirm the pinch current limitation effect in oxygen plasma focus, and consequently the soft X-ray yield. Figures 6, 7 represent I_{pinch} and X-ray limitation effects in oxygen plasma focus at 10 Torr as L_0 is reduced from 200 to 1 nH.

Looking at Tables 4 and 5, it is noticed that as L_0 was progressively reduced, to optimize 'a' had to be progressively increased and z_0 progressively decreased. Also the plasma pinch dimensions (pinch radius a_{min} and pinch length Z_{max}) increased as L_0 was reduced.

As the external inductance L_0 is lowered from 200 to 1 nH, the tube inductance ($L_a = 2 \times 10^{-7} \ln(b/a) Z_0$) is decreased and the focus pinch inductance ($L_p = \sim \ln(b/a_{min}) Z_{max}$) is increased [23, 39].

Based on the obtained results of these sets of numerical experiments on PF-SY2 with oxygen gas, we can say that to improve the soft X-ray yield, L_0 should be reduced to a value around 10–15 nH (which is an achievable range incorporating low inductance technology [10]), below which the pinch current I_{pinch} and the soft X-ray yield Y_{srx} would not be improved much, if at all. These experiments confirm the pinch current limitation effect, and consequently the soft X-ray yield for the oxygen plasma focus. Finally, we would like to emphasize that we, practically, have no intention (or ambition) to go below 10–15 nH (which is an achievable range), but in our numerical experiments using RADPF5.15 K we go down to low values (5–1 nH) just to find the pinch current limitation effect.

Conclusions

The required oxygen plasma thermodynamic parameters (the ion fraction, effective ionic charge number and effective specific heat ratio) were calculated at different temperatures and the X-ray emission properties of oxygen plasma were investigated using corona model.

The Lee model RADPF5.15a was modified to get RADPF5.15 K which includes oxygen gas and it was used to characterize the Rico Plasma focus (1 kJ) using its experimental parameters. The soft X-ray yield was found to be 0.04 mJ at the usual operating pressure $p_0 = 0.2$ Torr. By changing p_0 to 0.745 Torr the Y_{srx} will increase to 6.8 mJ, as the optimum value for Rico Plasma focus (1 kJ).

The optimum combination of p_0 , z_0 and 'a' for optimum soft X-ray yield was found to be: $p_0 = 3$ Torr, $z_0 = 1.5$ cm, $a = 1.29$ cm and $Y_{srx} = 43$ mJ. From these

Table 4 For each L_0 the optimization combination of z_0 and 'a' were found and are listed here

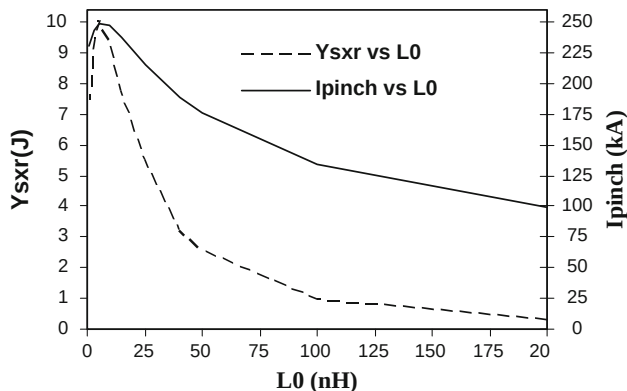
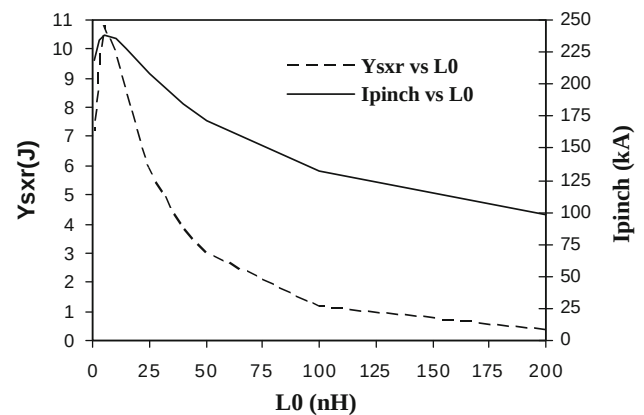
L_0 (nH)	z_0 (cm)	a (cm)	b (cm)	I_{peak} (kA)	I_{pinch} (kA)	$I_{\text{pinch}}/I_{\text{peak}}$	a_{min} (cm)	Z_{max} (cm)	Ysxr (J)
200.0	6.1	0.5900	2.0	143	99	0.692	0.06	0.8	0.36
100.0	4.5	0.7700	2.6	198	134	0.676	0.07	1.0	1.03
50.0	3.2	0.9900	3.3	271	175	0.645	0.09	1.4	2.64
40.0	2.8	1.1000	3.7	300	189	0.63	0.11	1.5	3.27
25.0	2.5	1.2000	4.0	365	215	0.589	0.12	1.7	5.52
15.0	2.1	1.3000	4.4	443	237	0.534	0.14	1.8	7.75
10.0	1.7	1.4000	4.7	510	247	0.484	0.17	2.0	9.44
5.0	1.6	1.4600	4.9	627	249	0.397	0.21	2.1	10.06
3.0	1.6	1.4500	4.9	699	244	0.349	0.23	2.1	9.45
1.0	1.6	1.3600	4.6	799	230	0.288	0.234	2.0	7.47

PF-SY2: Bank parameters: $L_0 = 200$ nH, $C_0 = 25$ μF , $r_0 = 14$ m Ω ; tube parameter: $c = b/a = 3.368$; model parameters: $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.1$, $f_{cr} = 0.7$; operating at 10 Torr oxygen gas, $V_0 = 15$ kV

Table 5 For each L_0 the optimization combination of z_0 and 'a' were found and are listed here

L_0 (nH)	z_0 (cm)	a (cm)	b (cm)	I_{peak} (kA)	I_{pinch} (kA)	$I_{\text{pinch}}/I_{\text{peak}}$	a_{min} (cm)	Z_{max} (cm)	Ysxr (J)
200.0	6.1	0.6700	2.3	141	98	0.695	0.06	0.9	0.43
100.0	4.7	0.8800	3.0	196	133	0.678	0.08	1.2	1.22
50.0	3.2	1.1000	3.7	265	172	0.649	0.10	1.5	3.02
40.0	3.0	1.2000	4.0	293	185	0.631	0.11	1.6	3.91
25.0	2.5	1.4000	4.7	358	209	0.583	0.14	1.9	5.90
15.0	2.1	1.5000	5.1	432	228	0.527	0.17	2.1	8.65
10.0	1.7	1.6000	5.4	494	236	0.477	0.20	2.24	9.97
5.0	1.4	1.6100	5.4	595	239	0.401	0.23	2.29	10.81
3.0	1.2	1.5700	5.3	656	234	0.356	0.24	2.26	9.87
1.0	1.2	1.4600	4.9	741	219	0.295	0.25	2.1	7.22

PF-SY2: Bank parameters: $L_0 = 200$ nH, $C_0 = 25$ μF , $r_0 = 14$ m Ω ; tube parameter: $c = b/a = 3.368$; model parameters: $f_m = 0.03$, $f_c = 0.7$, $f_{mr} = 0.08$, $f_{cr} = 0.7$; operating at 10 Torr oxygen gas, $V_0 = 15$ kV

**Fig. 6** The X-ray yield from PF-SY2 and I_{pinch} (computed) vs L_0 (200 to 1 nH), model parameters: $f_m = 0.05$, $f_c = 0.7$, $f_{mr} = 0.1$, $f_{cr} = 0.7$; operating at 10 Torr oxygen gas, $V_0 = 15$ kV**Fig. 7** The X-ray yield from PF-SY2 and I_{pinch} (computed) vs L_0 (200 to 1 nH), model parameters: $f_m = 0.03$, $f_c = 0.7$, $f_{mr} = 0.08$, $f_{cr} = 0.7$; operating at 10 Torr oxygen gas, $V_0 = 15$ kV

numerical experiments we expect to increase the oxygen Ysxr of PF-1 kJ thousand-fold from its present typical operation or hundred-fold from its pressure-optimized

present configuration; without changing its capacitor bank, merely by changing its electrode configuration and operating pressure.

The Lee model code RADPF5-15 K was also used to run numerical experiments on PF-SY2 with oxygen gas for optimizing soft X-ray yield with reducing L_0 , varying z_0 and 'a'. Contrary to the general expectation that performance of a plasma focus would progressively improve with progressive reduction of its external inductance L_0 , the pinch current limitation effect in plasma focus was confirmed with reducing L_0 , and consequently the maximum soft X-ray yield was computed as 10 J at $L_0 = 5$ nH; operating the inductance-reduced PF-SY2 at 15 kV, 10 Torr oxygen pressure.

From these numerical experiments we expect to increase the oxygen Y_{sxr} of PF-SY2 with reducing L_0 , from the present 0.4 J at $L_0 = 200$ nH to maximum value of near 8 J at an achievable $L_0 = 15$ nH. Because of the current limitation effect, there is little to gain to try to reduce L_0 to 5 nH (which is technically very difficult); and even a loss to reduce L_0 below 5 nH.

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NUCLEAR FUSION ENERGY- MANKIND'S GIANT STEP FORWARD

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Abstract

Estimates of energy supply versus consumption indicate the middle of this century as the critical point when world energy supply will no longer keep pace with the demand. The demand grows inexorably because of both the world population growth as well as the growth of average per capita energy consumption. Technological and economic progress are closely correlated with per capita energy consumption. Hence the inadequacy of energy supplies will limit the progress of human civilization, stifling its soaring spirit. Conservationism, making incremental improvements in this situation, is completely inadequate. What is needed is a giant step - the development of a new, limitless, clean source of energy - nuclear fusion energy. Nuclear fusion technology, when perfected to fusion-burn only deuterium, will have a fuel supply lasting millions of year, even with continuing energy consumption growth as in the past. Intensive efforts in five decades of Tokamak research has advanced the fusion product up by 10^7 times, to the point when breakeven is only a step away. The next step necessarily involves international collaboration on an unprecedented scale in ITER- the International Thermonuclear Experimental Reactor, on which work has started in Cadarache France. ITER and later Demo are envisioned to bring online the first commercial nuclear fusion energy reactor by 2050. Using this as the starting point and the history of the uptake of nuclear fission reactors as a guide, a scenario is described here which depicts a not unreasonable rapid take up of nuclear fusion energy starting after the middle of this century. Just into the next century fusion energy should be able to take up the slack and allow Mankind to continue its progress and growth. Because the development of fusion energy is such a complex technological task it is probable that there will be several decades when the constraints of energy shortage will be severely felt as shown by the flattening of the energy consumption from around 2040 to 2100. Such a period of stagnation seems unavoidable even with the envisaged development and rapid adoption of fusion energy. On the other hand without nuclear fusion energy the scenario depicts a severe downturn unavoidably in the fortunes of Mankind with world population shrinking below 5 billion and eventually even lower.

Keywords: Nuclear Fusion Energy, World Energy Consumption, World Energy Supplies, World Population

1. Introduction

When Christopher Columbus sailed into the Americas in 1492, world population was 450 million (7% of today's population) and world energy consumption was less than 1% of present day consumption. The world was sparsely populated. There were ample energy resources waiting to be developed to support a larger and more progressive human civilization.

The invention of the first reliable steam engine in 1775 started the Industrial Revolution, replacing manual labour with a variety of machines. Then the scientific discoveries of the early 1900's, led by Einstein, enabled Man to control the processes within the atom leading to electronics, lasers, computers, global communications, aerospace transportation, new materials, nanotechnology, biotechnology and nuclear energy. Thus continues the era of human prosperity on a greater scale than ever before with corresponding increase in energy consumption and population increase. World population grew from the 450 million in Columbus' time to 1.6 billion [1] at the start of Einstein's career around 1900 to 6.8 billion towards end 2009. In the past 100 years world population grew 4 times, whilst energy consumption grew 10 times. Thus energy consumption grew faster than population growth; in other words energy consumption per head also grew more than 2 times. This trend of per capita energy consumption growth is bound to continue as the rest of the world marches relentlessly in an attempt to catch up with the best standard of living in the world. As is well known, per capita consumption of energy is closely correlated with standard of living [2].

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In the past 100 years the doubling time of world population was about 50 years whilst the doubling time of energy consumption was 30 years. If this trend were to continue, world population would reach 27 billion in another hundred years whilst energy consumption would increase another 10 times. This is of course unsustainable as the world is already near the critical point when supply of energy barely meets the demand. Energy resources are limited and supply trends are estimated to peak in a few short decades from now. This is the reason underlying the demographers' projections that world population growth must slow down in the near future. The current debate on the unsustainability of population growth, energy consumption trends and the degradation of the environment, whilst important in raising public awareness, does not address the fundamental problem.

What is needed to safeguard Mankind's unimpeded progress is not incremental moves; but one giant bold step - the development of a new limitless source of energy, clean non-polluting energy which will not further aggravate the environment. This is not a pipe-dream. The technology is already nearly proven. Fifty two years of scientific and technological work have already shown that the technology is feasible. Moreover the last final push is set to begin with an international consortium comprising the major economic and scientific communities of the world. The project is ITER- the International Thermonuclear Experimental Reactor which is currently being built in France at Cadarache. The process involves nuclear fusion which is the same process occurring in the stars causing their glow and powering all the energetics of the universe, including all life on earth. In the stanza below the dream (any dream), the river, the glamour and the night of the first two lines are all dependant on the process mentioned in the last two lines:

And a dream lies on the river
And a glamour veils the night
Whilst above the white stars quiver
With nuclear fusion - bright.

Nature is thus showing the way, powering the whole universe with nuclear fusion. Man is in the process of emulating nature.

In 50-100 years time, with human control of this limitless clean non-polluting energy, Man's scientific and technological progress can continue to accelerate, human population can continue to grow. With limitless energy, materials can be created or mined in extra-terrestrial territories like our Moon or further afield from the planets. Living space can be extended by extra-terrestrial colonization which will also serve as energy production bases to avoid overheating the earth. Man's will to explore, up the mountains, down to the sea floors, to the heart of the atom, to the very fabric of space-time; to colonize, like the Americas, Australia in days gone by, and to grow, should not be stifled by a limit to energy or a limit to population. Man's spirit must, will remain indomitable. As Columbus reached for the Americas in the not too distant past 500 years ago, in the not too distant future, Man will reach for the stars.

Einstein's $E=mc^2$ enabled Man to understand the energy source of the universe. Man's control of $E=mc^2$ is demonstrated in the awesome power of his Hydrogen bomb. Man will liberate his destiny with $E=mc^2$ in nuclear fusion reactors - the Dawning of the Fusion Age.

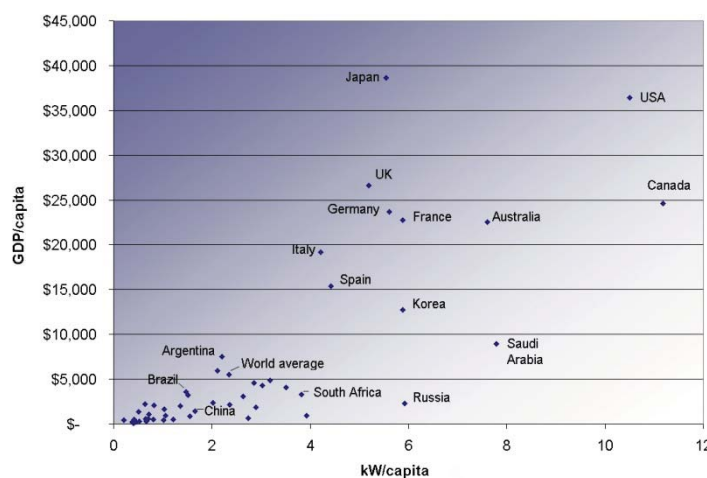


Figure 1. Showing the close correlation between economic wealth and energy consumption [expressed as equivalent power] per capita-2006 [2]

This subject of nuclear fusion is a key subject which will grow in world-wide importance as the ITER project progresses towards maturity. An international conference such as this is the ideal platform to present the basic science and issues of nuclear fusion in such a way as to interest and educate the world-wide community.

2. World Population, Energy Consumption and Supplies

2.1. Correlation of standard of living to energy consumption

There is a close fundamental correlation between the stage of development of a country and its energy consumption. Developed countries have the highest per capita consumption of energy. Poorest, least developed countries have the lowest per capita consumption. This can be seen from Figure 1 which plots GDP per capita against energy consumption per capita. There is generally good correlation between GDP per capita and energy consumption per capita.

Thus as less developed countries move up in their development, energy consumption per capita rises. The world energy consumption per capita will continue to rise in the foreseeable future, driven by the rapid development of China, India, Latin America [2].

2.2. Energy supplies

Energy used by today's industrial societies is derived from utilization of finite earth resources. These resources are an inheritance and their consumption involves the expenditure of materials accumulated by the earth over aeons of time [3].

The quantity of fossil fuels available to man has been the subject of intense interest and numerous estimates. A good review has been made recently [4]. In the context of this paper we are not attempting to present the most precise figures or the most reliable projections into the future. Since all projection scenarios point to the need for Mankind to take a giant step to solve its energy needs, a projection scenario not in wild variance with the consensus would be adequate for our purposes. Hence we present Figure 2 [3] which shows world energy production from 1850 with projection into the next century. Included in this figure are the fossil fuels coal, petroleum and natural gas. Also included are renewables such as hydro, wind, photovoltaic and biomass and nuclear fission energy production. In the same figure is included the world population and its projection into the next century (top line). The close correlation between the world population scenario with the total energy production scenario in all such estimates is no mere coincidence. We venture to say that it reflects the acceptance of the demographers to the fate that humanity is expected to endure - the limitation of growth and progress due to the shortage of energy resources. Pushing the projection further, one probable scenario would be the severe retardation of human civilization, even to the point of extinction.

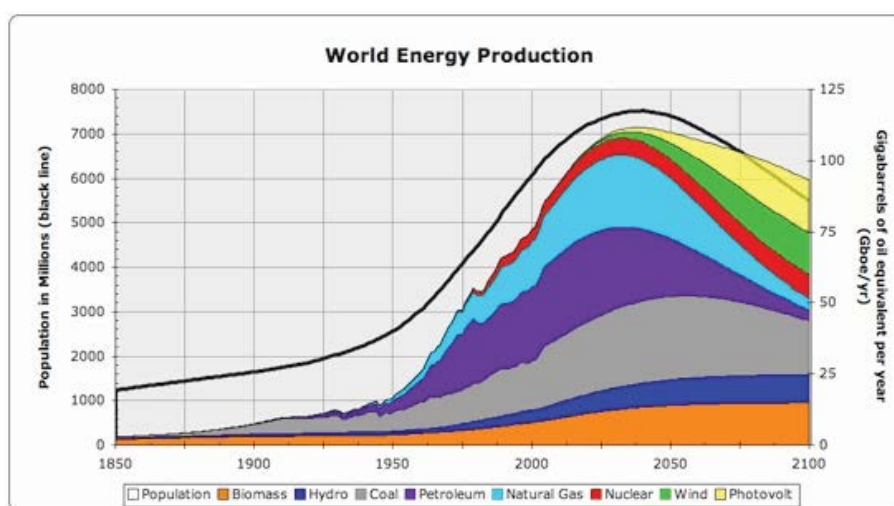


Figure 2. Estimates of world energy production projected into the future in a scenario [3]

2.3. World population growth

World population is growing; food, water, housing, education, medical care must grow correspondingly, in fact faster as less developed countries becomes more developed. Rate of population growth is about 1.5% per year and population is projected to grow to 10 billion by 2030 (see top line in the Figure 3) and can be expected to continue

growing beyond that time given no drastic limitations. If the needs of the growing population cannot be met, there will be economic, political and environmental upheavals. The key is the availability of energy. If energy supplies prove inadequate, then world population will stagnate and even drop as shown in the middle and bottom lines depicting scenarios of lower growth.

The most popular demographic projections nowadays appear to follow the middle line or a line somewhere between the top and middle line, projecting the world population to stabilize in the region of 10 billion.

It is likely that these popular projections are all based on the underlying assumptions of the inability of resources, principally the underlying resource, that is energy resources, to keep up with the growing demand. Thus as already mentioned in the previous section, the growth of the human race is assumed by the demographers to have reached a critical point at which the population will be limited, even start to seriously decline, dictated by the limitations of energy resources.

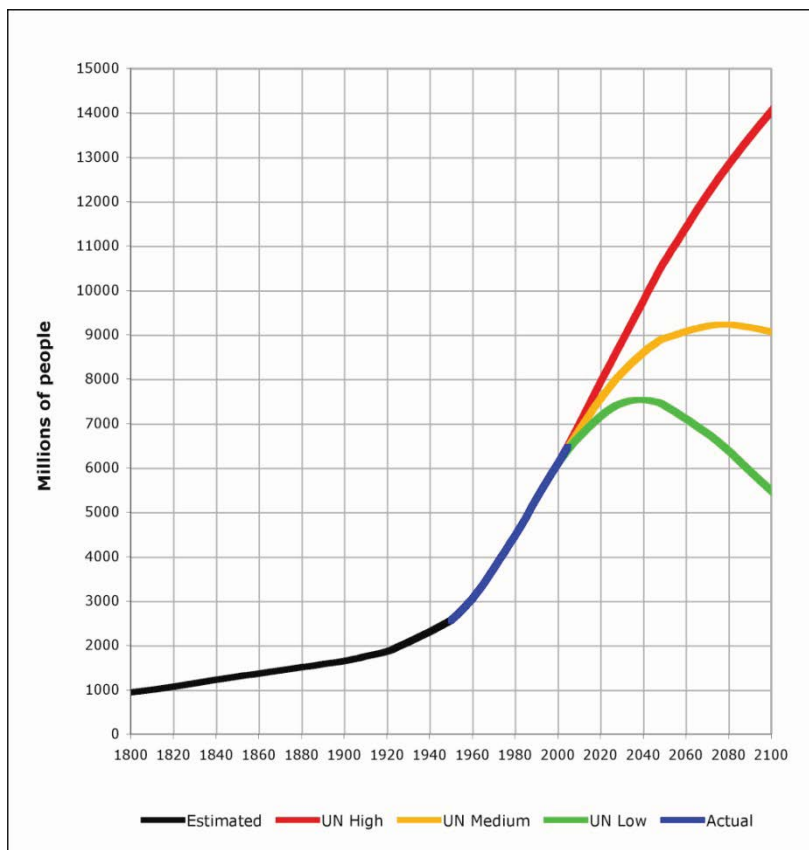


Figure 3. World population and projection [5]

3. Energy Consumption Scenario

Even if world population growth slows down to zero: world energy consumption will continue to rise due to the continuing development of the less developed nations and the corresponding need for a higher energy per capita consumption. Consider the following popular scenario: The world population growth slows down to zero and world population stabilizes at 10 billion at around 2030. However due to continued improvement in the standard of living of the world on average, the demand of energy continues to rise, until world consumption exceeds the supply of conventional energy sources. Then for energy consumption per head to rise on average to $\frac{3}{4}$ of US 1985 per head consumption (which was equivalent to 10kW per head power consumption, a comfortable level by any reckoning), alternative sources of energy have to make up the shortfall.

This is depicted in Figure 4. If the shortfall is not made up then obviously the less developed country will unfortunately continue to remain less developed and the average per head consumption will not rise to $\frac{3}{4}$ US 1985 consumption

4. The Dawning of the Fusion Age

Nuclear fusion technology, when perfected to fusion-burn only deuterium, will have a fuel supply lasting millions of year, even with continuing energy consumption growth as in the past. In nature, nuclear fusion energy powers all the stars and consequently all life in the Universe. Fusion energy has been demonstrated on earth in the hydrogen bomb. Intensive efforts have been made to harness fusion as an energy source. Fifty two years of Tokamak research has

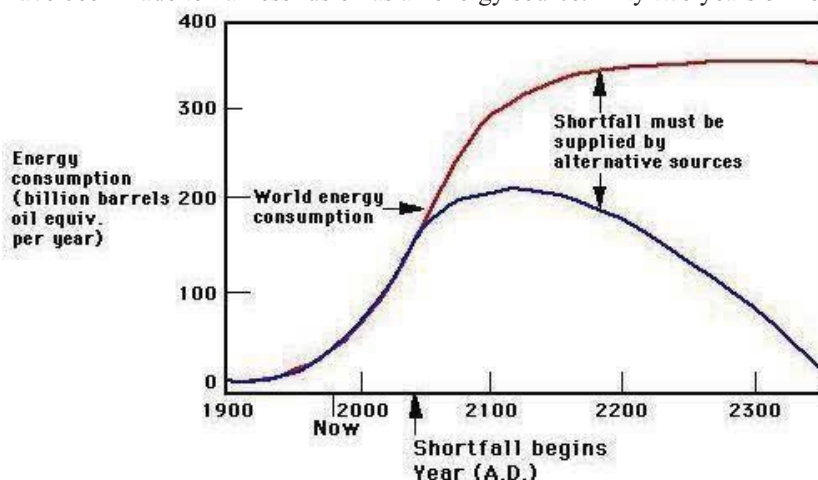


Figure 4. Energy consumption based on a stabilized population of 10 billion with average energy consumption per head taken as $\frac{3}{4}$ of 1985 US per capita consumption

identified a fusion product (density-confinement time-temperature $n\tau T$) of $10^{22} \text{ m}^{-3}\text{-sec-keV}$ as a minimum requirement for breakeven. In these 52 years, the research has pushed the fusion product up by 10^7 times, to the point when breakeven is only a step away [6,7]. Immense efforts in science and technology have already been expended. A glimpse of the scientific basis and technological achievements is provided by the Joint European Torus JET, the biggest fusion experiment yet, which reached $Q=0.6$ in 2000 [8] (In this context $Q=1$ is breakeven and $Q=10$ is a good target). The next step necessarily involves international collaboration on an unprecedented scale, to solve the greatest technological challenge we have yet faced. This takes the form of the International Thermonuclear Experimental Reactor, on which work has started in Cadarache France [9]. ITER aims to reach $Q=10$ by the 2020's, to demonstrate feasibility of nuclear fusion as an energy source. Beyond that lies DEMO which will deliver power into the grid by 2040 [6,7,9,10]. The scientific and engineering plans are well laid and with the support of the best scientific, engineering and political will of all the major world powers and the stakes being the very survival of human civilization, success must be guaranteed.

If energy supplies were to become unlimited then there is no need to restrict the growth of energy consumption, a better standard of living or the growth of population. This is depicted in the Figure 5 of energy consumption well into the 22nd century. Such unlimited growth (curve 4 of Figure 5) need not imply unbridled wasteful consumption. The best practice of environmental conservatism could, should be incorporated into growth, so that efficient and 'green' habits become part of the sustained culture of the human race.

From Figure 5, it is seen that the development of nuclear fusion energy is coming not a moment too soon. The critical point when total available energy starts to decline is seen to be reached just before the middle of this century; thereafter the consumption curve has to drop (light blue curve 1, available energy, without fusion energy) and Man will have to cope with a decreasing supply unless the increasing shortfall is made up by nuclear fusion energy (brown dot-dash curve 2, available energy, with fusion energy).

How fast can we expect the take-up of nuclear fusion energy? To set a reasonable scenario we look at the history of the uptake of nuclear fission energy. The first commercial nuclear fission power station was installed at Calder Hall in the UK in 1956 with 50 MW. By 1960 1 GW was installed. By the late 70's the installed nuclear fission power worldwide had reached 100 GW. Then in the 70's and 80's rising costs of fission reactors due to regulatory changes and pressure-group litigation [11] together with falling fossil fuel prices made fission plants less attractive and the industry slowed down so that by the late 80's installed fission power had only grown to 300 GW and by 2005 only to 370 GW, supplying some 15% of the world's electricity consumption and 2.1% of total power consumption. From looking at the history of the development of nuclear fusion power plants we learn that given the perceived necessity, in less than 20 years the installed capacity could grow 100 fold from a base of 1 GW (10-fold every 9 years for 18 years). The scenario of Figure 5 assumes an installed 1 GW of commercial nuclear fusion power plant in 2050 growing 1000-fold to 1000 GW by 2077 (10-fold every 9 years for 27 years) and then with a slower growth to 10000 GW by 2100 (10-fold in 23 years); thereafter tripling every 50 years. The initial rate of 1 GW to 1000 GW from 2050 to 2077 starts at the same base and is about the same rate as achieved by the nuclear fission industry from 1960 to the late 70's. This is not unreasonable as by 2050 the world would know with a certainty the desperation of its energy position.

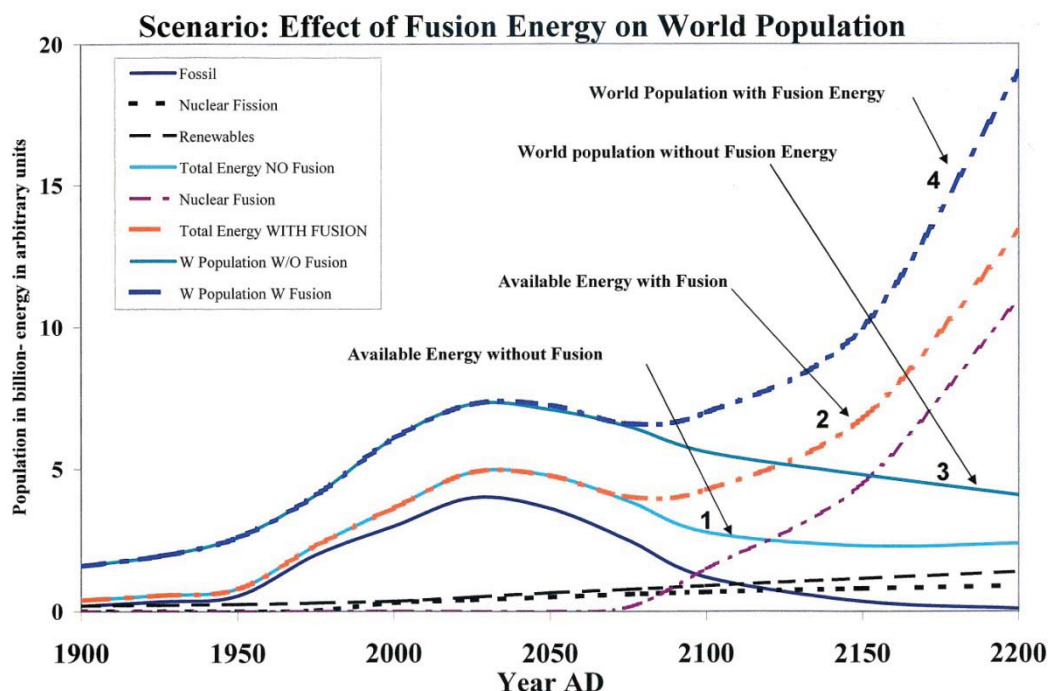


Figure 5. Energy consumption based on continuing population growth; provided no limit to energy supply

The scenario of Figure 5 thus depicts a not unreasonable rapid take up of nuclear fusion energy starting just after the middle of this century. Just into the next century fusion energy should be able to take up the slack and allow Mankind to continue its progress and growth (brown dot-dash curve 2, available energy, with fusion energy). Because the development of fusion energy is such a complex technological task it is probable that there will be several decades when the constraints of energy shortage will be severely felt as shown by the flattening of the energy consumption from around 2040 to 2100 as depicted in the scenario of Figure 5 (brown dot-dash curve 2, available energy, with fusion energy). Such a period of stagnation in energy and corresponding stagnation in world population (dark blue dot-dash curve 4 showing world population scenario, with fusion energy) seems unavoidable even with the development and rapid adoption of fusion energy. Without nuclear fusion energy the scenario is as shown by the light blue curve 1, depicting a severe downturn in the fortunes of Mankind with world population dropping below 5 billion (green curve 3, world population scenario, without fusion energy).

5. Conclusion

The human spirit, its will to explore, to always seek new frontiers, the next Everest, deeper ocean floors, the inner secrets of the atom: these are iconised into human consciousness by the deeds of Christopher Columbus, Edmund Hillary, Jacques Costeau, and Albert Einstein. In the background of the ever-expanding universe, this boundless spirit will be curbed by a requirement to limit growth. That was never meant to be. That should never be so. Man should have an unlimited destiny. To reach for the moon, as he already has; then to colonize it for its resources. Likewise to reach for the planets. Ultimately-the stars. Man's spirit must and will remain indomitable.

Conservationism is essential but probably not enough for the survival of the human race within the stunted limits which are popularly accepted in the present as depicted by the light blue line 1 of Figure 5. Conservationism is important but certainly not enough to secure the continued soaring of the human spirit. That requires a new limitless clean supply of energy - one giant step for Mankind - the Dawning of the Fusion Age.

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THE PLASMA FOCUS- TRENDING INTO THE FUTURE

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Abstract

The plasma focus is a promising small scale alternative to the huge Tokamak project in the development of nuclear fusion energy. Its strength lies in the characteristic that the plasma condition is the same whether the plasma focus is a small sub-kilojoule machine or a large one with thousands of kilojoules of stored energy; and the related constancy of the dynamic resistance. Yet this strength turns out to result in a weakness. The observed neutron 'saturation' is more correctly stated as a 'scaling deterioration' effect. This critical weakness is due to the same constancy of plasma condition intimately related to a constancy of the dynamic resistance. The understanding of this situation points to a new class of plasma focus devices to overcome the 'saturation' of the electric current. Plasma focus technology has to move to ultra high voltage technology and take advantage of circuit manipulation techniques in order to move into a new era of high performance. This paper will examine fundamental scaling properties of the plasma focus including speeds, temperatures, dimensions and times. It will link up these basic scaling characteristics with the crucial ideas of the inherent yield scaling deterioration, thus providing a clear understanding of its overall performance characteristics, paving the way for future exploitation.

Keywords: Plasma Focus, Nuclear Fusion, Plasma Focus Scaling, Plasma Focus Properties, Neutron Saturation

1. Introduction

The plasma focus is one of the smaller scale devices which complement the international efforts to build a nuclear fusion reactor [1]. It is an important device for the generation of intense multi-radiation including x-rays, particle beams and fusion neutrons. The physics underlying the mechanisms for the generation of these radiations in the plasma focus is still not completely known although there have been intensive investigations for the past five decades. Experimental and theoretical work on the focus has reached quite high levels. For example, detailed simulation, work on the plasma focus had been carried out since 1971 [2] and a large range of devices has been constructed from less than 100 J small focus to greater than 1 MJ large focus [3]. Advanced experiments have been carried out on the dynamics, radiation, instabilities and non-linear phenomena [4]. Yet despite all these intensive studies, very little regarding scaling appears to be documented with the exception of the scaling law for neutron yield. Other more recent work has thrown much needed light on other aspects of scaling such as how the dimensions of the dense focused plasma (the focus pinch) and the pinch lifetime scale with apparatus dimensions, the dominating dimension being the anode radius [5].

2. Neutron Scaling with Energy

Historically the most appealing quantity for use as the base for scaling is the stored energy used to drive the focus. Using the highest voltage technologically convenient all one needs to do to scale up energy ($E_0 = 0.5C_0V_0^2$) is to put in more capacitors in parallel, thus increasing the capacitance of the energy bank and incidentally also decreasing the static inductance L_0 of the bank to some extent. Thus this has become a major preoccupation of the larger laboratories, sometimes seemingly at the expense of other considerations. Along these lines, early work has shown that $Y_n \sim E_0^2$; and since under ideal conditions (minimized inductance L_0 and when the system is dominated by the generator impedance) the capacitor current I may have the relationship $I \sim E_0^{0.5}$, then it quickly follows that $Y_n \sim I^4$. This very simplistic view has been too smug and has led to the hold-up of the progress of large plasma focus devices. It was found that when the capacitor bank reached storage energies of only several hundred kJ, the neutron yield no longer increased, the so-called neutron saturation effect. It has been shown recently that whilst the discharge circuit is indeed dominated by the generator impedance at low energies, i.e. low capacitance so that indeed $I \sim E_0^{0.5}$, at a certain point when the C_0 i.e. E_0

gets sufficiently big, the generator impedance has dropped to such low values as to reach the value of the load impedance that the generator is driving. As E_0 is increased even further and further, the generator impedance eventually becomes negligible when compared to the load impedance which remains relatively constant, hardly affected by the decreasing generator impedance. Eventually at very large E_0 , the constant load impedance completely dominates and the circuit current reaches an asymptotic value and does not even increase at all for a further increase in E_0 at those already very large values. At this point typically in the high tens of MJ for the plasma focus, the capacitor current may be considered to have saturated, leading to neutron saturation. What is observed at hundreds of kJ and which has been termed as neutron saturation is based on very limited data. When more data from more experiments are put in together with data from rigorous systematic numerical experiments, then the global picture shows the scaling deterioration very clearly (see Figure 1). We will come back to this central problem again in the concluding section of this paper.

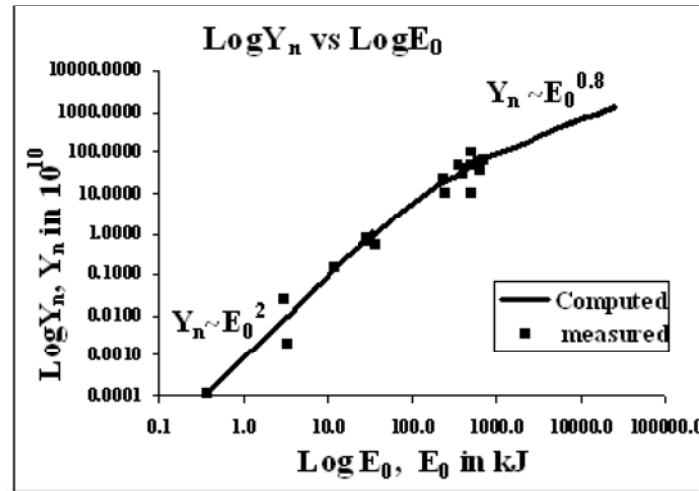


Figure 1. The global scaling law, combining experimental and numerical data. The global data illustrates Y_n scaling observed in numerical experiments from 0.4 kJ to 25 MJ (solid line) using the Lee model code, compared to measurements compiled from publications (squares) of various machines from 0.4 kJ to 1 MJ.

3. Scaling Properties of the Plasma Focus

3.1. Various plasma focus devices

In Figure 2a is shown the UNU ICTP PFF 3 kJ device [6] mounted on a 1m by 1m by 0.5m trolley, which was wheeled around the ICTP for the 1991 and 1993 Plasma Physics Colleges. The single capacitor is seen in the picture mounted on the trolley. In contrast, Figure 2b shows the 300-times larger PF1000, the 1 MJ device at the ICDMP in Warsaw Poland [7]. Only the chamber and the cables connecting the plasma focus to the capacitors are shown. The capacitor bank with its 288 capacitors, switches and chargers are located in a separate hall.



Figure 2a. 3 kJ UNU ICTP PFF



Figure 2b. 1 MJ PF1000 plasma focus

We show in Table 1 the characteristics of three plasma focus devices [8] computed using the Lee model code, fitted by comparing the computed current waveform to the measured current waveform. These computed characteristics are also in broad agreement with measured experimental values where available in the published literature [6,7,9].

Table 1.

	E_0	a	z_0	V_0	P_0	I_{peak}	v_a	ID	SF	Y_n
	kJ	cm	cm	kV	Torr	kA	cm/□s	kA/cm	(kA/cm) torr ^{0.5}	10 ⁸
PF1000	486	11.6	60	27	4	1850	11	160	85	1100
UNU ICTP	2.7	1.0	15.5	14	3	164	9	173	100	0.20
PF-400J	0.4	0.6	1.7	28	7	126	9	210	82	0.01

In Table 1 we look at the PF1000 and study its properties at typical operation with device storage at 500 kJ level. We compare this big focus with two small devices at the kJ level.

From Table 1 we note:

Voltage and pressure do not have any particular relationship to E_0 .

Peak current I_{peak} increases with E_0 .

Anode radius 'a' increases with E_0 .

Current per cm of anode radius (ID) I_{peak}/a is in a narrow range 160 to 210 kA/cm

SF (speed or drive factor) $(I_{peak}/a)/P_0^{0.5}$ is 82 to 100 (kA/cm) torr^{0.5} deuterium gas. Observed Peak axial speed v_a is in the narrow range 9 to 11 cm/us.

Fusion neutron yield Y_n ranges from 10⁶ for the smallest device to 10¹¹ for the PF1000.

We stress that whereas the ID and SF are practically constant at around 180 kA/cm and 90 (kA/cm) per torr^{0.5} deuterium gas throughout the range of small to big devices, Y_n changes over 5 orders of magnitude.

We emphasise that the data of Table 1 is generated from numerical experiments and most of the data has been confirmed by actual experimental measurements and observation.

Table 2.

	$c=b/a$	a	T_{pinch}	v_p	r_{min}	z_{max}	Pinch duration	r_{min}/a	z_{max}/a	Pinch duration/a
		cm	10 ⁶ K	cm/□s	cm	cm	ns			ns/cm
PF1000	1.4	11.6	2	13	2.2	19	165	0.17	1.6	14
UNU ICTP PFF	3.4	1.0	8	26	0.13	1.4	7.3	0.14	1.4	8
PF400J	2.6	0.6	6	23	0.09	0.8	5.2	0.14	1.4	9

Table 2 compares further the properties of the range of plasma focus devices. The properties being compared in this table are mainly related to the radial phase. These are computed from numerical experiments and found to be in close agreement with laboratory measurements [5,10].

The pinch temperature T_{pinch} is strongly correlated to the square of the radial pinch speed v_p .

v_p itself is closely correlated to the value of v_a and $c=b/a$; so that for a constant v_a , v_p is almost proportional to the value of c [11].

Notably the dimensions and lifetime of the focus pinch scales as the anode radius 'a'.

r_{min}/a (almost constant at 0.14-0.17)

z_{max}/a (almost constant at 1.5)

Pinch duration has a relatively narrow range of 8-14 ns/cm of anode radius.

The duration per unit anode radius is correlated to the inverse of T_{pinch} .

T_{pinch} itself is a measure of the energy per unit mass. It is quite remarkable that this energy density at the focus pinch varies so little (factor of 5) over a range of device energy of more than 3 orders of magnitude (factor of 1000).

This practically constant pinch energy density (per unit mass) is related to the constancy of the axial speed moderated by the effect of the values of $c=b/a$ on the radial speed.

The constancy of $r_{\min}/a$ suggests that the devices also produce the same compression of ambient density to maximum pinch density; with the ratio (maximum pinch density)/(ambient density) being proportional to $(a/r_{\min})^2$. So for two devices of different sizes starting with the same ambient fill density, the maximum pinch density would be the same.

From the above discussion, we may put down as rule-of-thumb the following scaling relationships, subject to minor variations caused primarily by the variation in c .

Axial phase energy density (per unit mass)	constant
Radial phase energy density (per unit mass)	constant
Pinch radius ratio	constant
Pinch length ratio	constant
Pinch duration per unit anode radius	constant

Summarising, the dense hot plasma pinch of a small E_0 plasma focus and that of a big E_0 plasma focus have essentially the same energy density, the same mass density. The big E_0 plasma focus has a bigger physical size and a bigger discharge current. The size of the plasma pinch scales proportionately to the current and to the anode radius, as does the duration of the plasma pinch. The bigger E_0 , the bigger I_{peak} , the bigger 'a' has to be, hence the larger the plasma pinch and the longer the duration of the plasma pinch. The larger size and longer duration of the big E_0 plasma pinch are essentially the properties leading to the bigger neutron yield compared to the yield of the small E_0 plasma focus.

We may also summarise the dimensions and lifetimes for deuterium and neon plasma focus pinch as follows [5,10]:

Table 3.		Deuterium	Neon (for SXR)
minimum radius	$r_{\min}$	0.15a	0.05a
max length (hollow anode)	z	1.5a	1.6a
radial shock transit	t_{comp}	$5 \times 10^{-6}a$	$4 \times 10^{-6}a$
pinch lifetime	t_p	$10^{-6}a$	$10^{-6}a$
Speed factor	SF	90	

where, for the times in sec, the value of anode radius, a, is in m. For the neon calculations radiative terms are included; and the stronger compression (smaller radius) is due to thermodynamic effects. The units of the speed factor SF are: $(\text{kA/cm})/(\text{torr}^{0.5})$

The above description of the plasma focus combines data from numerical experiments, consistent with laboratory observations [5,8,10,11].

The next section describes briefly the code.

4. Introduction to the Lee model code

The Lee model code couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling a realistic simulation of all gross focus properties. The basic model, described in 1984 [12], was successfully used to assist several projects [6,13,14]. Radiation-coupled dynamics was included in the five-phase code, leading to numerical experiments on radiation cooling [15]. The vital role of a finite small disturbance speed discussed by Potter in a Z-pinch situation [16] was incorporated together with real gas thermodynamics and radiation-yield terms. This version of the code assisted other research projects [17-22] and was web published in 2000 [23] and 2005 [24]. Plasma self-absorption was included in 2007 [23], improving the SXR yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [6,15,17,19,20-22,25], NX2 [18,26,27], and NX1 [27,28] and has been adapted for the Filippov-type plasma focus DENA [29]. A recent development is the inclusion of the neutron yield Y_n using a beam-target mechanism [30-34], incorporated in recent versions [8,11] of the code (versions later than RADPFV5.13), resulting in realistic Y_n scaling with I_{pinch} [30,31]. The versatility and utility of the model are demonstrated in its clear distinction of I_{pinch} from I_{peak} [32] and the recent uncovering of a plasma focus pinch current limitation effect [33-36], as static inductance is reduced towards zero. Extensive numerical experiments had been carried out systematically resulting in the uncovering of neutron [1,30,31, 37-39] and SXR [40-43] scaling laws over a wider range of energies and currents than attempted before. The numerical experiments also gave insight into the nature and cause of 'neutron saturation' [31,38,44]. The description, theory, code, and a broad range of results of this "Universal Plasma Focus Laboratory Facility" are available for download from [8,11].

A brief description of the 5-phase model is given in the following.

4.1 The 5-phases

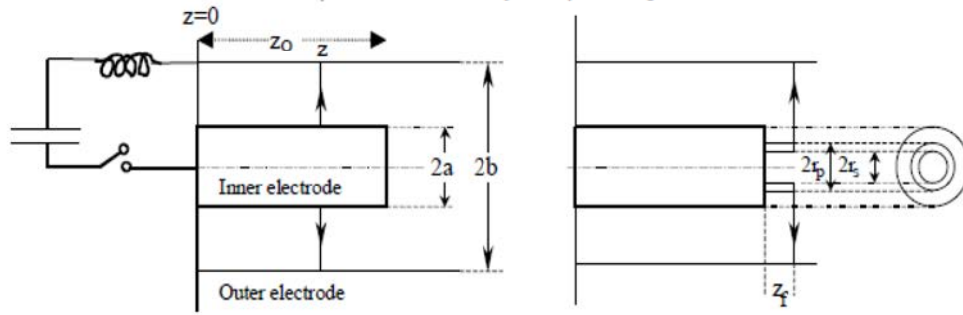


Figure 3. Schematic of the axial and radial phases. The left section depicts the axial phase, the right section the radial phase. In the left section, z is the effective position of the current sheath-shock front structure. In the right section r_s is the position of the inward moving shock front driven by the piston at position r_p . Between r_s and r_p is the radially imploding slug, elongating with a length z_f . The capacitor, static inductance and switch powering the plasma focus is shown for the axial phase schematic only.

The five phases (a-e) are summarised [11,35,38,45,46] as follows:

a. Axial Phase (see Figure 3 left part): Described by a snowplow model with an equation of motion which is coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c . The mass swept-up factor f_m accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheetshock front structure, boundary layer effects, and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor f_c accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall, and current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.

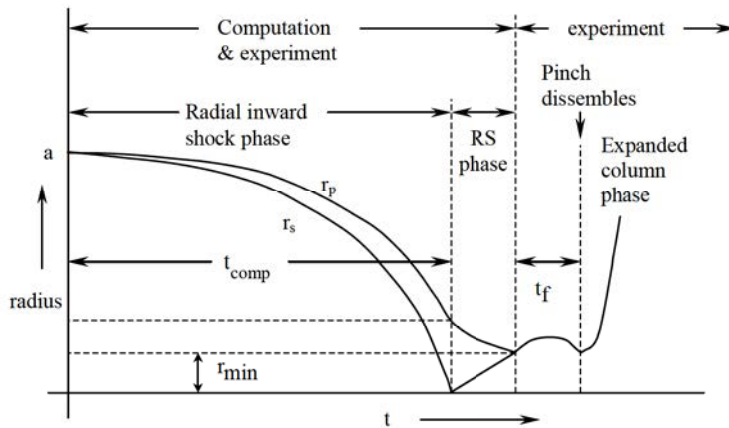


Figure 4. Schematic of radius versus time trajectories to illustrate the radial inward shock phase when r_s moves radially inwards, the reflected shock (RS) phase when the reflected shock moves radially outwards, until it hits the incoming piston r_p leading to the start of the pinch phase (t_f) and finally the expanded column phase.

b. Radial Inward Shock Phase (see Figure 3 right part, also Figure 4): Described by 4 coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, (magnetic piston), allowing the current sheath to separate from the shock front by applying an adiabatic approximation [16]. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being particularly important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase using shock-jump equations. A communication delay between shock front and current sheath due to the finite small disturbance speed [11,16, 46] is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors f_{mr} and f_{cr} are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor f_{cr} accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

c. Radial Reflected Shock (RS) Phase (See Figure 4): When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops which moves radially outwards, whilst the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the reflected shock moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the reflected shock undergoes a jump by a factor close to 2. Number densities are also computed using the reflected shock jump equations.

d. Slow Compression (Quiescent) or Pinch Phase (See Figure 4): When the out-going reflected shock hits the inward moving piston, the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.

e. Expanded Column Phase: To simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase the snow plow model is used, and two coupled equations are used similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

We note [45] that in radial phases *b*, *c* and *d*, axial acceleration and ejection of mass caused by necking curvatures of the pinching current sheath result in time-dependent strongly center-peaked density distributions. Moreover the transition from phase *d* to phase *e* is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These centre-peaking density effects and localized regions are not modeled in the code, which consequently computes only an average uniform density and an average uniform temperature which are considerably lower than measured peak density and temperature. However, because the four model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modeled. Hence the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

4.2 The model code as a general diagnostic tool

The model code now includes a sheet in which is displayed charts of the properties of the particular shot including: total discharge current and plasma current, tube voltage, axial trajectories and speeds, tube inductance and total inductive energy, piston work and Joule work related to the dynamic resistance, the dynamic resistance, ion and electron number density (spatial uniform averaged and peak), plasma temperature (spatial uniform averaged and peak), computed soft x-ray power; all these properties are displayed as functions of time, in the axial phase (where applicable) and in the radial phase. The model code thus provides information to guide experimental measurements and in this sense can be considered as a powerful diagnostics tool as well [38].

5. Neutron Saturation- Its Relationship with the Plasma Focus Properties

In Section 2 we had discussed the global scaling law for neutron yield as shown in Figure 1 which was compiled with data from experiments and numerical experiments. Figure 1 shows that whereas at energies up to tens of kJ the $Y_n \sim E_0^2$ scaling held, deterioration of this scaling became apparent above the low hundreds of kJ. This deteriorating trend worsened and tended towards $Y_n \sim E_0^{0.8}$ at tens of MJ. The global data of Figure 1 suggests that the apparently observed neutron saturation effect is overall not in significant variance with the deterioration of the scaling shown by the numerical experiments.

5.1. The cause of neutron ‘saturation’ is the dynamic resistance

A simple yet compelling analysis of the cause of this neutron saturation has been published [44]. In that paper it is shown that it is the interaction of dynamic resistance of the plasma focus axial phase with the generator impedance of the plasma focus capacitor bank that causes the deterioration of the scaling trend and eventual saturation of yield at high multi-MJ energies. This dynamic resistance is practically a constant over the range of plasma focus devices, depending as it is on the value of $\ln(c)$ (where radius ratio $c=b/a$) and on the axial speed; the latter being practically a constant and the c having only a small range typically from 1.5 to 3. The value of this dynamic resistance is typically 5

m $\square$. The value of the generator impedance of the capacitor bank on the other hand is $Z_0=(L_0/C_0)^{0.5}$, inversely proportional to $C_0^{0.5}$ and is much larger than the dynamic resistance for small plasma focus whilst being much smaller than the dynamic resistance for large plasma focus devices as C_0 is increased from the range of $\square$ F to the range of tens of thousands of $\square$ F. It turns out that in the region of kJ to tens of kJ the scaling of the current is controlled by the bank impedance. In this range of energy the current thus increases with $C_0^{0.5}$. On the other hand at very large MJ energies the capacitor bank impedance has dropped so much below the dynamic resistance that the circuit current is now controlled solely by the dynamic resistance. As this is a constant, increasing the capacitance or bank energy any further no longer affects the circuit current which then tends to an asymptotic value. The point at which the current scaling begins to deteriorate is in the range of low hundreds of kJ. As bank energy increases to this range of energies, the scaling of current begins to deteriorate; the deterioration worsens as MJ levels are approached; but it is not until the high tens of MJ that circuit current begins to approach saturation. Likewise the behaviour of the neutron yield. Thus the term neutron ‘saturation’ popularly referred to in the literature at several hundred kJ is a misnomer, mistakenly applied to what is in fact just a neutron yield scaling deterioration.

The asymptotic value at high tens of MJ is $I_{\text{peak}}=V_0/Z_{\text{total}} \sim V_0/DR_0$ where V_0 is charging voltage, Z_{total} is total circuit impedance which tends towards the value of the axial phase dynamic resistance DR_0 since at very large values of C_0 the value of $Z_0=(L_0/C_0)^{0.5} \ll DR_0$. Thus DR_0 causes current ‘saturation’. This is shown in Figure 5.

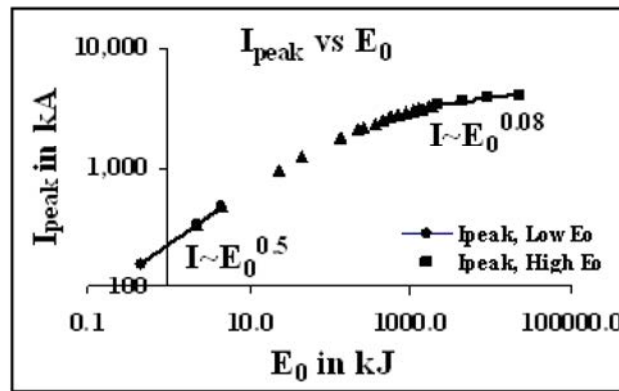


Figure 5. I_{peak} versus E_0 on log-log scale, illustrating I_{peak} ‘saturation’ at large E_0

In other extensive numerical experiments we had shown the following relationships between Y_n and I_{peak} and I_{pinch} as follows:

$$Y_n \sim I_{\text{pinch}}^{4.5} \quad (1)$$

$$Y_n \sim I_{\text{peak}}^{3.8} \quad (2)$$

Hence saturation of I_{peak} will lead to saturation of Y_n .

Thus we have shown that neutron ‘saturation’ is inevitable as E_0 is increased to very large values by an increase in C_0 ; simply due to the dominance of the axial phase dynamic resistance. However the apparently observed neutron ‘saturation’ at low hundreds of kJ is more accurately represented as a neutron scaling deterioration.

5.2. Relationship with plasma focus scaling properties

Now we link up this neutron scaling law deterioration and subsequent saturation with the scaling properties of the plasma focus discussed in Section 3. This scaling law deterioration and saturation is due to the constancy of the speed factor SF and energy density, as E_0 increases. The constancy of the axial speed or SF caused the deterioration of current scaling, requiring that the anode radius ‘a’ is not increased as much as it would have been increased if there were no deterioration. This implies that the size and duration of the focus pinch are also restricted by the scaling deterioration. Ultimately at high tens of MJ, I_{peak} saturates, the anode radius of the focus should not be increased anymore with E_0 . The size and duration of the focus pinch no longer increase with E_0 and Y_n also saturates. We now have the complete picture.

6. Conclusions

This paper has reviewed the global scaling law for neutron yield as a function of storage energy. First, the scaling deterioration and eventual ‘saturation’ of circuit current are ascribed to the energy density constancy manifested in the form of a constancy in dynamic resistance of the axial phase. Second, the deterioration of current scaling implies that the anode radius ‘a’ is not increased as much as it would have been if there were no deterioration. Third, this implies

that the size and duration of the focus pinch are also restricted by the scaling deterioration. Ultimately at high tens of MJ, I_{peak} saturates, the anode radius of the focus should not be increased anymore with E_0 , the size and duration of the focus pinch no longer increase with E_0 .

The restriction on the plasma pinch size and duration has a corresponding effect on the neutron yield Y_n . The neutron yield Y_n scales with E_0^2 at low energies up to tens of kJ, begins to exhibit scaling deterioration around low hundreds of kJ and approaches 'saturation' at high tens of MJ.

In this manner this paper has for the first time connected the global scaling laws for the current and the neutron yield to the scaling properties of the plasma focus. This more complete picture will facilitate the further development of the plasma focus as a fusion device.

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Seminar on Plasma Focus Experiments SPFE2010

**Use of current measurements to interpret
plasma focus properties and mechanisms**

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Nanyang Technological University, NIE, Singapore

Outline

- The Lee model code and its contributions to landmark plasma focus understanding
- Use of the code to reference diagnostics
- Current waveforms show necessity to classify PF's into T1 and T2
- T2 shows necessity to model 6th phase ie Phase 4a
- Product of this fitting is the magnitude and temporal form of anomalous resistance.

Insights 1/3

- The Lee model code has produced ground-breaking insights no other plasma focus codes has been able to produce



For Example: Ground-breaking Insights published 2/3

- Limitation to Pinch Current and Yields- Appl Phys Letts. 92 (2008) S Lee & S H Saw: **an unexpected, important result**
- Neutron Yield Scaling-sub kJ to 1 MJ-J Fusion Energy 27 (2008) S Lee & S H Saw- multi-MJ- PPCF 50 (2008) S Lee
- Neon Soft x-ray Scaling- PPCF 51 (2009) S Lee, S H Saw, P Lee, R S Rawat
- Neutron Yield Saturation- Appl Phys Letts. 95 (2009) S Lee
simple explanation of major obstruction to progress for a generation

Others papers we published using the Lee model code numerical experiments

3/3

Techniques: of Pinch current from total current measurements, in-situ determination of bank parameters (2 papers 2008, 2010) ; numerical experiments applied to specific machines (3 papers 2009); soft x-rays from nitrogen and oxygen (4 papers with Syrian AEC 2009, 2010)



The 5-phase Lee Model code 1/4

- Includes electrodynamical- and radiation-coupled equations to portray the REGULAR mechanisms of the axial (phase 1), radial inward shock (phase 2), radial RS (phase 3), slow compression radiation phase (phase 4) & the expanded axial post-pinch phase (phase 5)
- Relates to physical reality through a measured current trace

Given any plasma focus machine: e.g. PF1000 (biggest PF in the world) 2/4

- Bank parameters: $L_0=33\text{nH}$; $C_0=1332\mu\text{F}$; $r_0=6\text{ m}\Omega$
- Tube parameters: $b=16\text{ cm}$, $a=11.55\text{ cm}$, $z_0=60\text{cm}$
- Operation parameters: $V_0=27\text{kV}$, $P_0=3.5\text{ Torr}$ in D2

The UPFLF (Lee code) is configured (by keying figures into the configuration panel on the EXCEL sheet) as the PF1000

Lo	Co	b	a	zo	ro mOhm
33.5	1332	16	11.55	60	6.1
massf	currf	massfr	currfr	Model Parameters	
0.13	0.7	0.35	0.65		
Vo	Po	MW	A	At-1 mol	Operational
27	3.5	4	1	2	Parameters

Output of code: PF1000 current waveform

PF1000 dynamics & electrodynamics

PF1000 plasma pinch dimensions & characteristics

PF1000 neutron & Radiation yields

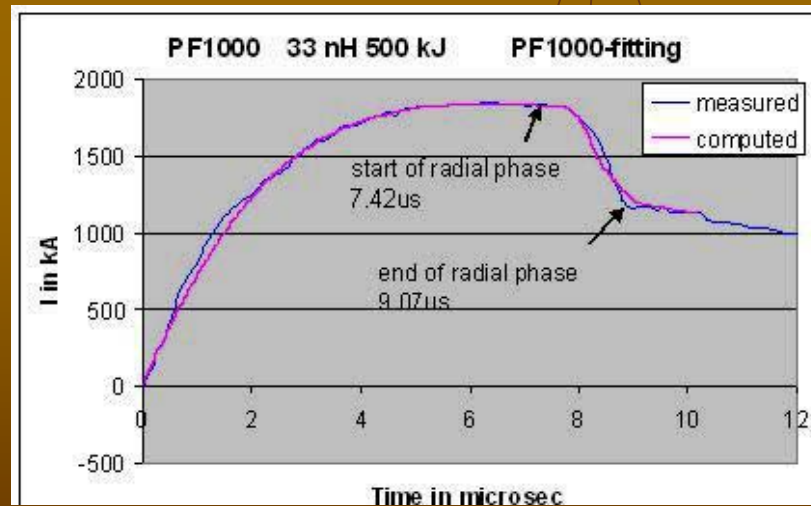


1997 ICDMP (International Centre for Dense Magnetised Plasmas) established in Warsaw-now operates one of biggest plasma focus in the world, the PF1000



Connection of model with Reality is by means of a measured current waveform 3/4

- The computed current waveform is fitted to the measured current waveform by adjusting 4 physical parameters:
- f_m & f_c : mass swept up & current factor; axial phase
- f_{mr} & f_{cr} : mass swept up & current factors: radial phase



5-point fit: rising slope, topping profile, peak value, slope of dip, depth of dip



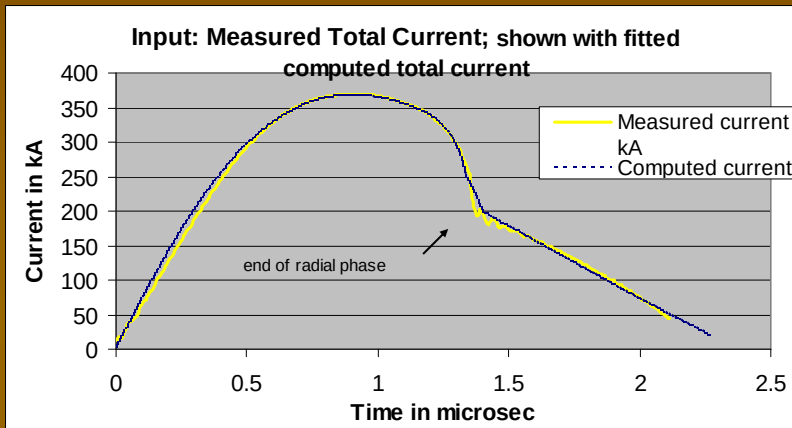
Once fitted: model is energy-wise & mass-wise
& equivalent to the physical situation 4/4

- All dynamics, electrodynamics, radiation, plasma properties and neutron yields are realistically simulated; so that the code output of these quantities may be used as reference points for diagnostics

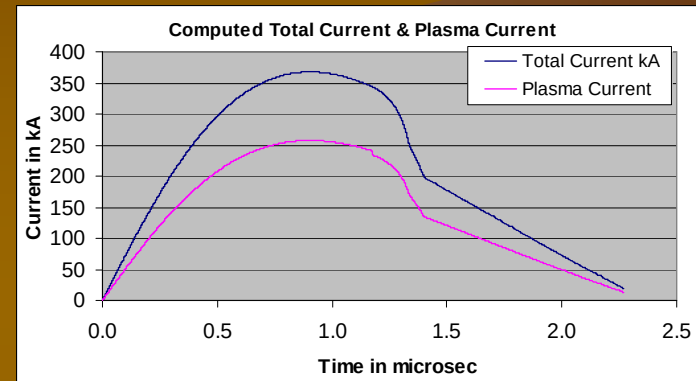
Diagnostics-Time histories of dynamics, energies and plasma properties computed by the code

Last adjustment, when the computed I_{total} trace is judged to be reasonably well fitted in all 5 features, computed times histories are presented (NX2 operated at 11 kV, 2.6 Torr neon)

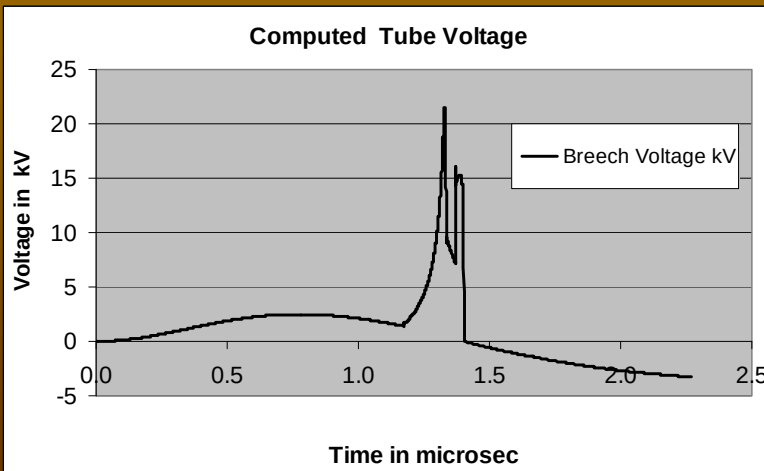
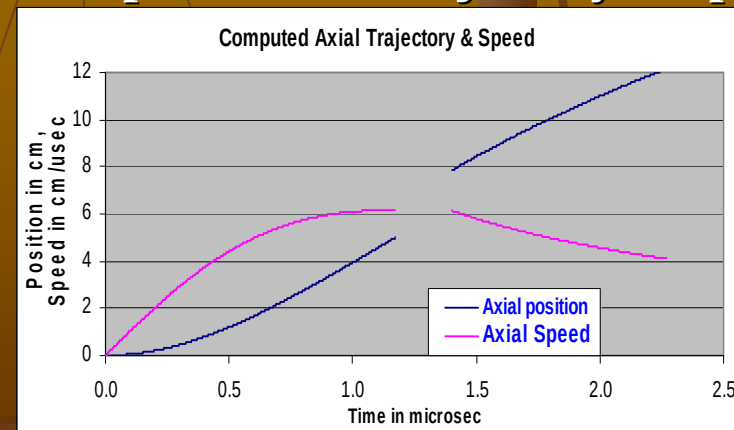
Computed I_{total} waveform fitted to measured



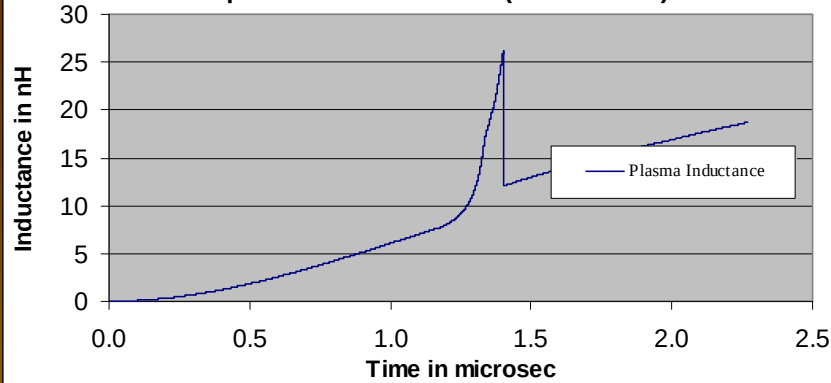
Computed I_{total} & I_{plasma}



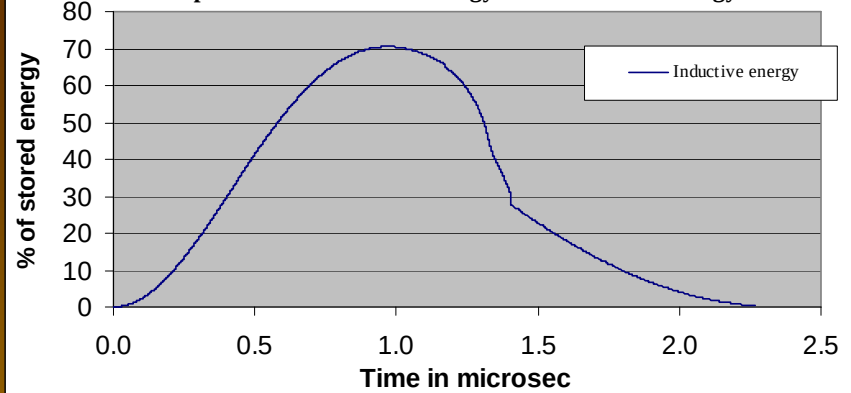
Computed axial trajectory & speed



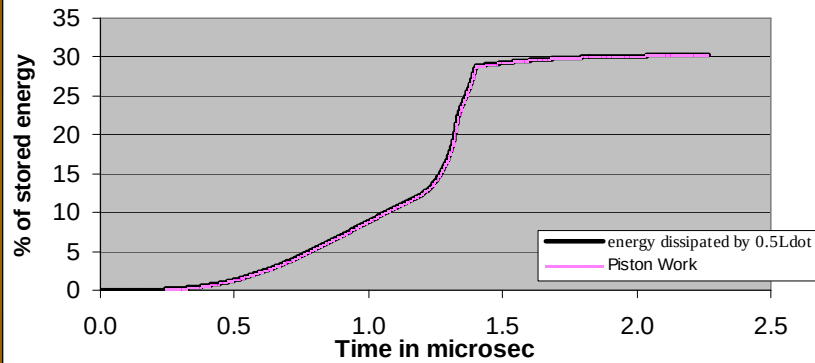
Computed tube Inductance (axial + radial)



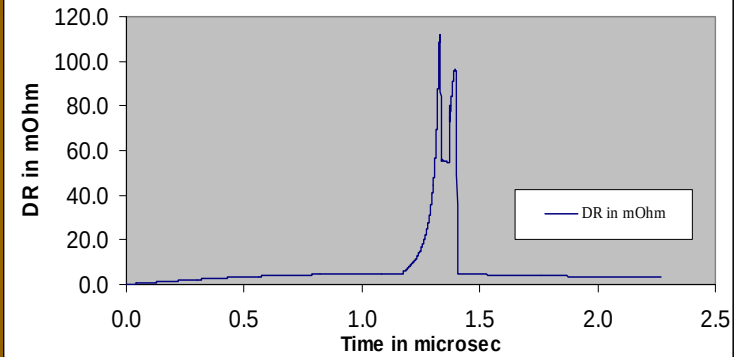
Computed total inductive energy as % of stored energy



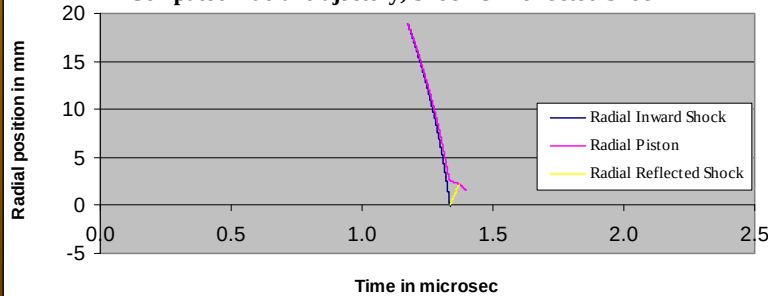
Compare energy dissipated by 'dynamic resistance' with piston work



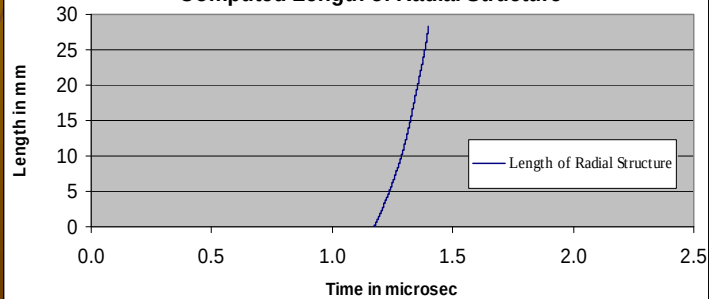
Dynamic Resistance in mOhm



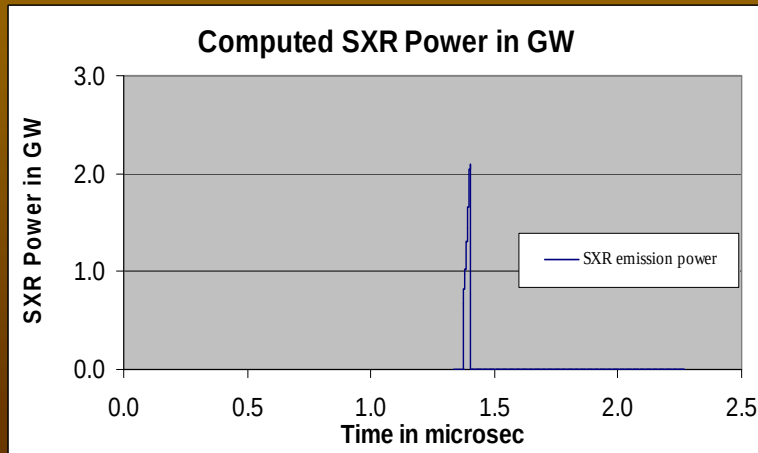
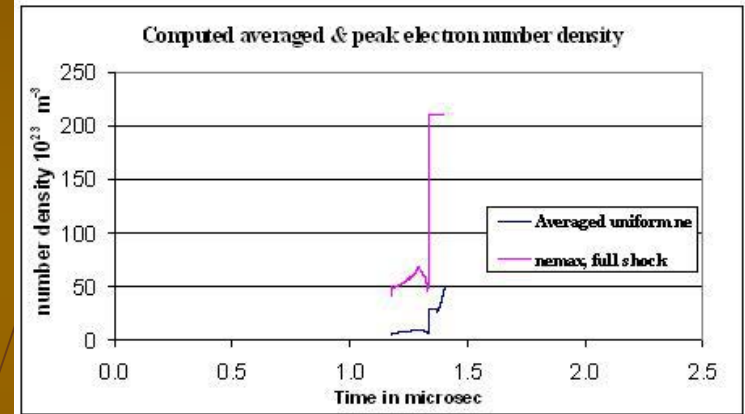
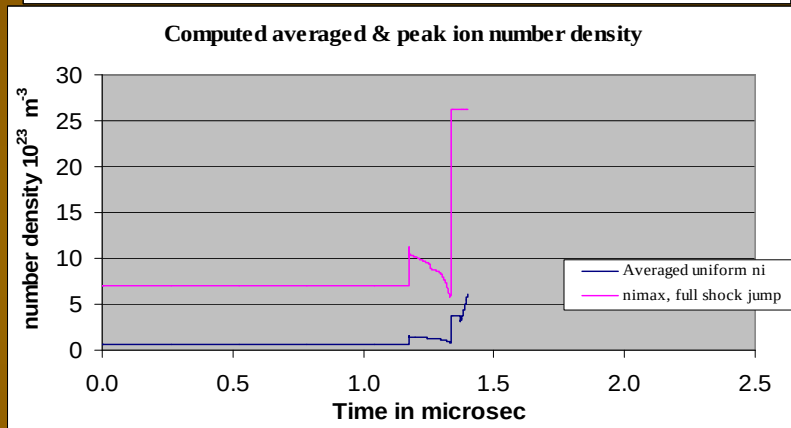
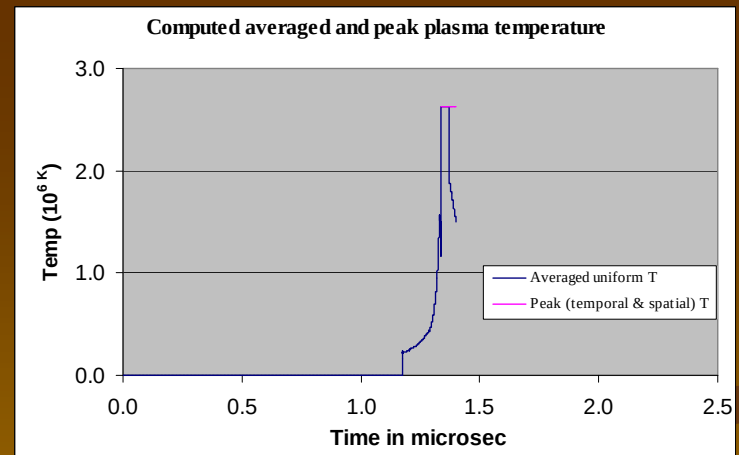
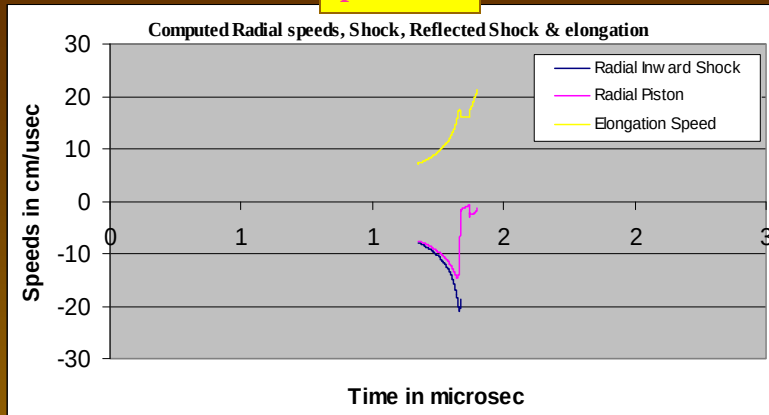
Computed Radial trajectory, Shock & Reflected Shock



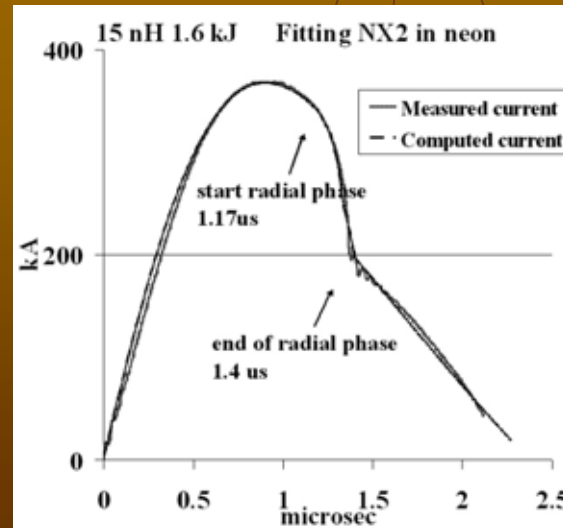
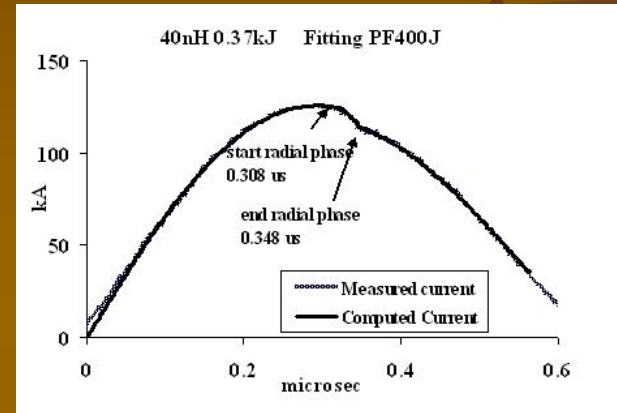
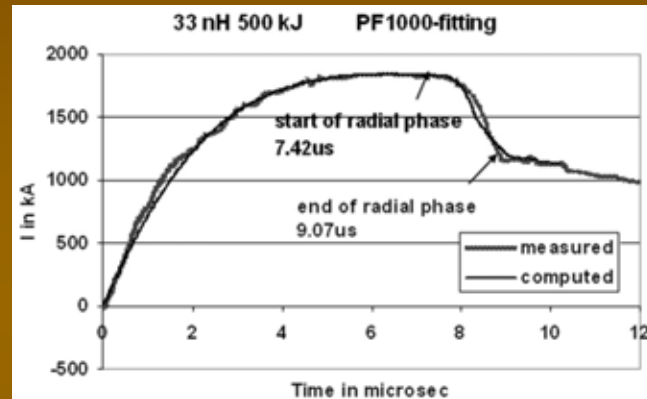
Computed Length of Radial Structure



piston

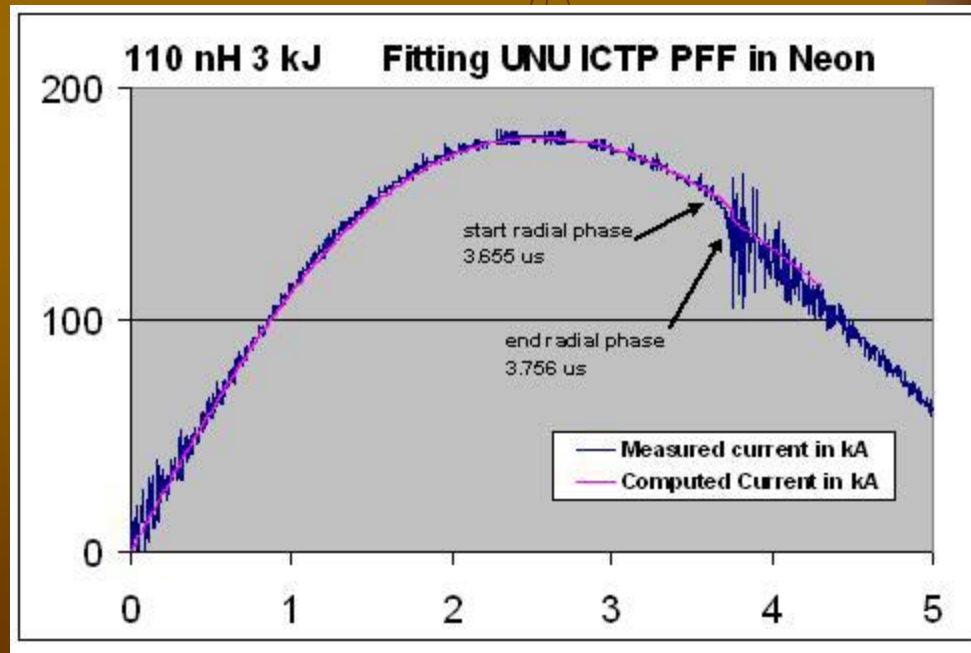


All well-published PF machines are well-fitted: see following examples and many others;
note: the fit for the axial phase, and for the radial phase 1/4



Except one 2/4

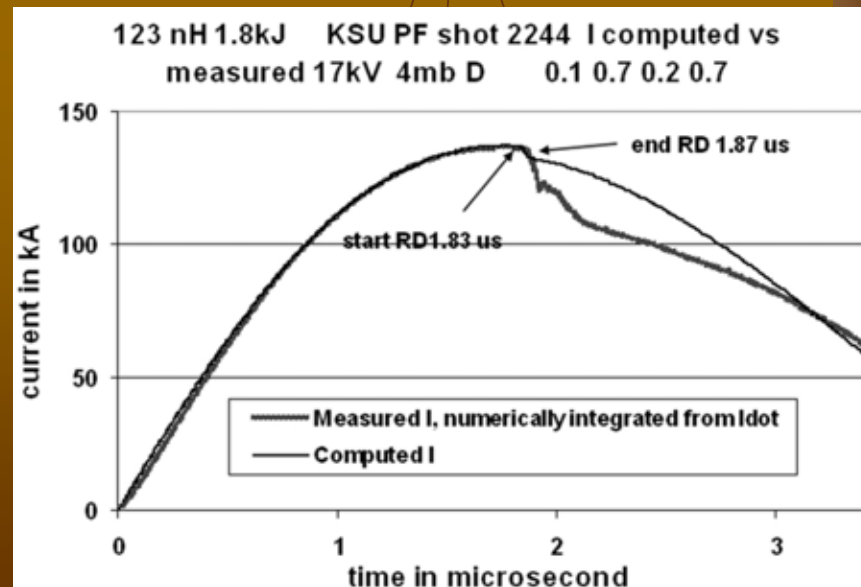
- UNU ICTP PFF- famed low-cost sharing network; current signal noisy and dip is small; difficult to judge the fitting-suspected ill-fit
- Low cost- necessitates single capacitor- hence high inductance L_0



Recently KSU commissioned a machine; a modernised version of the UNU ICTP PFF 3/4

- A good Rogowski system was developed to measure dI/dt ; which was then numerically integrated resulting in a clean current signal-

Best fit nowhere near the fit of the well-published machines- in fact clearly could only fit a small portion of the radial phase



A study followed; resulting in classifying plasma focus devices into T1 & T2 4/4

Table 1. Classification of Plasma Focus Machines (D₂ operation).

PF name	L_0 (nH)	C_0 (μF)	I_{peak} (kA)	R_L	R_{EL}	RD dip (%)	Type
Poseidon	17.7	156	3205	0.9	2.5	32	T1
PF1000	33.5	1332	1845	1	1.6	34	T1
DPF78	55	17.2	869	4.1	12.8	11	T1
FN-II	75	7.5	309	4.3	8.5	10	T1
FMPF1	31	2.4	81	6.9	8.6	14.5	T1
PF-400J	40	1	126	8.8	17.3	8	T1
UNUICTP	110	30	163	16.7	29.5	1.9	T2
KSU	123	12.5	137	21.4	40	1.5	T2

Differentiator:

$$L_0$$

Better Differentiators:

$$R_L = (L_0 + L_a) / L_p$$

$$R_{EL} = (E_{L0} + E_{La}) / E_{LPinch}$$



Physical explanation 1/2

- RD mechanism for pinch purely compressive
- At end of RD (call this REGULAR DIP), expts show other effects eg instabilities leading to anomalous resistance- these mechanisms not modelled by 5-phase Lee code
- These anomalous resistive effects will absorb further energy from pinch; will result in further current dips- called EXTENDED DIP, ED



Our studies further concluded 2/2

- T1: Small L_0 lead to big RD and relatively small ED
- T2: Big L_0 lead to small RD and relatively big ED

This explains why the 5-phase model:

For T1: the model parameters can be stretched for the RD to 'absorb' the ED

For T2: the model parameters, stretch how one likes, the RD cannot 'absorb' the ED

Need to develop 6th phase ie Phase 4a 1/2

- We have simulated using anomalous resistance of following form:

$$R=R_0[\exp(-t/t_2)-\exp(-t/t_1)]$$

Where R_0 is of order of 1 Ohm, t_1 controls rise time of the anomalous resistance and t_2 controls the fall time (rate)

Use one term to fit one feature; terminate the term

Then use a 2nd term to fit a 2nd feature and so on

Simulated Anomalous Resistance Term 2/2

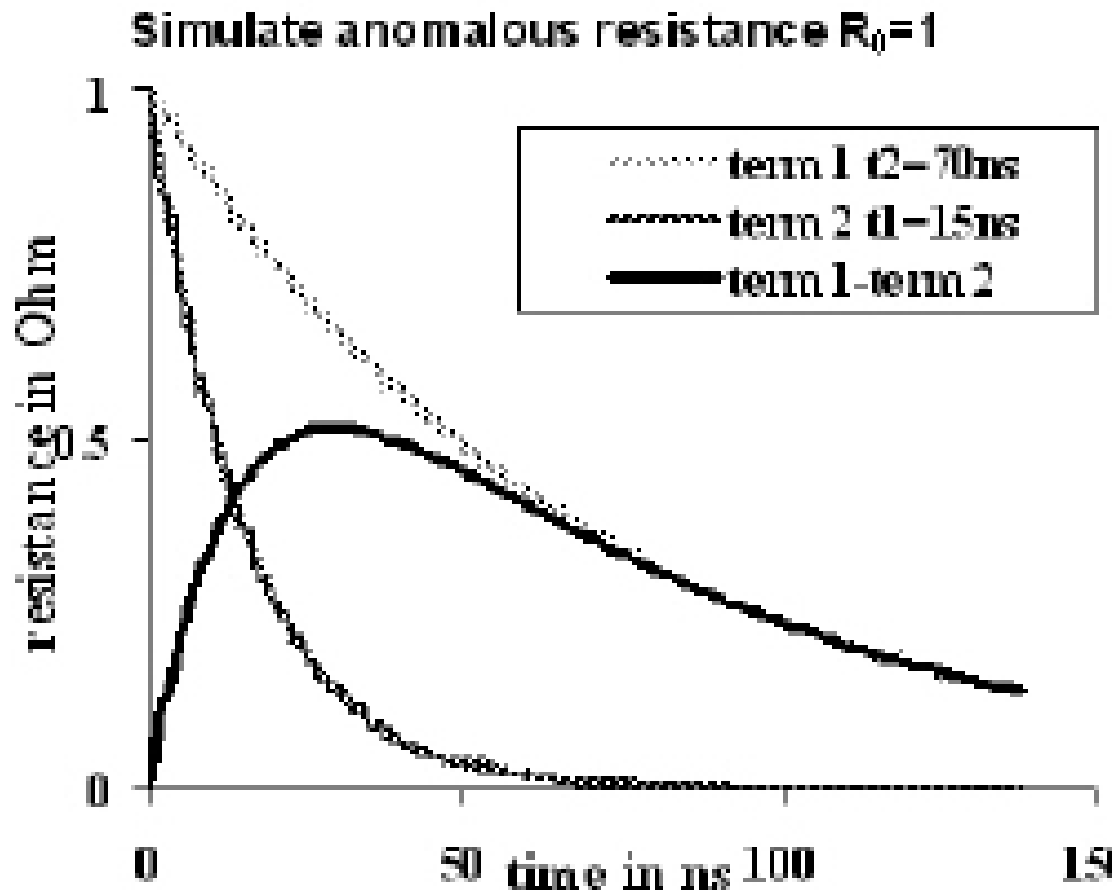


Figure 6. Simulating anomalous resistance.

Result of Phase 4a fitting applied to KSU Current Trace 1/3

Fitting computed current trace to measured trace for RD and ED

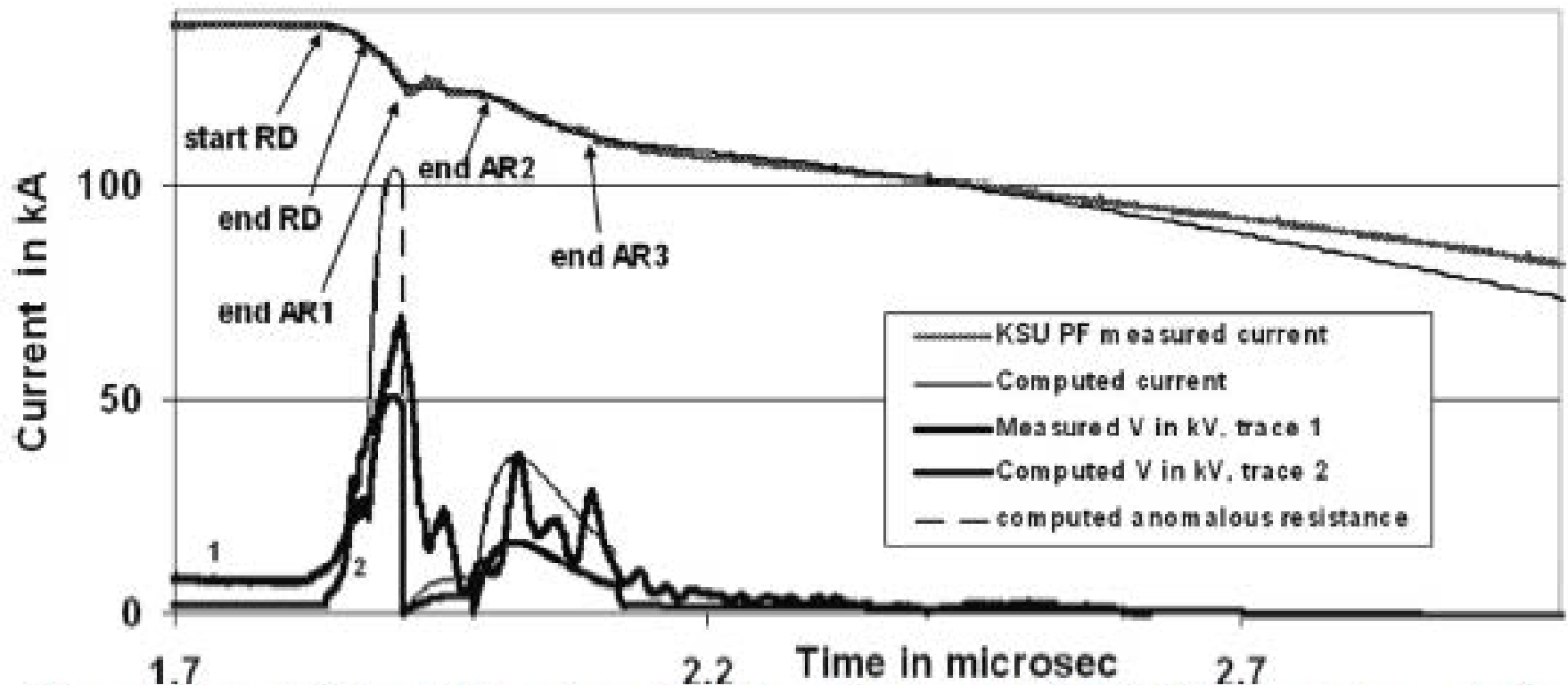


Figure 7. Computed Current (dip region only and expanded to see details) fitted to measured current with inclusion of Phase 4a.



Result of Phase 4a fitting applied to KSU Current Trace 2/3

Table 3. Anomalous resistances used for the fitting.

	R_0 (Ω)	t_1 (ns)	t_2 (ns)	<i>endfraction</i>
Dip 1 <i>ED</i>	1.0	70	15	0.53
Dip 2	0.2	70	40	0.4
Dip 3	0.5	70	25	1.0

Features 3/3

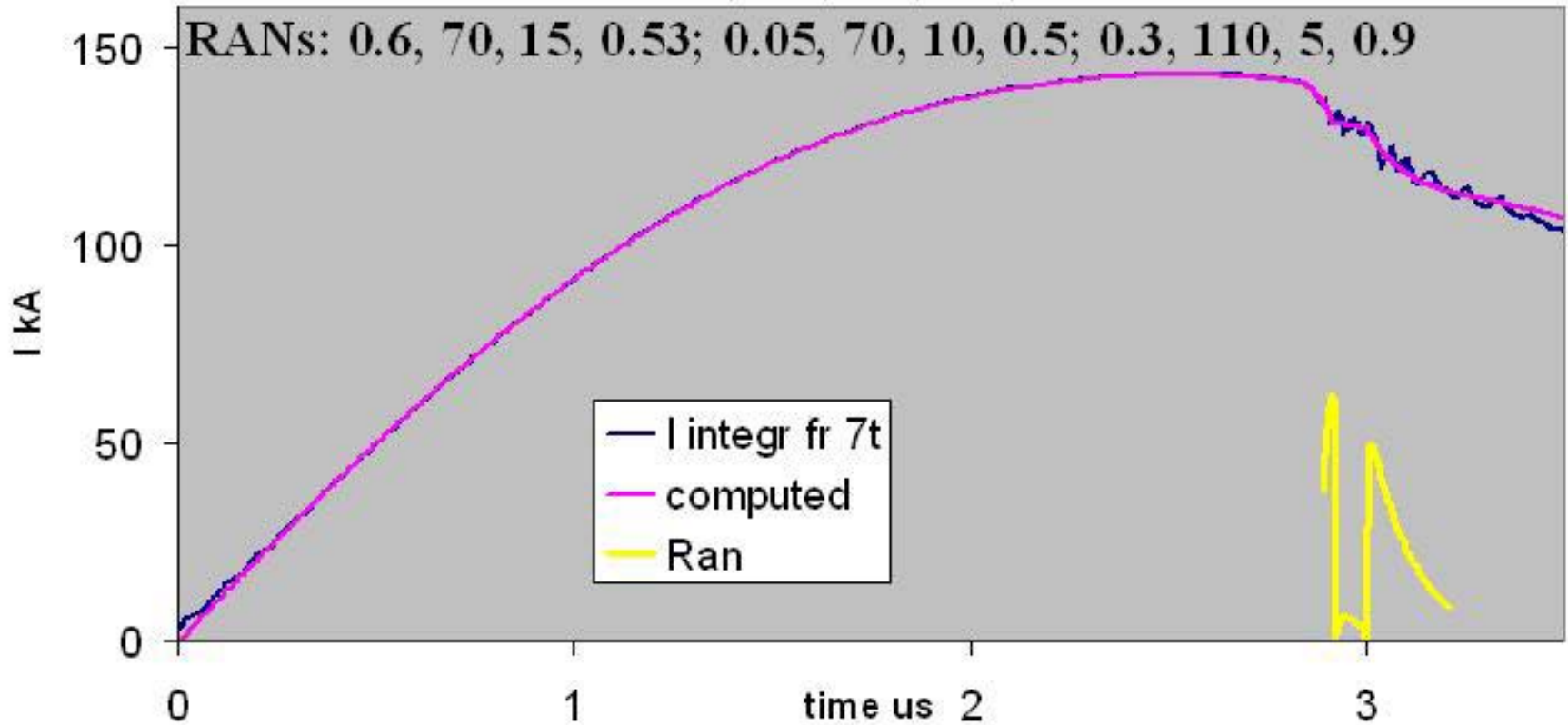
- Current ED now fitted very well
- Fig also shows the form of the fitted anomalous resistance (3 terms)
- Figure shows that the computed tube voltage waveform also shows features in agreement with the measured tube voltage waveform
- The product of this Phase 4a fitting is the magnitude and temporal form of the anomalous resistance. This is an important experimental result. The information is useful to elaborate further on the instability mechanisms.
- Moreover even for the T1 current waveforms, we should fit by first just fitting the RD using the 5-phase model; ie the part that fits well with the computed is the RD; the rest of the dip is then fitted using phase 4a.



UNU ICTP PFF (INTI) 6-Phase Fit

Fitting s6 160810, 12 kV 1 Torr, 114nH, 13mOhm,
0.0408, 0.7, 0.3, 0.7,

RANs: 0.6, 70, 15, 0.53; 0.05, 70, 10, 0.5; 0.3, 110, 5, 0.9



Conclusion

In this paper we have discussed:

- The Lee model code and its contributions to landmark plasma focus understanding
- Use of the code to reference diagnostics
- Current waveforms show necessity to classify PF's into T1 and T2
- T2 shows necessity to model 6th phase ie Phase 4a
- Product of this fitting is the magnitude and temporal form of anomalous resistance.

A Paper has been submitted:

Characterizing plasma focus devices- role of the static inductance- instability phase fitted by anomalous resistances

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Use of current measurements to interpret plasma focus properties and mechanisms

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Remembering Abdus Salam- Thank You



Seminar on Plasma Focus Experiments SPFE2010

In-situ measurement of Capacitor Bank Parameters Using Lee model code

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- **Back-ground to 2-step method**
- **Step 1: High Pressure shot: Static model-uses L-C-R to estimate L_0 and r_0**
- **Intermediate step: Show that even in high pressure, there is significant plasma motion.**
- **Step 2: Use Lee Model code to fit- in the fitting obtain L_0 and r_0**



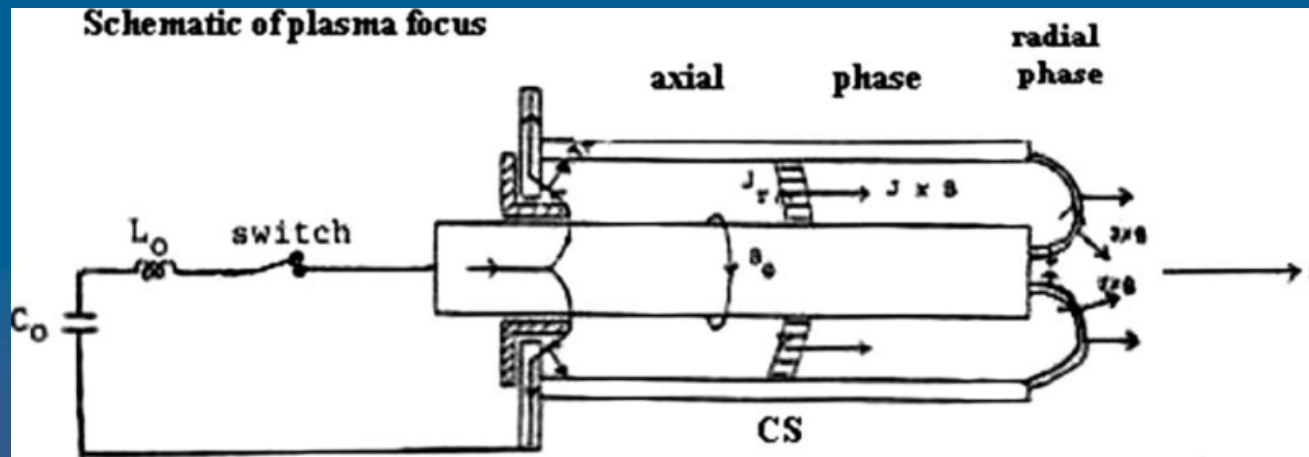


FIG. 1. Showing the plasma focus with representative current sheet positions in the axial phase and the radial phase. The plasma focus acts in the following way: A capacitor bank C_0 discharges a large current into the coaxial tube. The current flows in a current sheath CS which is driven by the $J \times B$ force, axially down the tube. At the end of the axial phase the CS implodes radially, forming an elongating pinch. The static resistance r_0 of the discharge circuit is not shown.

Background



- Parameters of capacitor bank L_0 , r_0 , C_0 are important to be determined, most cases given the value of C_0
- SC Method: Short-circuit bank at its output-then discharge is L-C-R with time-constant parameters
Analyse by L-C-R theory to obtain L_0 and r_0
- In-situ method: If short-circuit difficult to apply, discharge at high pressure. Then assuming no plasma motion, L_0 & r_0 may be estimated using L-C-R theory

However even at 'safe' high pressure, there is still significant motion and L_0 , r_0 estimated using L-C-R not accurate. Hence fitting by the Lee Model code is necessary to account for motion and accurately determine L_0 and r_0 .

In-situ method: Step 1

A shot is fired at high pressure 1/3

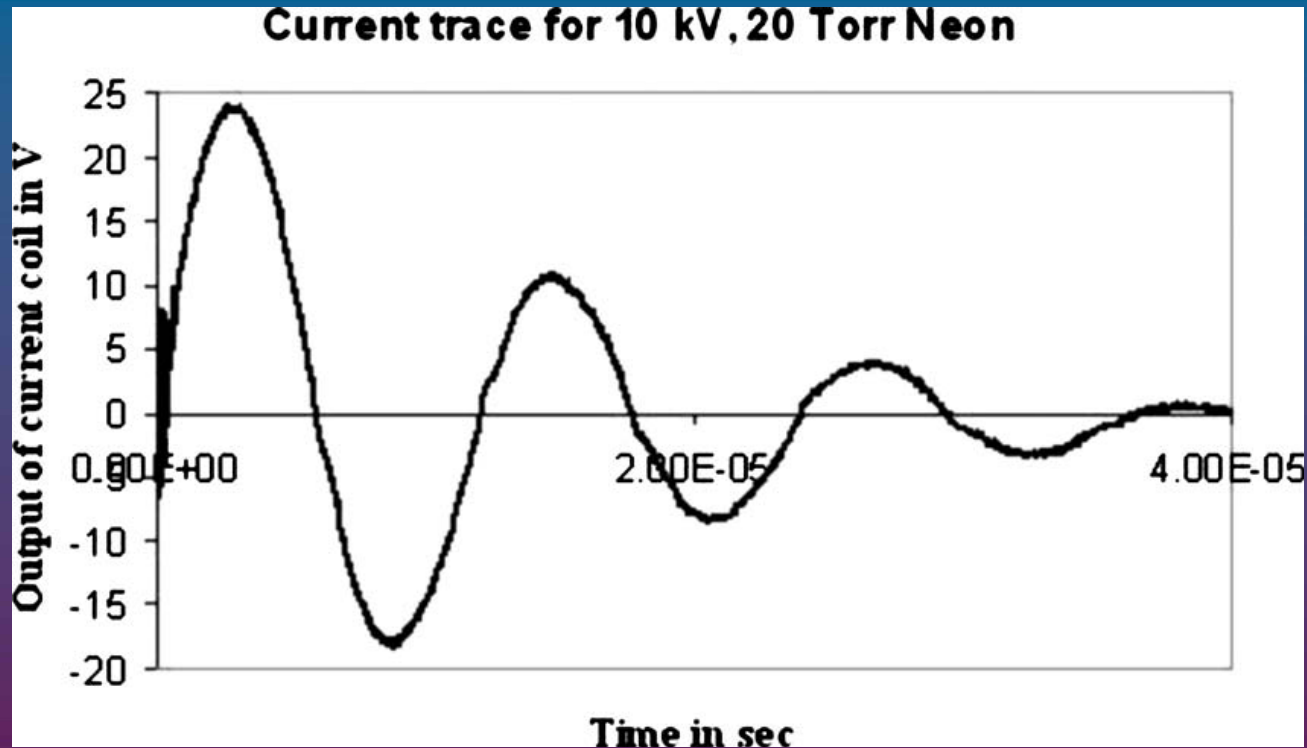


FIG. 2. Measured discharge current waveform at 10 kV, 20 Torr neon; for INTI PF with $C_0=30\mu\text{F}$



Assume discharge current is that from a lightly damped L_0 - C_0 - r_0 circuit

2/3

The waveform may be treated as sinusoid with period T the following approximate equns hold:

- $L_0 = T^2/4\pi^2 C_0$ [1]

- $r_0 = - (2/\pi)[(\ln f)](L_0/C_0)^{0.5}$ [2]

- $I_0 = C_0 V_0(1 + f)/T$ [3]

where f is the reversal ratio obtained from the successive current peaks I_1, I_2, I_3, I_4 , and I_5 with $f_1 = I_2 / I_1$, $f_2 = I_3 / I_2$, $f_3 = I_4 / I_3$, $f_4 = I_5 / I_4$, and $f = (1/4)(f_1 + f_2 + f_3 + f_4)$; and I_0 the highest peak current is written here as I_1 , the peak current of the first half cycle



From Fig 2, we estimate: 3/3



$3T = 36.6 \mu\text{s}$, (measured from Fig 2)
giving $T = 12.2 \mu\text{s}$ and with $C_0 = 30 \mu\text{F}$

- $L_0 = 126 \text{ nH}$.

Also $f_1 = 0.737$, $f_2 = 0.612$, $f_3 = 0.760$, and $f_4 = 0.524$, (measured from Fig 2)

giving $f = 0.658$; and

- $r_0 = 17.1 \text{ m}\Omega$
and peak current $I_0 = 128 \text{ kA}$.

The coil gives a peak first half cycle output of 24 V (measured from Fig 2)

- Thus additionally the coil sensitivity is obtained as $128/24 = 5.3 \text{ kA/V}$.



Checking validity of Step 1: Correction required if the current sheet had moved 1/3



- If the current sheet had moved, then movement add to the circuit inductance; and the value of the inductance measured will have a part which is due to inductance increment as a result of motion.
- To estimate motion at this pressure we fire a shot at a low enough pressure to obtain a strong focus in order to obtain axial speed at the lower pressure. This is shown in Fig. 3.



A shot at low pressure to estimate axial speed 2/3

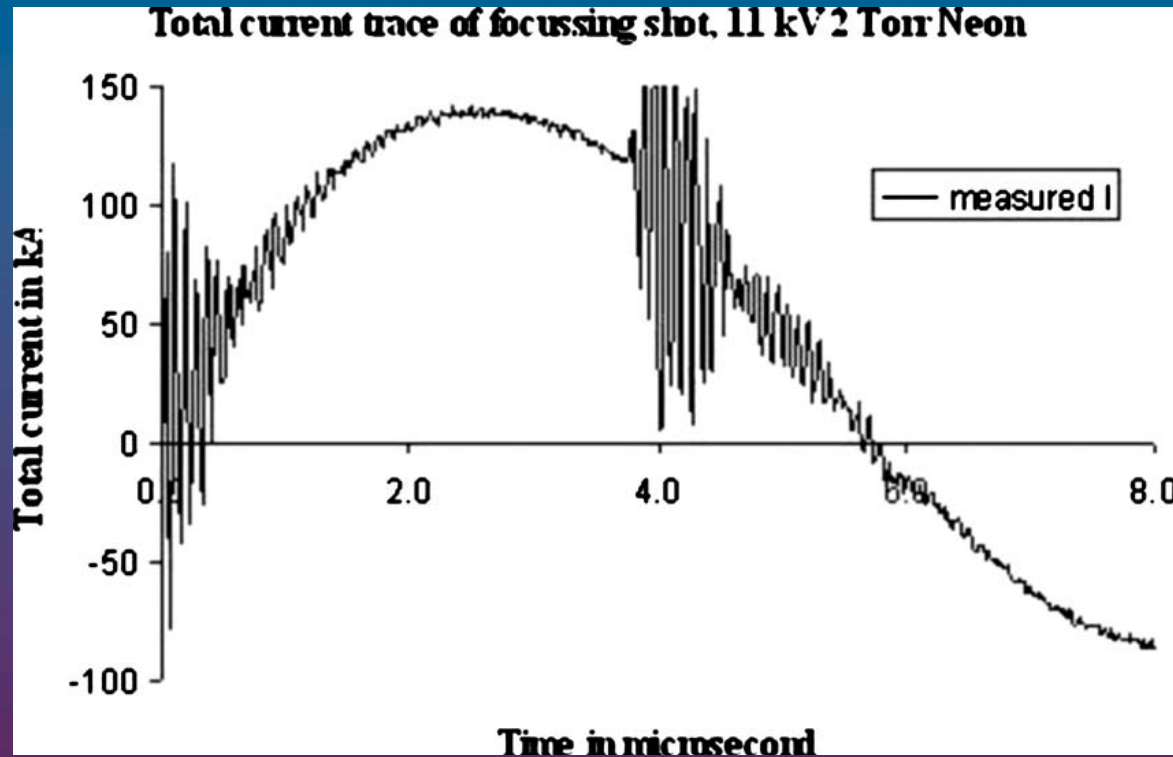


FIG. 3. Discharge current waveform at 11 kV, 2 Torr neon.



Measure the speed at low pressure hence deduce speed at high pressure 3/3



- The current dip starts at the point which is approximately the end of the axial phase. So it takes $3.9\mu\text{s}$ for the current sheet to travel 16 cm, the length of the axial phase. **The average speed in this axial phase is $4.1\text{ cm}/\mu\text{s}$ for this shot at 11 kV, 2 Torr neon.**
- From these data estimate the average speed of the high pressure shot at 10 kV 20 Torr neon; using the **scaling relation for electromagnetic drive speed $\sim I/(a/\rho)^{0.5}$**
- For both shots, anode radius a is constant, and note that the current I is approximately proportional to the charging voltage. Here ρ is the density which is proportional to the pressure for a fixed gas.
- Hence the **average axial speed for the 10 kV, 20 Torr neon shot** is estimated from the relationship to be $(10/11)(2/20)^{0.5}(4.1)=1.2\text{ cm}/\mu\text{s}$.
- So for the high pressure shot in the first half cycle of $6.1\mu\text{s}$, current sheet would have moved 7 cm. The coaxial tube has inductance of $2.4\text{ nH}/\text{cm}$. **So current sheet movement could have added 17 nH to the measured inductance.** This is not a negligible amount.
- **Hence the motion should be taken into account for the measurement of L_0 .**

Step 2: Fitting the computed current trace to the measured current trace to determine the static inductance L_0 1/7

- The Lee model code couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling realistic simulation of all gross focus properties.
- For the high pressure shot involving only the axial phase, the code is very precise in its computation of the discharge current, including the interaction of the circuit with the current sheet dynamics.
- The Lee model code is used in order to generate a computed current trace for fitting to the measured current trace. In this fitting, adjustments are normally made to four model parameters, f_m , f_c for the axial phase, and f_{mr} and f_{cr} for the radial phase.
- In this case we are fitting the 20 Torr neon shot, which has no radial phase, current sheet does not move fast enough to reach end of anode during drive time of the current pulse.
- Our experience has shown us that the three features to be fitted for the axial phase, i.e., the rising part of the current trace, the topping profile, and the peak current have to be fitted by adjusting the value of f_m , f_c . (see Fig 4)

Fitting computed current to measured current waveform 2/7

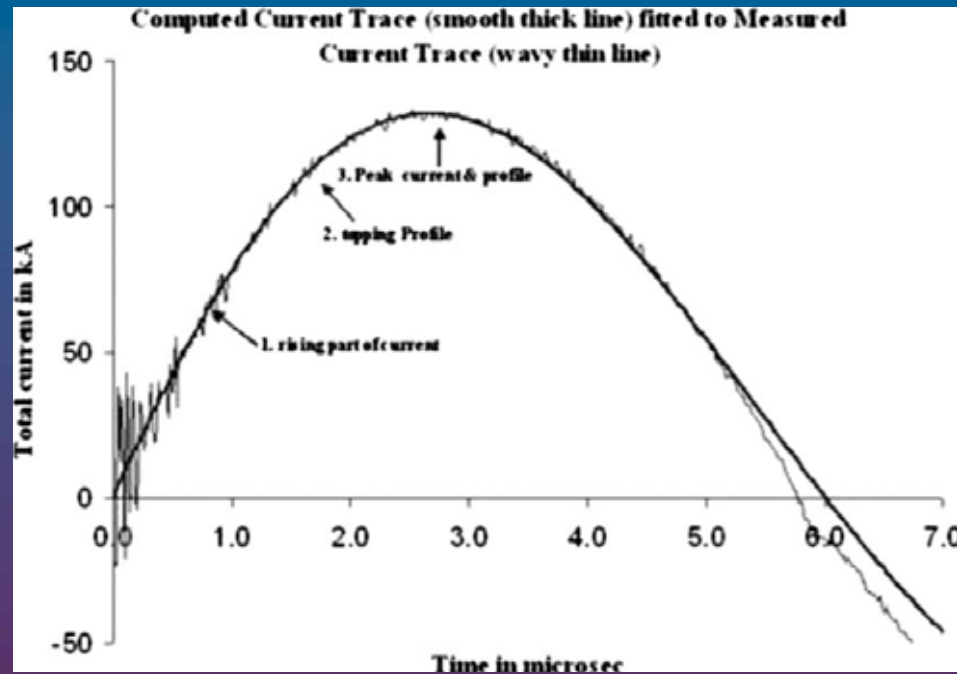


FIG. 4. Fitting the computed current trace to the measured current trace by varying model parameters f_m and f_c . This good fit was obtained after several operations described in the following



Procedure for fitting

3/7

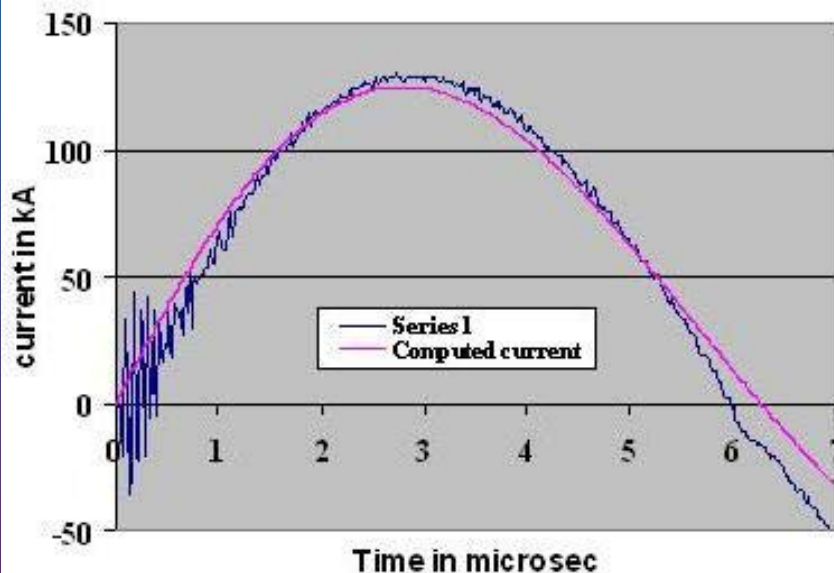


- The method sensitive enough that the static inductance and circuit stray resistance also need to be correctly fitted; otherwise no good fit would be obtained.
- Further, it may also be necessary to move the whole measured current trace a small amount in time; otherwise no good fit may be obtained. This is due to the non-perfect switching characteristics of the spark gap which typically takes a little time to go from nonconducting to conducting, whereas the code assumes instantaneous switching at time zero. This is apparent when the early rising slope of the high pressure shot is examined. The rising section of the measured current does not rise as fast as the computed current trace no matter how the model parameters f_m and f_c are adjusted and no matter how L_0 and r_0 are adjusted.
- The correct way is to shift either current trace relative to the other by the addition of a small time delay.

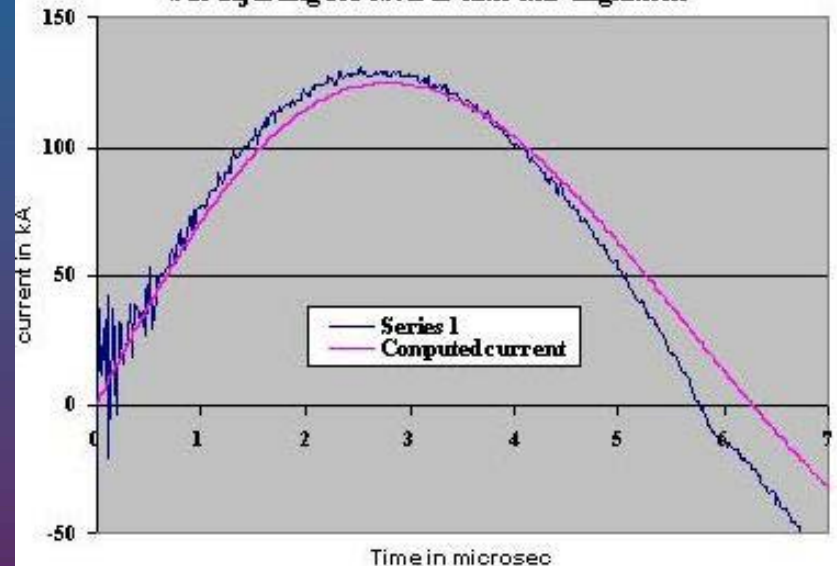


Time mis-alignment is corrected 4/7

First attempt to fit using parameters obtained from L-C-R



First attempt to fit using parameters obtained from L-C-R, but adjusting for obvious time mis-alignment



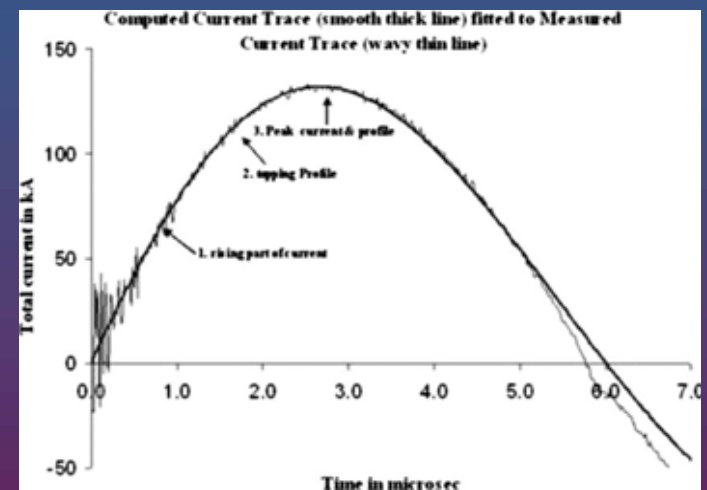
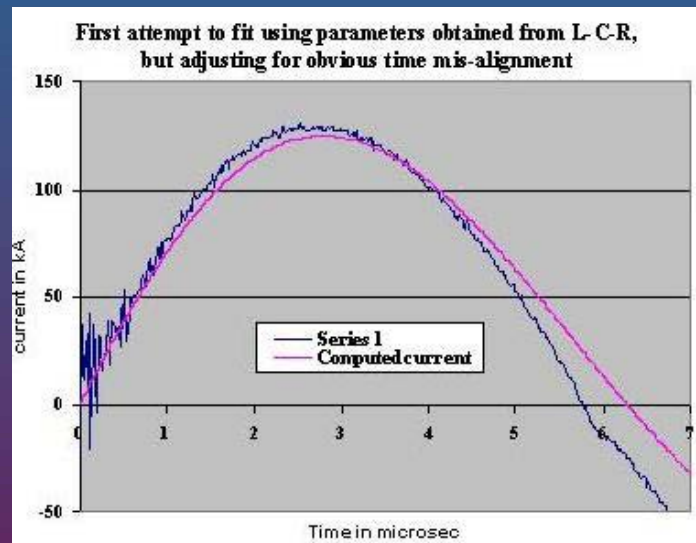
Details of operations for this Shot 5/7



- Figure 4 shows the results of the fitting of the computed current trace to the measured trace according to the procedures summarized in the earlier paragraph, and discussed in greater detail below.
- We start the fitting process by using the static inductance and resistance estimated earlier assuming no plasma dynamics; i.e., $L_0=126$ nH and $r_0=17.1$ m Ω , with capacitance $C_0=30$ μ F.
- Since for this high pressure shot the current sheet does not move beyond the end of the anode, we only need the first two of the model parameters, which we initialize by taking $f_m=0.08$ and $f_c=0.7$.
- By comparing the computed with the measured current waveforms, it was clear that the first part of the current trace was not going to fit except by shifting the whole measured current trace. This was done, and the required shift was 0.2 μ s to make the measured current trace come earlier in time relative to the computed.
- Next it was found that no suitable fit could be found unless we change the value of L_0 to $L_0=114$ nH, $r_0=15$ m Ω .
- Finally in small incremental steps the following values of model parameters for the axial phase were found to be $f_m=0.05$ and $f_c=0.71$. The resultant fit is shown in Fig. 4.



Computed rising slope too shallow indicating L_0 too large (also consistent with too low a current peak)[left figure] Correct by increasing L_0 in steps until best fit; then small adjustments needed for r_0 & f_m & f_c [right figure] 6/7



Note: The method is so sensitive that it picks up (see deviation feature at lower right of fitted figure) a current looping feature that occurs as the voltage across the tube drops to zero at $t \sim 5$ us. The closing of that loop suddenly removes the remnant flux of the original current, from the capacitor bank circuit, thus reducing total inductance; resulting in shortening the discharge periodic time starting at $t \sim 5$ us.



The fitting resulted in the determination of L_0 and r_0 : 7/7



- $L_0=114$ nH,
- $r_0=15$ m Ω .
- With model parameters for the axial phase found to be $f_m=0.05$ and $f_c=0.71$.
- Coil sensitivity also more accurately determined as: 5.4 kA/V

Conclusion



- A two-step method is discussed to determine the capacitor bank static parameters of a plasma focus.
- In the first step, the assumption is made that there is no current sheet movement for the high pressure discharge. The discharge current is analyzed by equations that assume a lightly damped sinusoid generated from an L_0 - C_0 - r_0 discharge circuit, where C_0 is known, and L_0 and r_0 are constant in time.
- The second step takes into account the current sheet motion. This step involves fitting the current trace computed with the Lee model code to the measured current trace, using the estimated values of L_0 and r_0 obtained from the first step. For this fitting, it is found that the static inductance L_0 has to be adjusted before the fitting is possible, with the adjustment of r_0 also playing a significant role.
- Finally the model parameters are adjusted for the best fit.
- This two-step process enables the values of L_0 and r_0 to be correctly measured in-situ without having to short-circuit the capacitor bank. At the same time the current measuring device is also more accurately calibrated.
- This in-situ method may be applied to all plasma focus with clear advantages.

Seminar on Plasma Focus Experiments SPFE2010



In-situ measurement of Capacitor Bank Parameters Using Lee model code

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THANK YOU

International Workshop on Plasma Science and Applications

25-28 October 2010, Xiamen China

Plasma Focus Numerical Experiments- Trending into the Future

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(Parts I & II)

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Plasma Focus Numerical Experiments- Trending into the Future

Part I: Scaling Properties & Scaling Laws

Outline to part I

Recent numerical experiments uncovered new **insights** into plasma focus devices including :

- (1) **Plasma current limitation effect**, as device static inductance L_0 tends towards 0
- (2) **Scaling laws** of neutron yield and soft x-ray yield as functions of E_0 & I

These effects & scaling laws are a consequence of the scaling properties

- (3) A by-product of the numerical experiments are diagnostic reference points.



The Plasma Focus

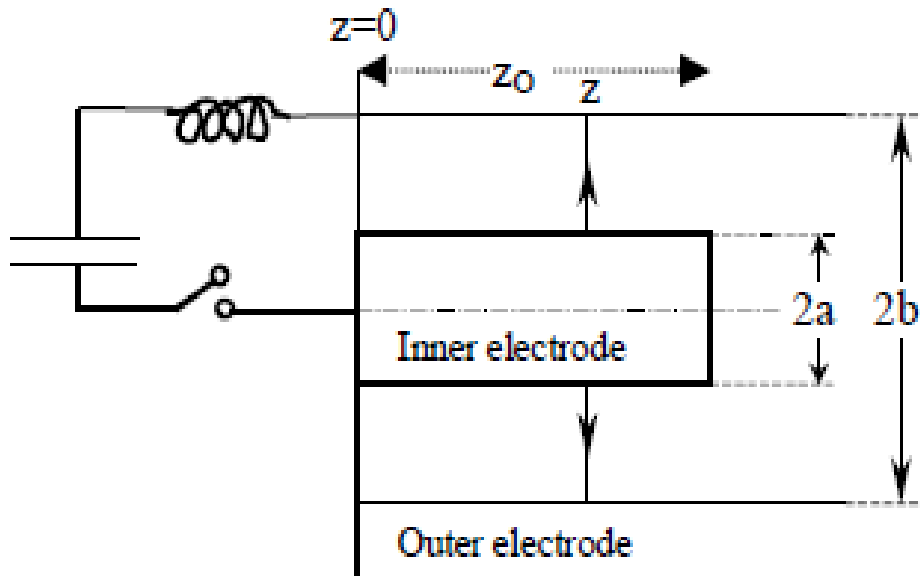
1/2

- Plasma focus: small fusion device, complements international efforts to build fusion reactor
- Multi-radiation device - x-rays, particle beams and fusion neutrons
- Neutrons for fusion studies
- Soft XR applications include microelectronics lithography and micro-machining
- Large range of device-from J to thousands of kJ
- Experiments-dynamics, radiation, instabilities and non-linear phenomena

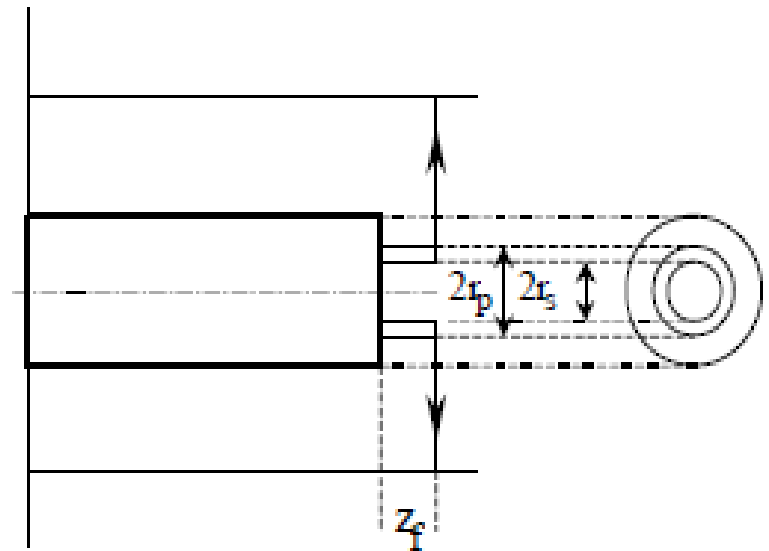
The Plasma Focus

2/2

Axial Phase



Radial Phases



The 5-phases of Lee Model code

Includes electrodynamical- and radiation- coupled equations to portray the REGULAR mechanisms of the:

- axial (phase 1)
- radial inward shock (phase 2)
- radial RS (phase 3)
- slow compression radiation phase (phase 4)
- the expanded axial post-pinch phase (phase 5)

Crucial technique of the code: Current Fitting

The Lee Model code- Comprehensive Numerical Experiments

This is the approach of the Lee Model code

- To model the plasma dynamics & plasma conditions
- Then obtain insights into scaling properties
- Then scaling laws

Critical to the approach:

Model is linked to physical reality by the current
waveform



Insights 1/2

- The Lee model code has produced ground-breaking insights no other plasma focus codes has been able to produce



Insights 2/2

Ground-breaking Insights published

- Limitation to Pinch Current and Yields- Appl Phys Letts. 92 (2008) S Lee & S H Saw: **an unexpected, important result**
- Neutron Yield Scaling-sub kJ to 1 MJ-J Fusion Energy 27 (2008) S Lee & S H Saw- multi-MJ- PPCF 50 (2008) S Lee
- Neon Soft x-ray Scaling- PPCF 51 (2009) S Lee, S H Saw, P Lee, R S Rawat
- Neutron Yield Saturation- Appl Phys Letts. 95 (2009) S Lee

Simple explanation of major obstruction to progress



From Measured Current Waveform to Modelling for Diagnostics

1/2

Procedure to operate the code:

Step 1: Configure the specific plasma focus

Input:

- **Bank parameters, L_0 , C_0 and stray circuit resistance r_0 ;**
- **Tube parameters b , a and z_0 and**
- **Operational parameters V_0 and P_0 and the fill gas**



Step 2: Fitting the computed current waveform to the measured waveform-(connecting with reality) 2/2

- A measured discharge current I_{total} waveform for the specific plasma focus is required
- The code is run successively. At each run the computed I_{total} waveform is fitted to the measured I_{total} waveform by varying model parameters f_m , f_c , f_{mr} and f_{cr} one by one, one step for each run, until computed waveform agrees with measured waveform.

The 5-Point Fit:

- First, the axial model factors f_m , f_c are adjusted (fitted) until
 - (1) computed rising slope of the I_{total} trace and
 - (2) the rounding off of the peak current as well as
 - (3) the peak current itselfare in reasonable (typically very good) fit with the measured I_{total} trace.
- Next, adjust (fit) the radial phase model factors f_{mr} and f_{cr} until
 - (4) the computed slope and
 - (5) the depth of the dipagree with the measured I_{total} waveform.



Example : NX2-Plasma SXR Source 1/4

- **NX2**
- 11.5kV, 2 kJ
- **16 shots /sec; 400 kA**
- 20J SXR/shot (neon)
- 10^9 neutrons/shot (D)



Example of current fitting: Given any plasma focus : e.g. NX2 16 shots/sec Hi Rep 2/4

- Bank parameters: $L_0=15\text{nH}$; $C_0=28\mu\text{F}$; $r_0=2\text{ m}\Omega$
- Tube parameters: $b=4.1\text{ cm}$, $a=1.9\text{ cm}$, $z_0=5\text{cm}$
- Operation parameters: $V_0=11\text{kV}$, $P_0=2.6\text{ Torr}$ in Neon

The UPFLF (Lee code) is configured (by keying figures into the **configuration panel** on the EXCEL sheet) as the NX2

Lo	Co	b	a	zo	ro mOhm
15	28	4.1	1.9	5	2
massf	currf	massfr	curfr	Model Parameters	
0.098	0.7	0.14	0.69		
Vo	Po	MW	A	At-1 mol-2	Operational
11	2.6	20	10	1	Parameters

INPUT:

OUTPUT: NX2 current waveform

NX2 dynamics & electrodynamics

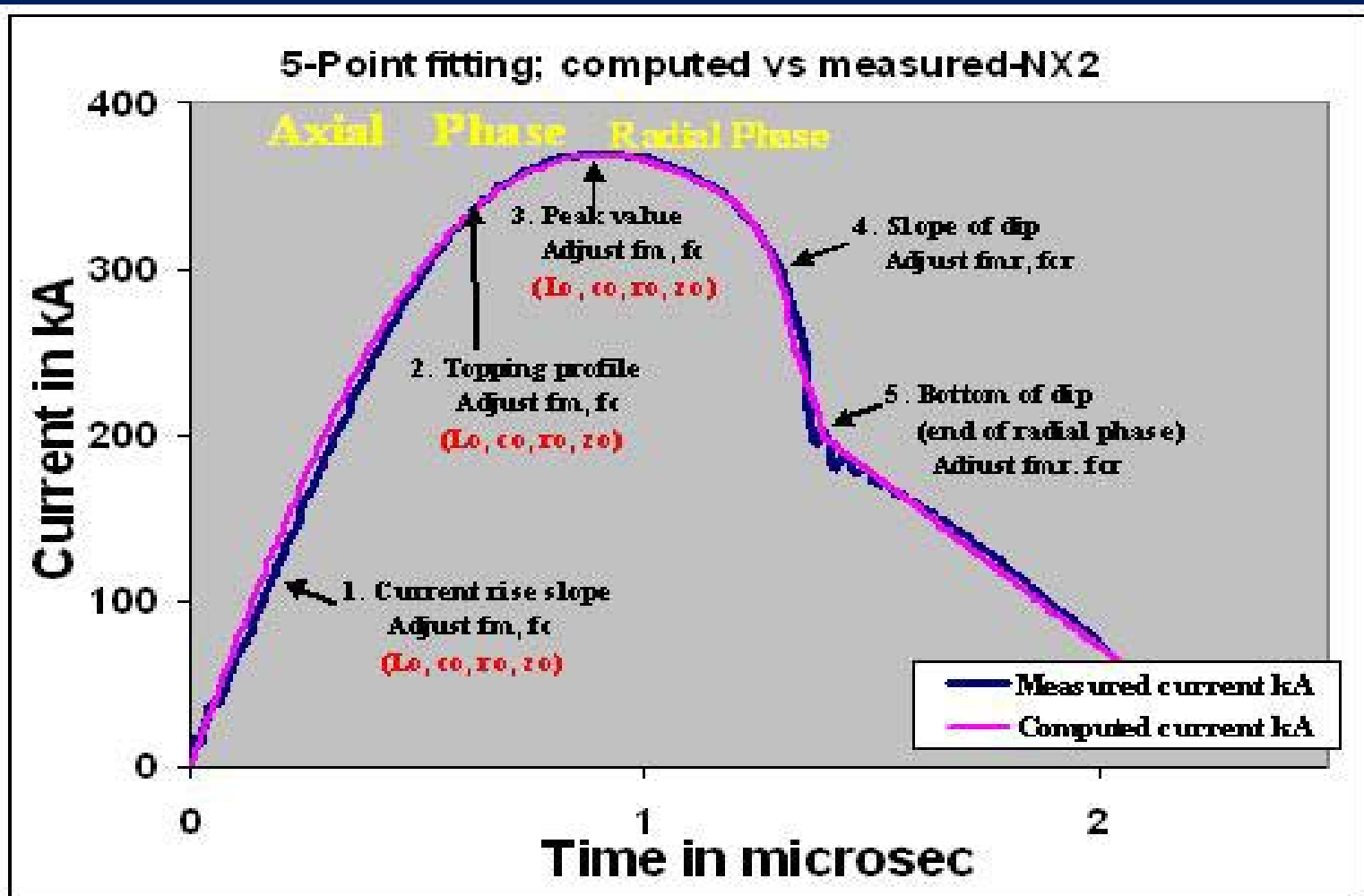
NX2 plasma pinch dimensions & characteristics

NX2 Neon SXR yield



Fitting computed I_{total} waveform to measured I_{total} waveform: the 5-point fit

3/4



Once fitted: model is energy-wise & mass-wise equivalent to the physical situation 4/4

- All dynamics, electrodynamics, radiation, plasma properties and neutron yields are realistically simulated; so that the code output of these quantities may be used as reference points for diagnostics

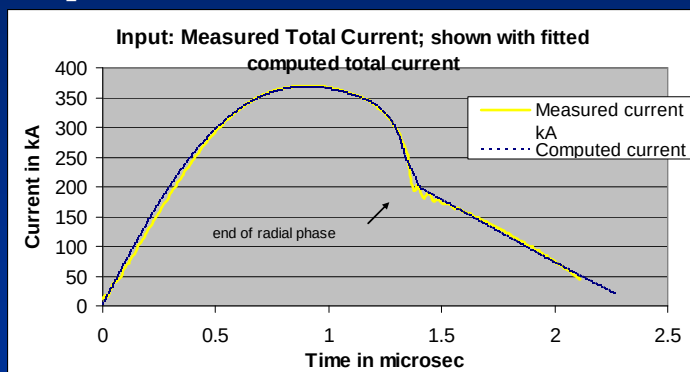
Numerical Diagnostics- Example of NX2

Time histories of dynamics, energies and plasma properties computed by the code

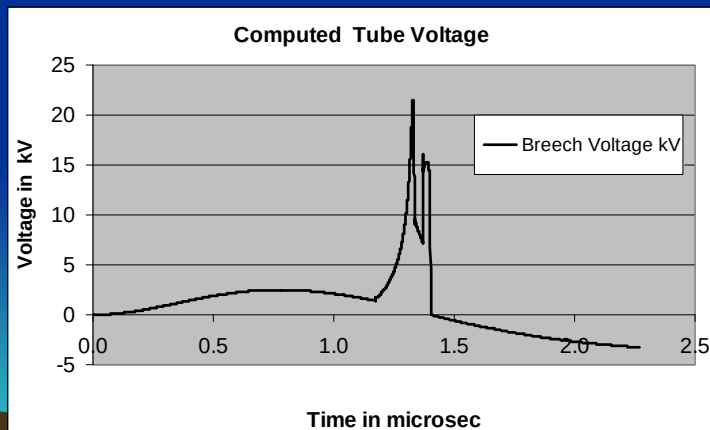
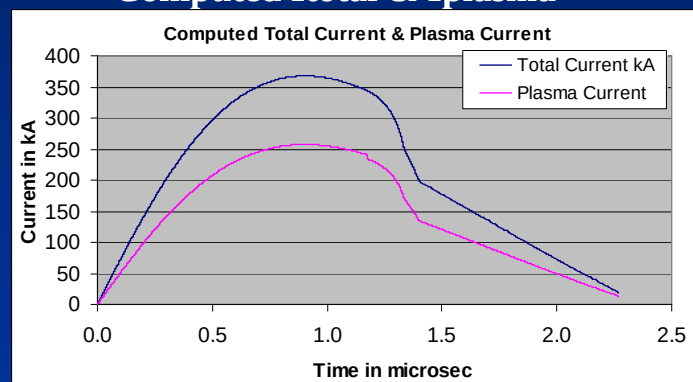
1/3

Last adjustment, when the computed I_{total} trace is judged to be reasonably well fitted in all 5 features, computed times histories are presented (NX2 operated at 11 kV, 2.6 Torr neon)

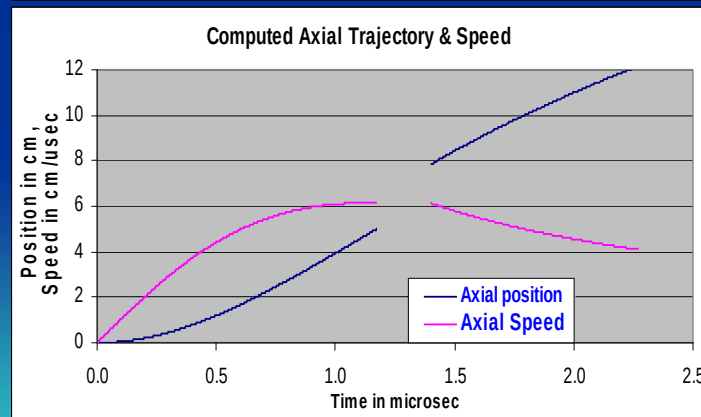
Computed I_{total} waveform fitted to measured



Computed I_{total} & I_{plasma}



Computed Tube voltage

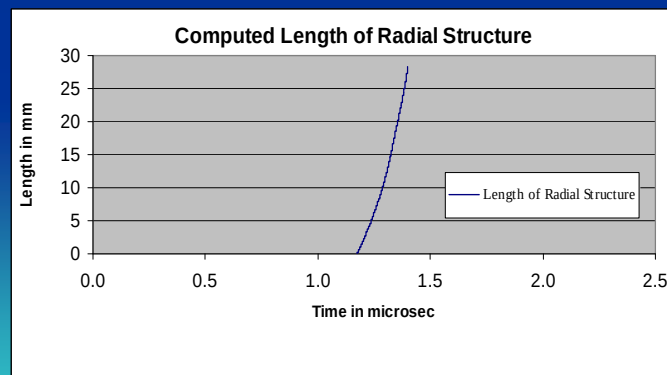
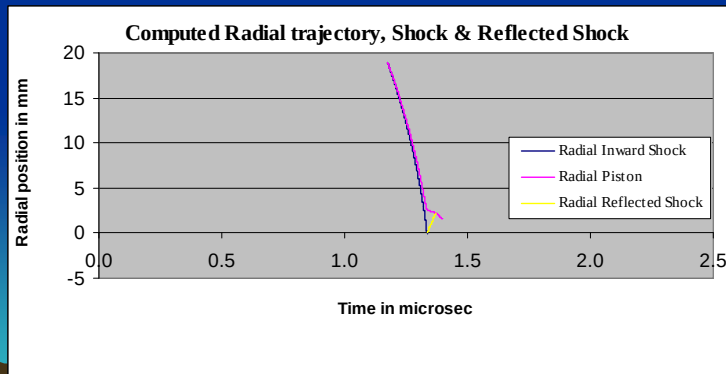
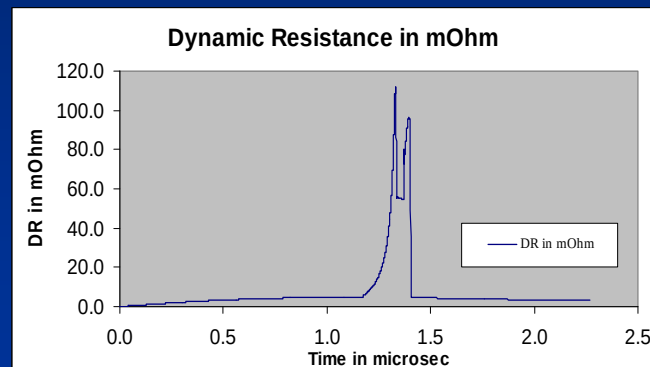
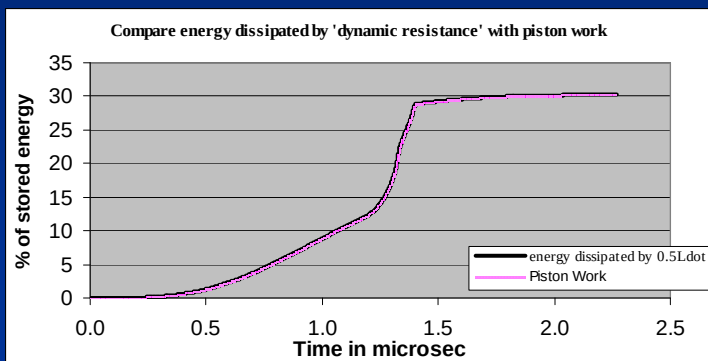
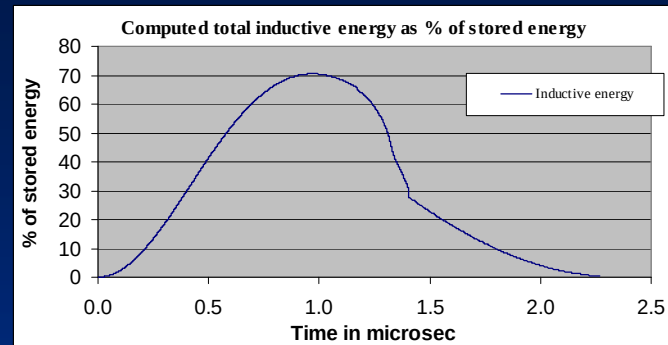
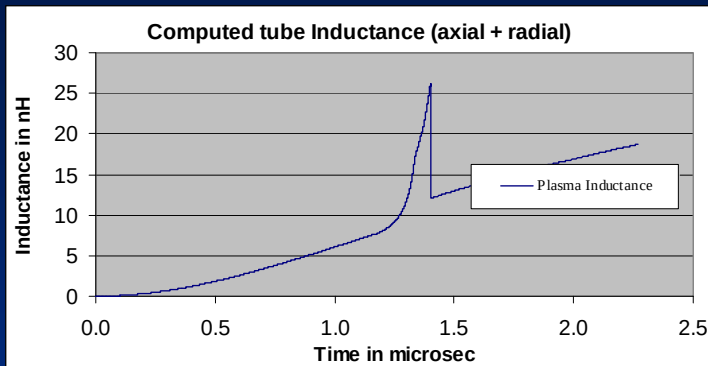


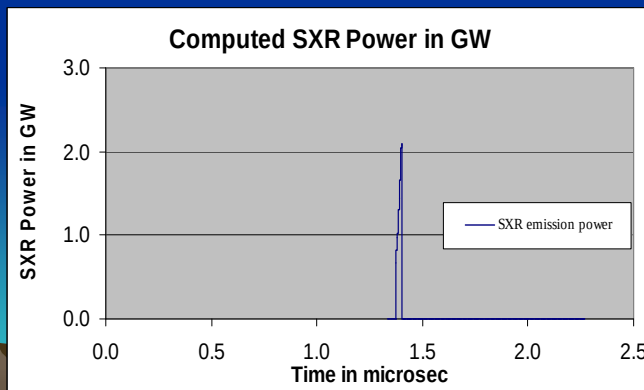
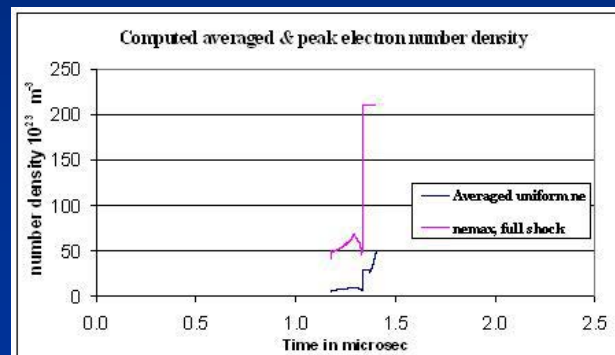
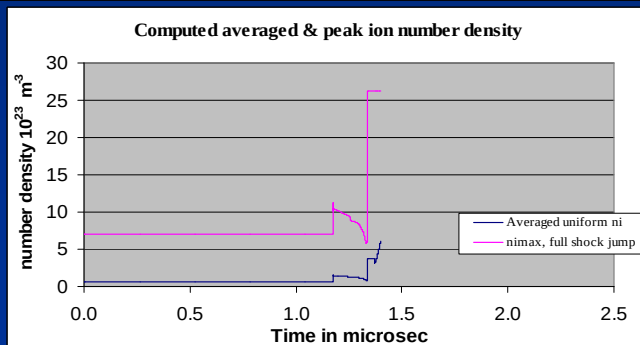
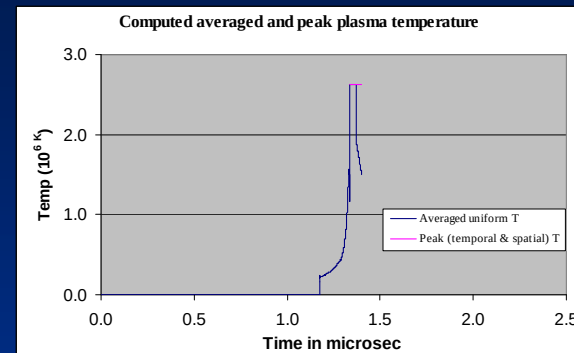
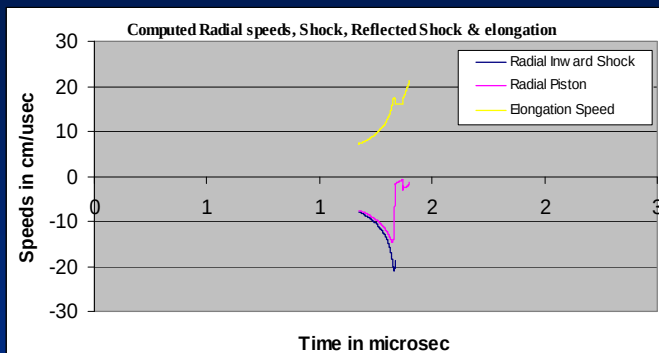
Computed axial trajectory & speed



Numerical Diagnostics- Example of NX2

2/3





Scaling Properties



3 kJ machine

Small Plasma Focus



PF1000 40kV 1332 μ F 9nH 1.1MJ I_0 = 15MA

1000 kJ machine

Big Plasma Focus



Comparing small (sub kJ) and large (thousand kJ) Plasma Focus

Scaling Properties: size (energy) , current, speed and yield

Scaling properties-mainly axial phase 1/3

Table 1.

	E_0	a	z_0	V_0	P_0	I_{peak}	v_a	ID	SF	Y_n
	kJ	cm	cm	kV	Torr	kA	cm/ μ s	kA/cm	(kA/cm) torr ^{0.5}	10 ⁸
PF1000	486	11.6	60	27	4	1850	11	160	85	1100
UNU ICTP	2.7	1.0	15.5	14	3	164	9	173	100	0.20
PF-400J	0.4	0.6	1.7	28	7	126	9	210	82	0.01



Scaling of anode radius, current and Y_n with energy E_0

Scaling properties-mainly axial phase

2/3

- Peak current I_{peak} increases with E_0 .
- Anode radius 'a' increases with E_0 .
- Current per cm of anode radius (ID) I_{peak} / a :
narrow range 160 to 210 kA/cm
- SF (speed factor) $(I_{\text{peak}} / a) / P^{0.5}$:
narrow range 82 to 100 (kA/cm) per Torr^{0.5} D
Observed Peak axial speed v_a : 9 to 11 cm/us.
- Fusion neutron yield Y_n :
10⁶ for PF400-J to 10¹¹ for PF1000.



Variation of ID SF and Y_n

Scaling properties-mainly axial phase 3/3

- ID and SF are practically constant at around 180 kA/cm and 90 (kA/cm) per torr^{0.5} deuterium gas throughout the range of small to big devices (1996 Lee & Serban IEEE Trans)
- Y_n changes over 5 orders of magnitude.



Comparing small (sub kJ) & large (thousand kJ) Plasma Focus

Scaling Properties: size ('a'), T, pinch dimensions & duration

Scaling properties-mainly radial phase 1/2

Table 2.

	c= b/a	a	T _{pinch}	v _p	r _{min}	z _{max}	Pinch duration	r _{min} /a	z _{max} /a	Pinch duration/a
		cm	10 ⁶ K	cm/μs	cm	cm	ns			ns/cm
PF1000	1.4	11.6	2	13	2.2	19	165	0.17	1.6	14
UNU ICTP PFF	3.4	1.0	8	26	0.13	1.4	7.3	0.14	1.4	8
PF400J	2.6	0.6	6	23	0.09	0.8	5.2	0.14	1.4	9



Focus Pinch T, dimensions & lifetime with anode radius 'a'

Scaling properties-mainly radial phase 2/2

- Dimensions and lifetime scales as the anode radius 'a'.
- $r_{\min}/a$ (almost constant at 0.14-0.17)
- $z_{\max}/a$ (almost constant at 1.5)
- Pinch duration narrow range 8-14 ns/cm of 'a'
- T_{pinch} is measure of energy per unit mass.

Quite remarkable that this energy density varies so little (factor of 5) over such a large range of device energy (factor of 1000).



- Scaling Properties: Pinch Dimensions & Duration:
Compare D & Ne
- (Lee, Kudowa 1998, Cairo 2003)

Table 3.

		Deuterium	Neon (for SXR)
minimum radius	$r_{\min}$	0.15a	0.05a
max length (hollow anode)	z	1.5a	1.6a
radial shock transit	t_{comp}	$5 \times 10^{-6} \text{a}$	$4 \times 10^{-6} \text{a}$
pinch lifetime	t_p	10^{-6}a	10^{-6}a
Speed factor	SF	90	



Rule-of-thumb scaling properties, (subject to minor variations caused primarily by the variation in $c=b/a$) over whole range of device

- **Axial phase energy density (per unit mass)** **constant**
- **Radial phase energy density (per unit mass)** **constant**
- **Pinch radius ratio** **constant**
- **Pinch length ratio** **constant**
- **Pinch duration per unit anode radius** **constant**



Further equivalent Scaling Properties

- Constant axial phase energy density (Speed Factor $(I/a)/\rho^{0.5}$, speed) equivalent to **constant dynamic resistance**
- I/a approx constant since ρ has only a relatively small range for each gas
- Also strong relationship requirement between plasma transit time and capacitor time $t_0 = (L_0 C_0)^{0.5}$
- E.g. strong interaction between t_0 and 'a' and I for a given bank.

The Lee Model Code

1/3

- Realistic simulation of all gross focus properties
- Couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation (Lee 1983, 1984)
- 5-phase model; axial & radial phases
- Includes plasma self-absorption for SXR yield (Lee 2000)
- Includes neutron yield, Y_n , using a beam–target mechanism (Lee & Saw 2008, J Fusion energy)

Numerical Experiments (1/2)

As shown earlier, Procedure is as follows:

- The Lee code is configured to work as any plasma focus:
- **Configure**
 - bank parameters: L_0 , C_0 and stray circuit resistance r_0 ;
 - tube parameters: b , a and z_0
 - operational parameters: V_0 and P_0 and the fill gas.
- **FIT:** the computed total current waveform to an experimentally measured total current waveform using four model parameters :
 - mass swept-up factor f_m ;
 - the plasma current factor f_i ,
 - for the axial phase; and
 - factors f_{mr} and f_{cr} for the radial phases.



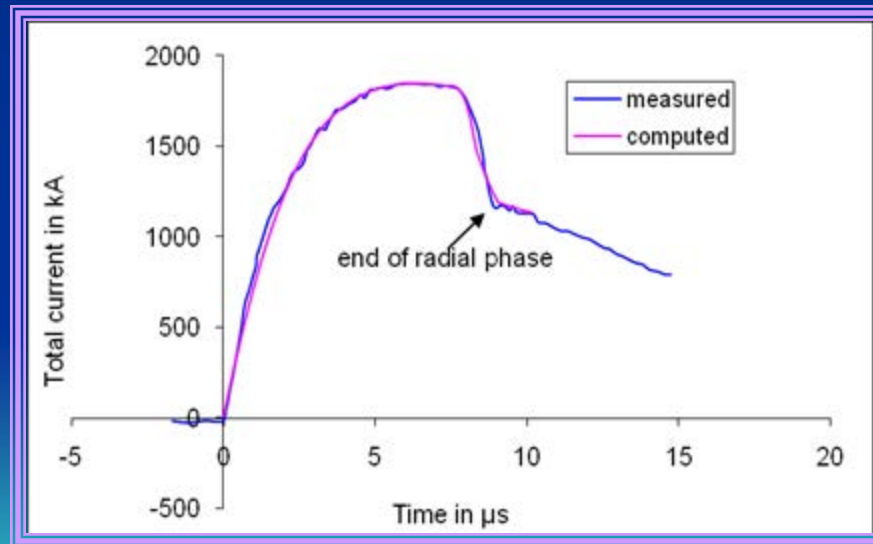
Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (1/4)

- To study the neutrons emitted by PF1000-like bank energies from 10kJ to 25 MJ.
- 1) Apply the Lee model code to fit a measured current trace of the PF1000:
 $C_0 = 1332 \mu\text{F}$, $V_0 = 27 \text{ kV}$, $P_0 = 3.5 \text{ torr D}_2$; $b = 16 \text{ cm}$, $a = 11.55 \text{ cm}$ or $c=1.39$; $z_0 = 60 \text{ cm}$; external (or static) inductance $L_0 = 33.5 \text{ nH}$ and; damping factor $\text{RESF} = 1.22$ (or stray resistance $r_0 = 6.1 \text{ m}\Omega$).
- 2) Apply the Lee code over a range of C_0 ranging from $14 \mu\text{F}$ (8.5 kJ) to $39960 \mu\text{F}$ (24 MJ):
 - Voltage, $V_0 = 35 \text{ kV}$; $P_0 = 10 \text{ torr deuterium}$; $\text{RESF} = 1.22$; ratio $c=b/a$ is 1.39.
 - For each C_0 , anode length z_0 is varied to find the optimum z_0 .
 - For each z_0 , anode radius a_0 is varied to get end axial speed of $10 \text{ cm}/\mu\text{s}$.



Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (2/4)

- Fitted model parameters : $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$.
- Computed current trace agrees very well with measured trace through all the phases: axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase as shown below.



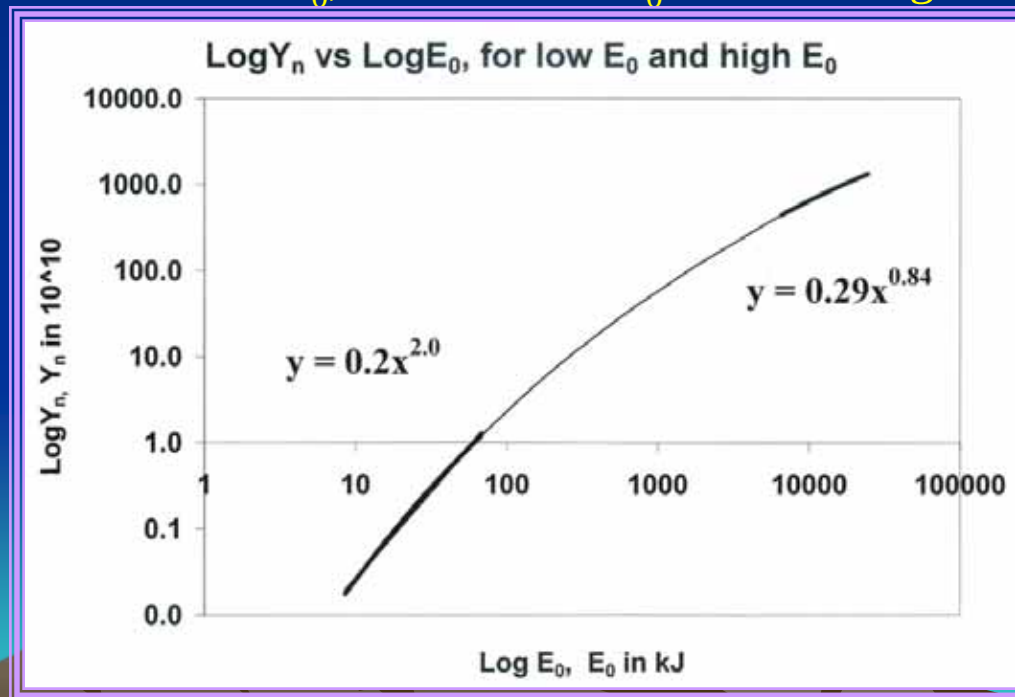
PF1000:

$C_0 = 1332 \mu\text{F}$; $V_0 = 27 \text{ kV}$;
 $P_0 = 3.5 \text{ Torr D}_2$; $b = 16 \text{ cm}$;
 $a = 11.55 \text{ cm}$; $z_0 = 60 \text{ cm}$;
 $L_0 = 33.5 \text{ nH}$; $r_0 = 6.1 \text{ m}\Omega$ or
 $\text{RESF} = 1.22$.



Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (3/4)

- Voltage, $V_0 = 35$ kV; $P_0 = 10$ torr deuterium; RESF = 1.22; ratio $c=b/a$ is 1.39.
- Numerical experiments: C_0 ranging from 14 μF (8.5 kJ) to 39960 μF (24 MJ)
- For each C_0 , anode length z_0 is varied to find the optimum z_0 .
- For each z_0 , anode radius a_0 is varied to get end axial speed of 10 cm/ μs .



Y_n scaling changes:

- $Y_n \sim E_0^{2.0}$ at tens of kJ

$Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ)



Scaling laws for neutrons from numerical experiments over a range of energies from 10kJ to 25 MJ (4/4)

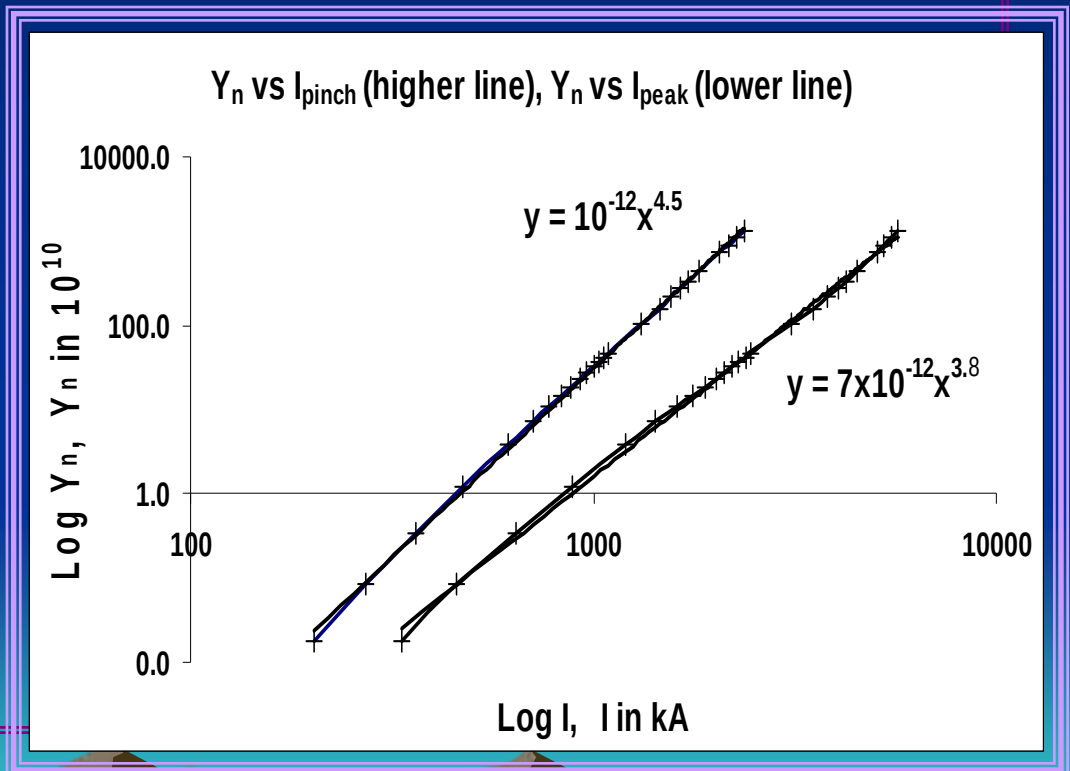
Scaling of Y_n with I_{peak} and I_{pinch} :

$$\blacksquare Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5}$$

and

$$\blacksquare Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8}$$

where $I_{\text{peak}} = (0.3-0.7)\text{MA}$
and $I_{\text{pinch}} = (0.2-2.4)\text{MA}$.



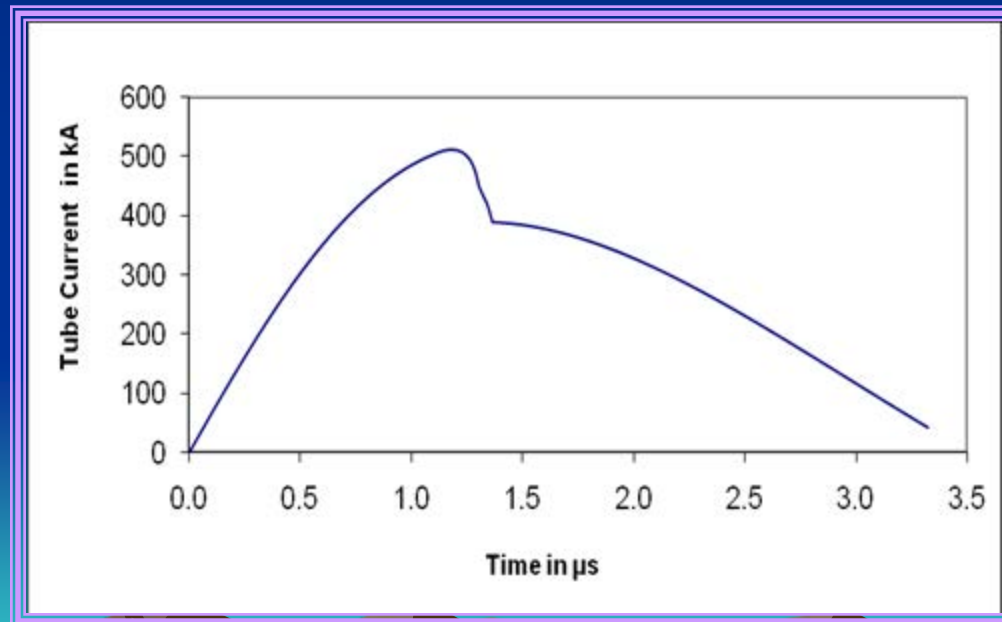
Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (1/4)

- To study the neon SXR emitted by a modern fast bank energies from 0.2 kJ to 1 MJ.
- Apply the Lee model code to a proposed modern fast plasma focus machine:
 - 1) With optimised values:
 $c=b/a = 1.5$
 $V_0 = 20 \text{ kV}$
 $L_0 = 30 \text{ nH}$
 $\text{RESF} = 0.1$
Model parameters : $f_m=0.06$, $f_c=0.7$, $f_{mr}=0.16$, $f_{cr}=0.7$.
 - 2) For C_0 varying from $1 \mu\text{F}$ (0.2 kJ) to $5000 \mu\text{F}$ (1MJ):
For each C_0 , vary P_0 , z_0 , and a_0 to find the optimum Y_{sxr}

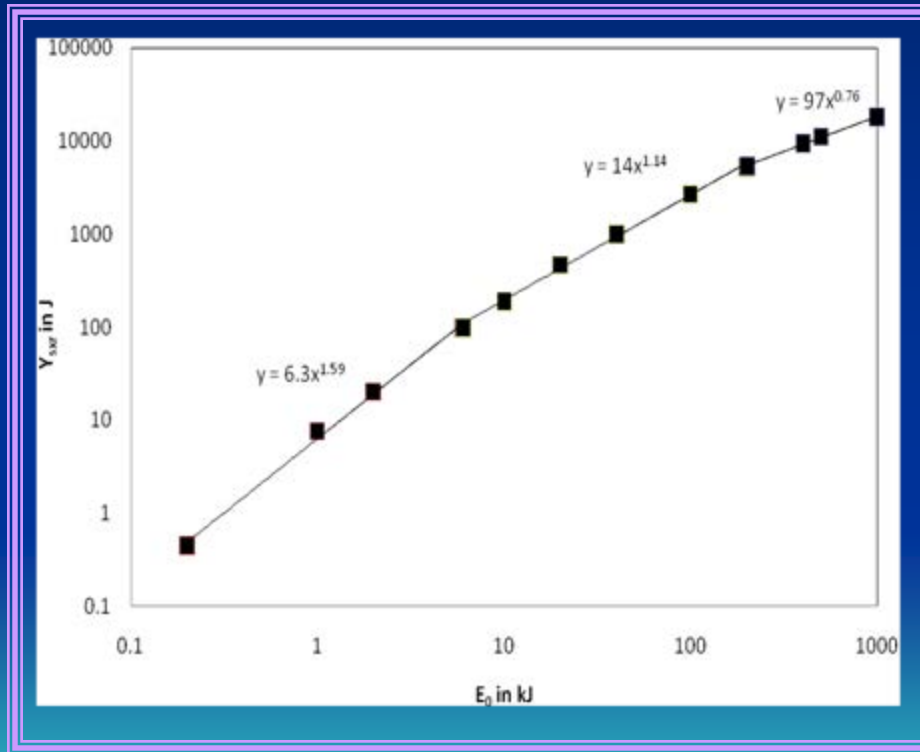


Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (2/4)

- Computed Total Current versus Time
- For $L_0 = 30\text{ nH}$; $V_0 = 20\text{ kV}$; $C_0 = 30\text{ }\mu\text{F}$; $\text{RESF} = 0.1$; $c=1.5$
- Model parameters : $f_m = 0.06$, $f_c = 0.7$, $f_{mr} = 0.16$, $f_{cr} = 0.7$
- Optimised $a=2.29\text{ cm}$; $b=3.43\text{ cm}$ and $z_0=5.2\text{ cm}$.



Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (3/4)

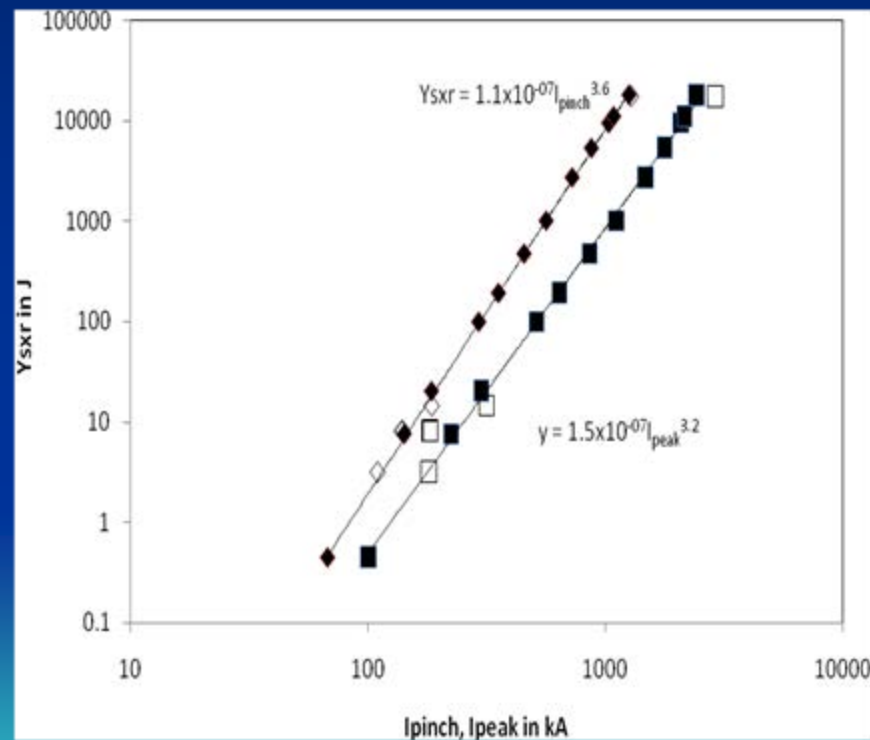


Y_{sxr} scales as:

- $E_0^{1.6}$ at low energies in the sub-kJ to several kJ region.
- $E_0^{0.76}$ at high energies towards 1MJ.



Scaling laws for neon SXR from numerical experiments over a range of energies from 0.2 kJ to 1 MJ (4/4)



- Scaling with currents
- $Y_{sxr} \sim I_{peak}^{3.2}$ (0.1–2.4 MA) and
- $Y_{sxr} \sim I_{pinch}^{3.6}$ (0.07–1.3 MA)
- Black data points with fixed parameters RESF=0.1; $c=1.5$; $L_0=30\text{nH}$; $V_0=20\text{ kV}$ and model parameters $f_m=0.06$, $f_c=0.7$, $f_{mr}=0.16$, $f_{cr}=0.7$.
- White data points are for specific machines with different values for the parameters : c , L_0 , V_0 etc.



Summary-Scaling Laws (1/2)

- The scaling laws obtained (at optimized condition) for Neutrons:
- $Y_n \sim E_0^{2.0}$ at tens of kJ to
- $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25MJ)
- $Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5}$ (0.2-2.4 MA)
- $Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8}$ (0.3-5.7MA)



Summary-Scaling Laws (2/2)

- The scaling laws obtained (at optimized condition) for neon SXR:
- $Y_{\text{sxr}} \sim E_0^{1.6}$ at low energies
- $Y_{\text{sxr}} \sim E_0^{0.8}$ towards 1 MJ
- $Y_{\text{sxr}} \sim I_{\text{peak}}^{3.2}$ (0.1–2.4 MA) and
- $Y_{\text{sxr}} \sim I_{\text{pinch}}^{3.6}$ (0.07–1.3 MA)



Plasma Focus Numerical Experiments- Trending into the Future

Part I: Scaling Properties & Scaling Laws

Conclusion to Part I

Recent numerical experiments uncovered new **insights** into plasma focus devices including :

- (1) **Plasma current limitation effect**, as device static inductance L_0 tends towards 0
- (2) **Scaling laws** of neutron yield and soft x-ray yield as functions of E_0 & I

These effects & scaling laws are a consequence of the scaling properties

- (3) A by-product of the numerical experiments are diagnostic reference points.



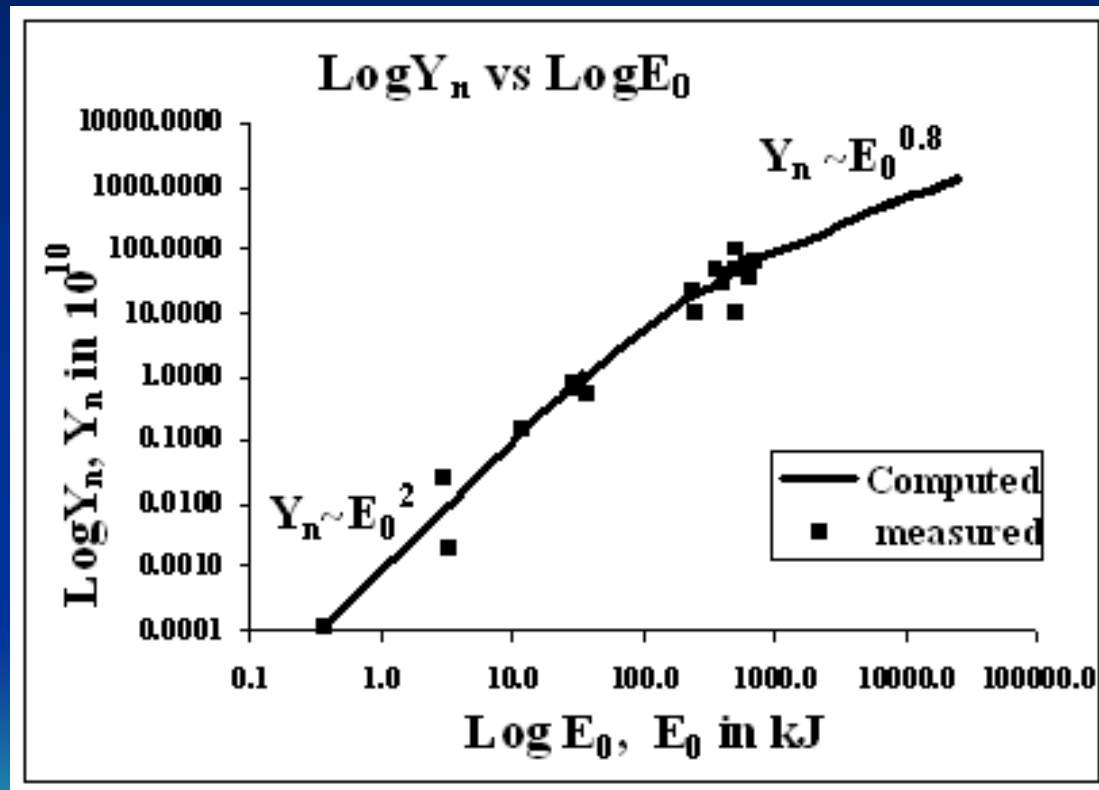
Plasma Focus Numerical Experiments- Trending into the Future

Part II: Concepts into the Future

- Global Neutron scaling law
- Yield deterioration & saturation
- Dynamic Resistance-Cause of “Neutron Saturation”
- Beyond present saturation?
- New classification of plasma focus devices into T1 (Low L_0) & T2 (High L_0)
- T2 requires instability phase modeling
- Simulate by means of anomalous resistance(s)
- Result in new quantitative data of anomalous resistance



Global scaling law, combining experimental and numerical data- Y_n scaling , numerical experiments from 0.4 kJ to 25 MJ (solid line), compared to measurements compiled from publications (squares) from 0.4 kJ to 1 MJ.



What causes the deterioration of Yield scaling?



What causes current scaling deterioration and eventual saturation? 1/3

- The axial speed loads the discharge circuit with a dynamic resistance
- The same axial speed over the range of devices means the same dynamic resistance constituting a load impedance DR_0
- Small PF's : have larger generator impedance $Z_0 = [L_0/C_0]^{0.5}$ than DR_0
- As energy is increased by increasing C_0 , generator impedance Z_0 drops



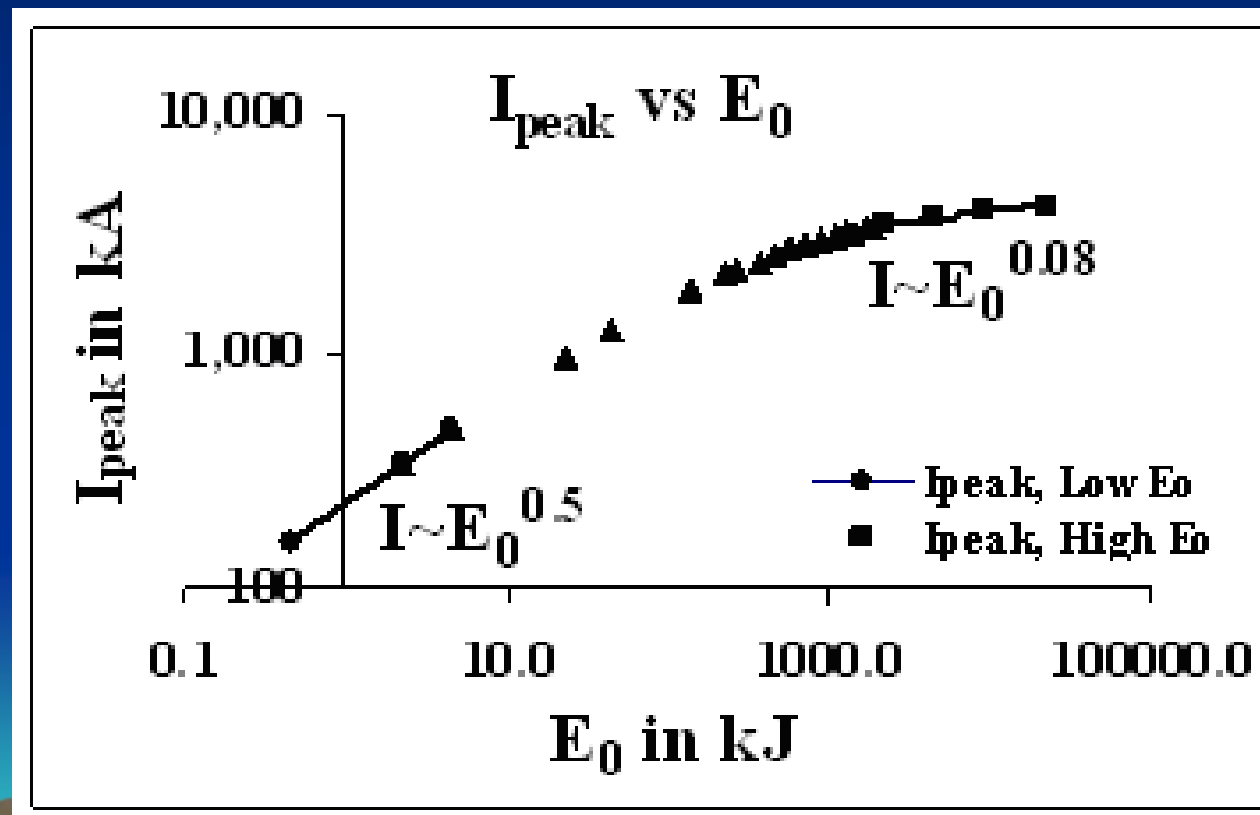
What causes current scaling deterioration and eventual saturation? 2/3

- At E_0 of kJ and tens of kJ the discharge circuit is dominated by Z_0
- Hence as E_0 increases, $I \sim C_0^{-0.5}$
- At the level typically of 100 kJ, Z_0 has dropped to the level of DR_0 ; circuit is now no longer dominated by Z_0 ; and current scaling deviates from $I \sim C_0^{-0.5}$, beginning of current scaling deterioration.
- At MJ levels and above, the circuit becomes dominated by DR_0 , current saturates



Deterioration and eventual saturation of I_{peak} as capacitor energy increases

- Axial phase dynamic resistance causes current scaling deterioration as E_0 increases



In numerical experiments we showed:

- $Y_n \sim I_{\text{pinch}}^{4.5}$
- $Y_n \sim I_{\text{peak}}^{3.8}$
- Hence deterioration of scaling of I_{peak} will lead to deterioration of scaling of Y_n .

What causes current scaling deterioration and eventual saturation? 3/3

- Analysis using the Lee model code has thus shown that the constancy of the dynamic resistance causes the current scaling deterioration resulting in the deterioration of the neutron yield and eventual saturation.
- This puts the global scaling law for neutron yield on a firmer footing

Connecting the scaling properties with the global scaling law (1/3)

- At kJ level; experimentally observed $Y_n \sim E_0^2$
 - Ideal scaling at the highest convenient voltage V_0 :
 $I \sim V_0 / Z_0$ at low energy level where Z_0 dominates
 - leading to $I \sim E_0^{0.5}$ for optimised low L_0
 - and $Y_n \sim I_0^4$
-
- At higher energy around 100kJ, Z_0 domination ends and current deterioration starts



Connecting the scaling properties with the global scaling law (2/3)

- Lower current increase than the ideal leads to lower increase in anode radius 'a'
- This leads to lower increase in pinch volume and pinch duration
- Which leads to lower increase in yield



Connecting the scaling properties with the global scaling law (3/3)

- Finally at very high energies, current hardly increases anymore with further increase in energy
- The anode radius should not be increased anymore.
- Hence pinch volume and duration also will not increase anymore.

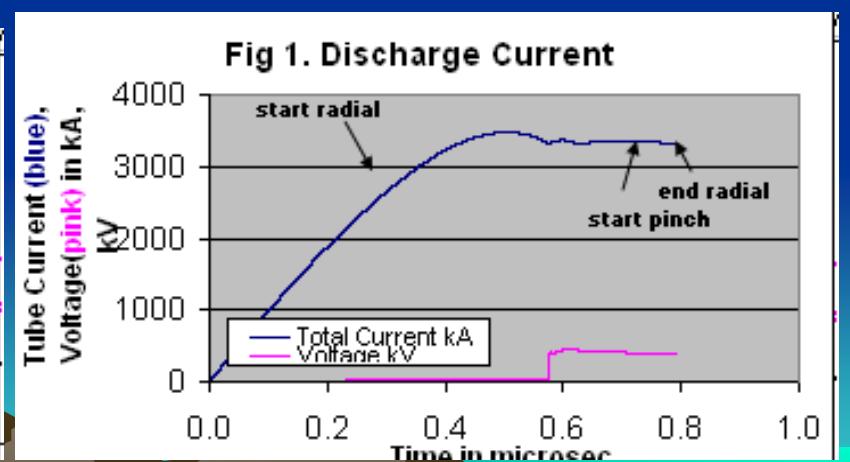
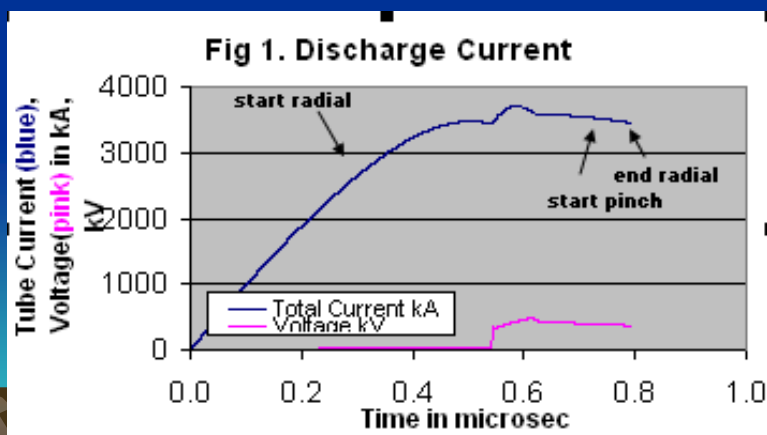
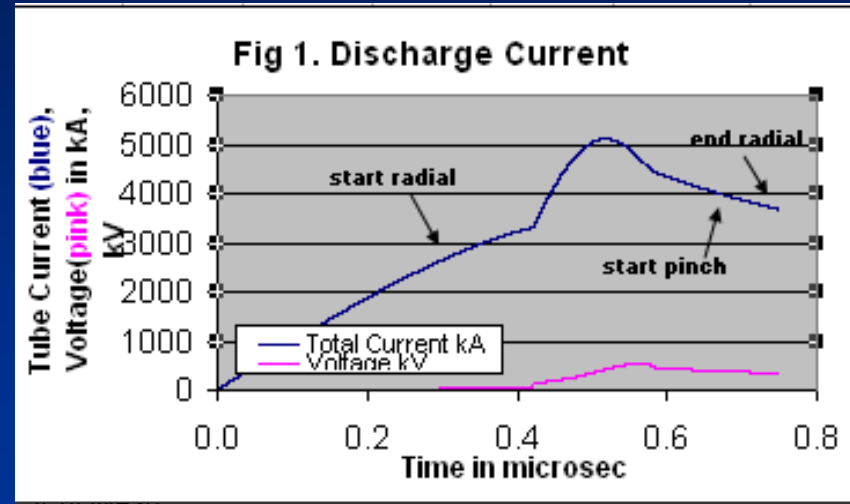
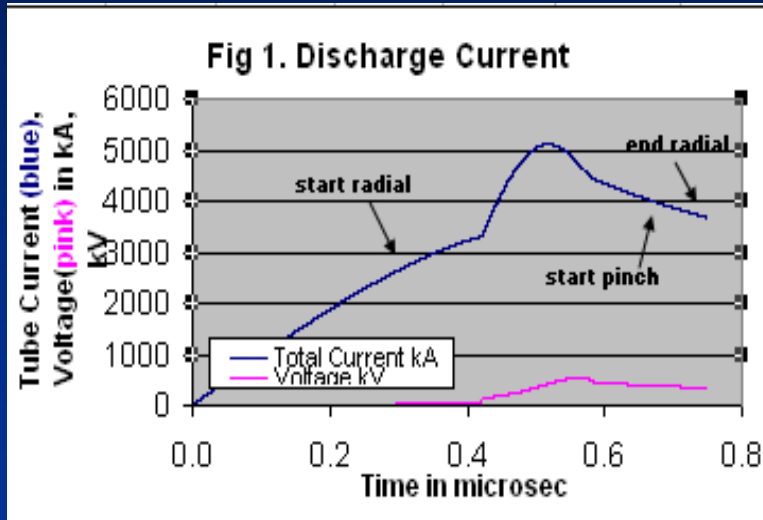
Thus we relate yield scaling deterioration & yield saturation to scaling properties, the fundamental one being the dynamic resistance.

Into the Future-Beyond Saturation Plasma Focus?

Current Stepped pinch: $b = 12\text{cm}$, $a = 8\text{cm}$, $z_0 = 2\text{cm}$; 2 capacitor banks: $L_1 = 30\text{nH}$, $C_1 = 8\mu\text{F}$, $r_0 = 6\text{m}\Omega$, $V_1 = 300\text{kV}$;
 $L_2 = 15\text{nH}$, $C_2 = 4\mu\text{F}$, $r_0 = 6.3\text{m}\Omega$, $V_2 = 600\text{kV}$; $P_0 = 12\text{Torr D}$

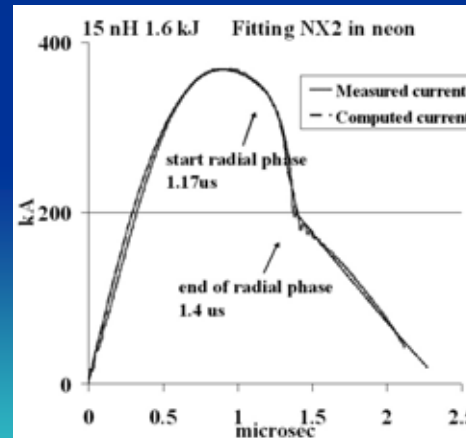
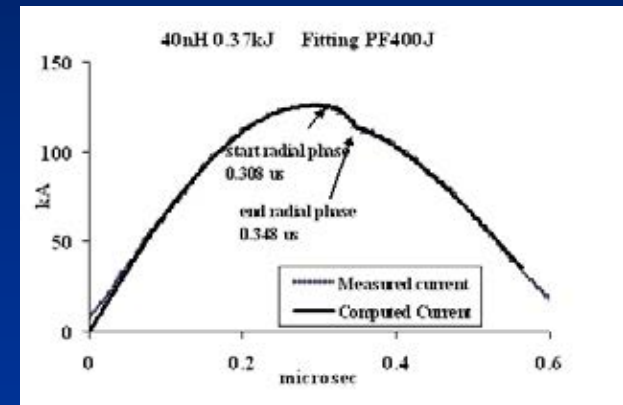
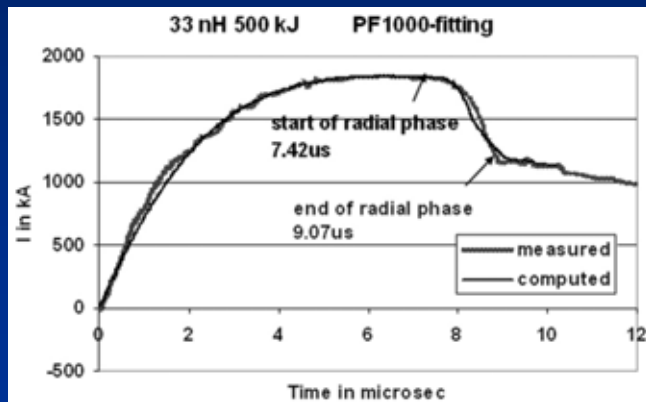
C_2 switched after radial start when $r = 0.8a$, $Y_n = 1.2\text{E}12$; $r = 0.6a$, $Y_n = 1.5\text{E}12$; $r = 0.5a$, $Y_n = 1.8\text{E}12$; $r = 0.4a$, $Y_n = 1.9\text{E}12$

IPFS-INTI Series 10, 10 October 2010 RADPF15.15d CS



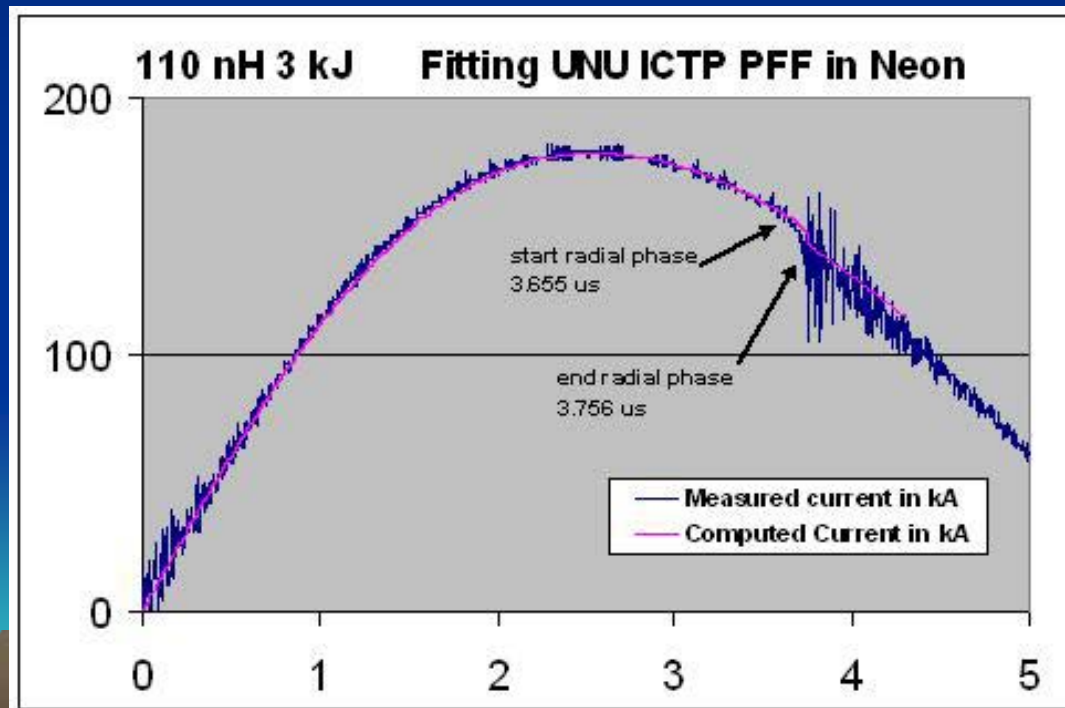
A New Development- 6 Phase Model 1/4

All well-published PF machines are well-fitted:
see following examples and many others;
note: the fit for the axial phase, and for the radial phase



Only one well-published machine did not fit

- UNU ICTP PFF- famed low-cost sharing network; current signal noisy and dip is small; difficult to judge the fitting-suspected ill-fit
- Low cost- necessitates single capacitor- hence high inductance L_0

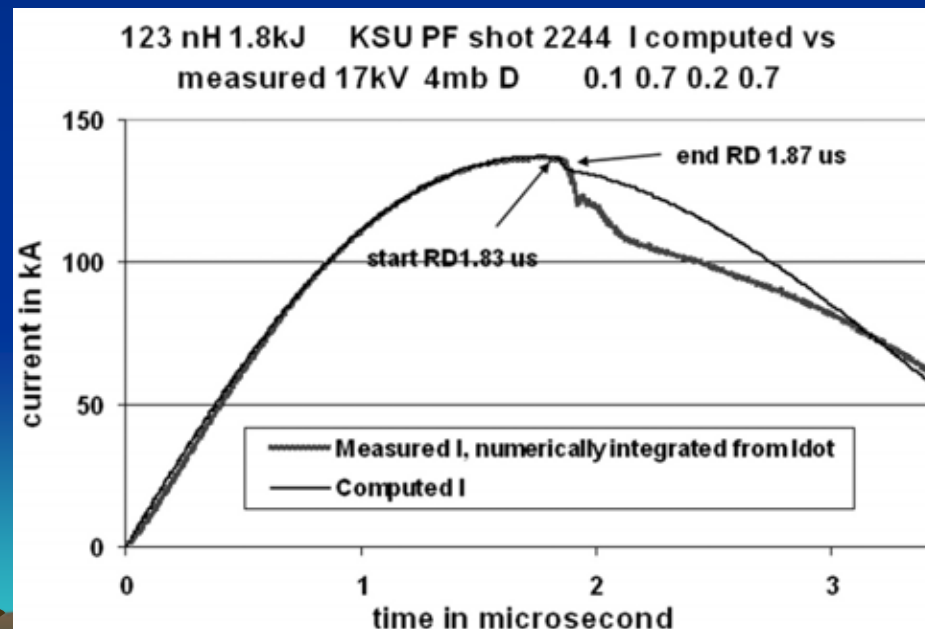


A New Development- 6 Phase Model 3/4

Recently KSU commissioned a machine; a modernised version of the UNU ICTP PFF

- A good Rogowski system was developed to measure dI/dt ; which was then numerically integrated resulting in a clean current signal-

Best fit nowhere near the fit of the well-published machines- in fact clearly could only fit a small portion of the radial phase



A New Development- 6 Phase Model

4/4

A study followed; resulting in classifying plasma focus devices into T1 & T2

Table 1. Classification of Plasma Focus Machines (D₂ operation).

PF name	L_0 (nH)	C_0 (μF)	I_{peak} (kA)	R_L	R_{EL}	RD dip (%)	Type
Poseidon	17.7	156	3205	0.9	2.5	32	T1
PF1000	33.5	1332	1845	1	1.6	34	T1
DPF78	55	17.2	869	4.1	12.8	11	T1
FN-II	75	7.5	309	4.3	8.5	10	T1
FMPF1	31	2.4	81	6.9	8.6	14.5	T1
PF-400J	40	1	126	8.8	17.3	8	T1
UNUICTP	110	30	163	16.7	29.5	1.9	T2
KSU	123	12.5	137	21.4	40	1.5	T2

Differentiator:

$$L_0$$

Better Differentiators:

$$R_L = (L_0 + L_a) / L_p$$

$$R_{EL} = (E_{L0} + E_{La}) / E_{LPinch}$$



Physical explanation 1/2

- RD mechanism for pinch purely compressive
- At end of RD (call this REGULAR DIP), expts show other effects eg instabilities leading to anomalous resistance- these mechanisms not modelled by 5-phase Lee code
- These anomalous resistive effects will absorb further energy from pinch; will result in further current dips- called EXTENDED DIP, ED

Physical explanation 2/2

Our studies further concluded

- T1: Small L_0 lead to big RD and relatively small ED
- T2: Big L_0 lead to small RD and relatively big ED

This explains why the 5-phase model:

For T1: the model parameters can be stretched for the RD to 'absorb' the ED

For T2: the model parameters, stretch how one likes, the RD cannot 'absorb' the ED

Development of the 6th phase 1/2 ie Phase 4a, between 4 and 5

- We have simulated using anomalous resistance of following form:

$$R=R_0[\exp(-t/t_2)-\exp(-t/t_1)]$$

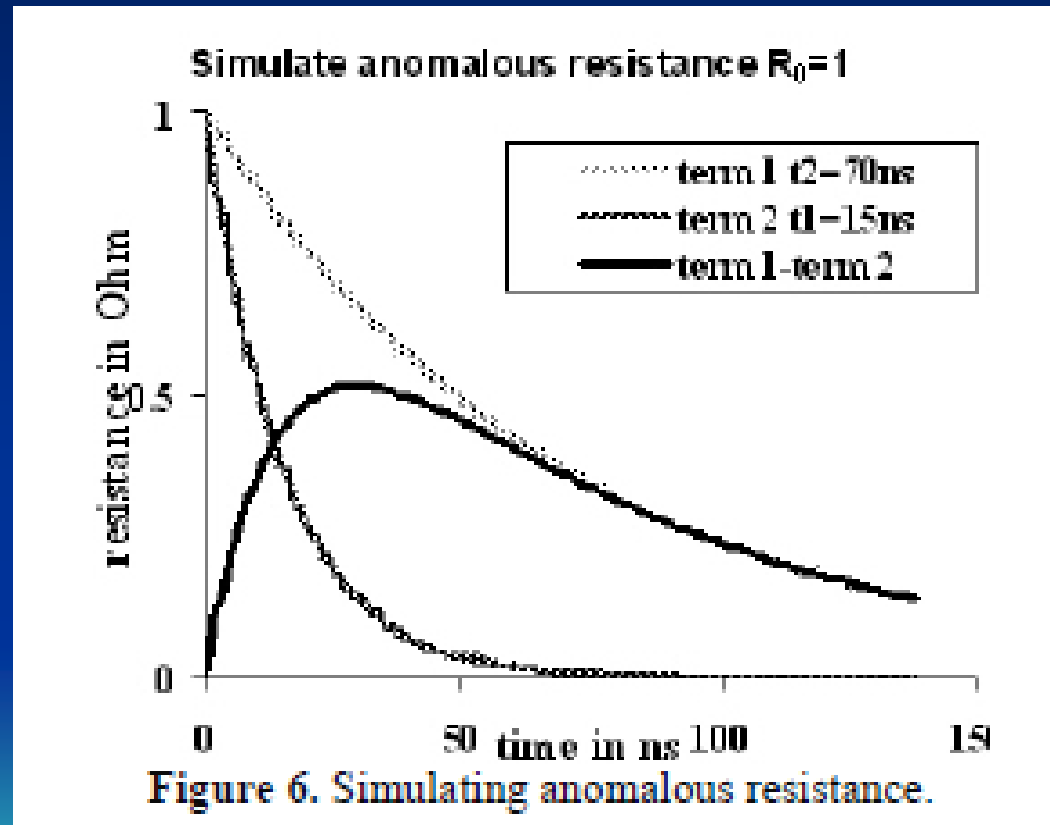
Where R_0 is of order of 1 Ohm, t_1 controls rise time of the anomalous resistance and t_2 controls the fall time (rate)

Use one term to fit one feature; terminate the term
Then use a 2nd term to fit a 2nd feature and so on



Development of the 6th phase 2/2

Simulated Anomalous Resistance Term



Result of Phase 4a fitting 1/3 applied to KSU Current Trace

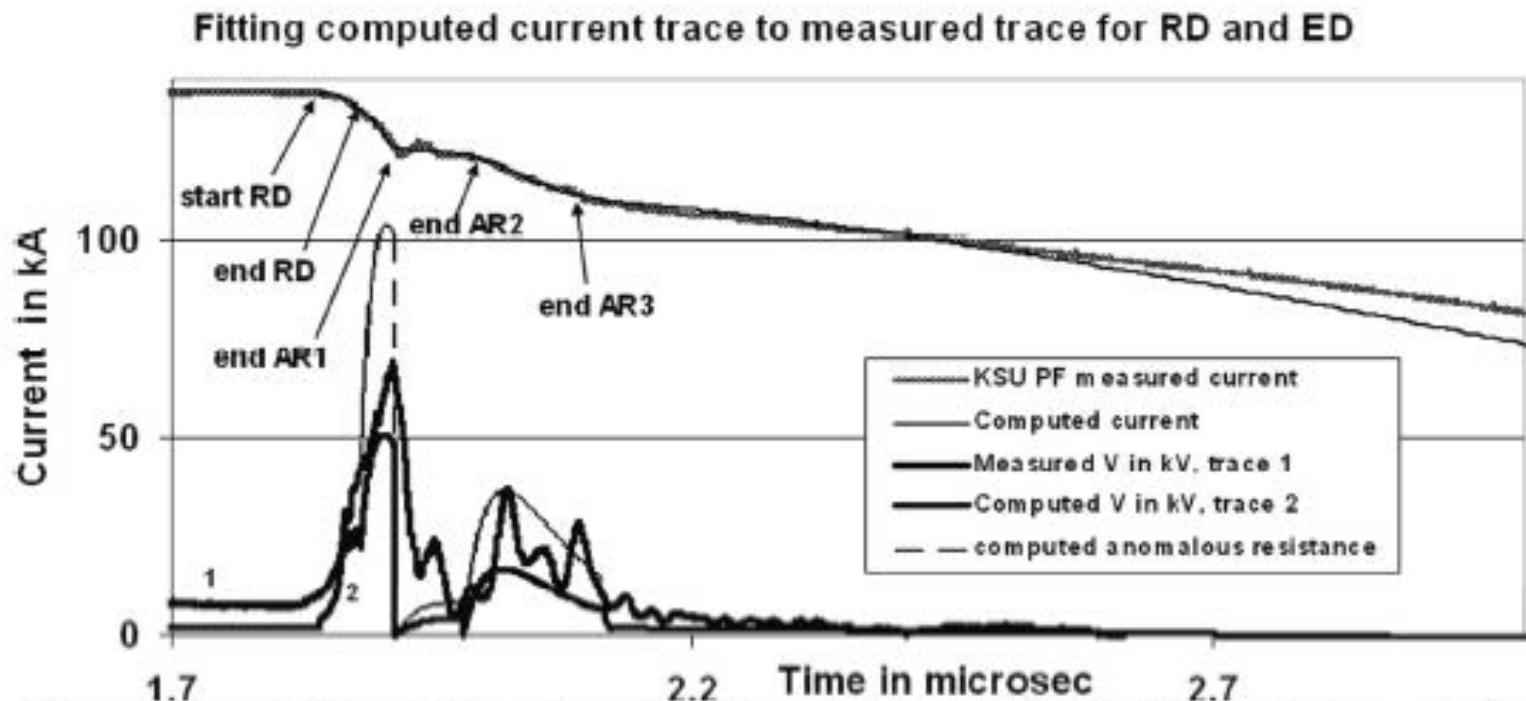


Figure 7. Computed Current (dip region only and expanded to see details) fitted to measured current with inclusion of Phase 4a.



Result of Phase 4a fitting 2/3

Table 3. Anomalous resistances used for the fitting.

	R_0 (Ω)	t_1 (ns)	t_2 (ns)	<i>endfraction</i>
Dip 1 <i>ED</i>	1.0	70	15	0.53
Dip 2	0.2	70	40	0.4
Dip 3	0.5	70	25	1.0



Result of Phase 4a fitting 3/3

- Current ED now fitted very well
- Fig also shows the form of the fitted anomalous resistance (3 terms)
- Figure shows that the computed tube voltage waveform also shows features in agreement with the measured tube voltage waveform
- The product of this Phase 4a fitting is the magnitude and temporal form of the anomalous resistance. This is an important experimental result. The information is useful to elaborate further on the instability mechanisms.
- Moreover even for the T1 current waveforms, we should fit by first just fitting the RD using the 5-phase model; ie the part that fits well with the computed is the RD; the rest of the dip os then fitted using phase 4a.



From Scaling Properties to Scaling Laws

Conclusion

We have looked at: numerical experiments deriving

- **Scaling properties of the Plasma Focus**
- **Scaling laws for neutrons & Neon SXR**
- **Neutron scaling law deterioration and saturation-and**
- **Connected the behaviour of the scaling laws to the scaling properties of the plasma focus**
- **The 6-phase model**
- **Resulting in new quantitative data of anomalous resistance**



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Papers from Lee model code

- S Lee and S H Saw, “Pinch current limitation effect in plasma focus,” Appl. Phys. Lett. 92, 2008, 021503.
- S Lee and S H Saw, “Neutron scaling laws from numerical experiments,” J Fusion Energy 27, 2008, pp. 292-295.
- S Lee, P Lee, S H Saw and R S Rawat, “Numerical experiments on plasma focus pinch current limitation,” Plasma Phys. Control. Fusion 50, 2008, 065012 (8pp).
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ASST Workshop on Engineering Education

Present Status, Trends and Challenges

19-21 July 2010, Damascus, Syria

Admission Policies in the Far-East

19 July 2010

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Content

- **Admission policies and student selection**
- **MQA-Student Selection and Support Services (AREA 4 COPIA & COPPA)**
- **COPIA Standards for Area 4.1 Admission and Selection**
- **COPIA Standards for Area 4.2 Articulation Regulations, Credit Transfer and Credit Exemption**
- **COPIA Standards for Area 4.3 Transfer of Students**
- **COPPA - Information on Area 4.1 to 4.3 : Student Selection**
- **Admission Criteria and guidelines for Engineering Degree Studies**

Admission Policies and Student Selection - 1/3

Admission Policies ensure that institutions enroll only applicants who:

- Possess the required pre-requisite knowledge and ability
- Are most likely to successfully achieve the stated learning outcomes of the programme
- Are most likely to successfully complete the studies

Admission Policies and Student Selection - 2/3

Admission Policies:

- Guide the development of degree curriculum (or vice-versa) to ensure that there is a smooth learning transition from high school into the degree study
- Guide the introduction of appropriate bridging courses where necessary to address knowledge gaps between high school and degree curriculum for applicants from particular region or nationality
- Provide information to potential applicants on the expectations of the degree programme with the aim to facilitate the applicants to make informed decisions and to better prepare themselves for degree studies

Admission Policies and Student Selection - 3/3

Student admission is:

- Governed by policies formulated by Ministry of Education/ Higher Education and together with the BEM-EAC for Engineering programme
- One of the areas of Quality Assurance for HEIs

MQA - Student Selection and Support Services (AREA 4 COPIA & COPPA) - 1/2

- Student admission is governed by policies formulated by the Ministry of Higher Education (MOHE) Malaysia
- There are varying views on the most appropriate way to select students into a higher education institution; there is no one best method of student selection
- Whatever method is selected, the HEI must be able to defend it consistently
- Appropriate developmental or remedial support must be made available to assist students

MQA - Student Selection and Support Services (AREA 4 COPIA & COPPA) - 2/2

- The number of students to be admitted is determined by the resources and capacity of the HEI as well as the number of qualified applicants
- The admission and retention policies of the HEI must not be compromised for the sole purpose of maintaining a desired enrolment
- If an HEI operates geographically separated campuses, or if a programme is a collaborative one, the selection and assignment of all students must be equally consistent with national policies.

COPIA Standards for Area 4.1

Admission and Selection - 1/4

Benchmarked Standards (10)

- The HEI must have clear criteria and processes of student admission --- including those affecting transfer and exchange students --- and select students whose capabilities are consistent with these criteria and processes
- The criteria and processes of selection must be published, disseminated and publicly accessible, especially to students
- Prerequisite knowledge and skills for purposes of student entry into each programme must be appropriate and clearly stated

COPIA Standards for Area 4.1

Admission and Selection - 2/4

Benchmarked Standards (10) –continue

- If a selection interview is utilised, the process must be structured, objective and fair
- Student selection must be fair and transparent
- There must be a clear policy on, and appropriate mechanism for, appeal
- The HEI must offer appropriate development or remedial support to assist students who need such support

COPIA Standards for Area 4.1

Admission and Selection - 3/4

Benchmarked Standards (10) –continue

- The number for each student intake must be related to the resources, capacity and capability of the HEI to effectively deliver its programmes
- Visiting, exchange and transfer students must be accounted for to ensure the adequacy of the HEI's resources to accommodate them
- The admission policy must be monitored and reviewed periodically to continuously improve the selection processes

COPIA Standards for Area 4.1

Admission and Selection - 4/4

Enhanced Standards (4)

- Student performance should be monitored as a feedback mechanism to assist in improving selection processes
- The review of the admission policy and processes should be in consultation with relevant stakeholders, nationally and internationally
- Student intake should incorporate social responsibility by privileged consideration for people with special needs
- There should be a relationship between student selection, programmes, and the desired learning outcomes

COPIA Standards for Area 4.2

Articulation Regulations, Credit Transfer and Credit Exemption

Benchmarked Standard

- HEP must have well defined and effectively disseminated policies, regulations and processes concerning articulation practices, credit transfers and credit exemptions

Enhanced Standard

- The HEI should be in touch with the latest development and thinking about the processes of articulation, credit transfers and credit exemptions including cross-border collaborative provisions

COPIA Standards for Area 4.3

Transfer of Students - 1/3

- In this age of increased cross-border education and student mobility, nationally and globally, the question of the transfer of students and credits and the articulation of accumulated learning has become a very important aspect of higher education
- Thus, sufficient attention must be given to ensure that transfer students are smoothly assimilated into the institution without undue disruption to his studies

COPIA Standards for Area 4.3

Transfer of Students - 2/3

Benchmarked Standards (2)

- The HEI must have a well-disseminated policy with clear criteria, mechanisms and processes, both academic and non-academic, to enable qualified students to transfer to another programme, within it or in another HEI
- Incoming transfer students must have comparable achievement in their previous institution of study

COPIA Standards for Area 4.3

Transfer of Students - 3/3

Enhanced Standard

The HEI should have in place policies and mechanisms that facilitate student mobility between programmes and institutions, within the country or cross- border, through articulation arrangements, joint degrees, exchange semesters, advanced standing arrangements, and the like

COPPA - Information on Area 4.1 to 4.3:

Student Selection - 1/6

4.1 Admission and Selection

Information on Benchmarked Standards

- Who is responsible for student selection? State the academic criteria and the mechanisms for admission to the programme and any other additional requirements. Show evidence that the admission policy and mechanism are free from discrimination and bias
- Provide evidence that the students selected fulfill the admission policies
- Describe the admission mechanisms and criteria for students with other equivalent qualifications (where applicable)
- Show how the criteria and mechanisms are published and disseminated

COPPA - Information on Area 4.1 to 4.3: Student Selection - 2/6

4.1 Admission and Selection

Information on Benchmarked Standards (continue)

- Describe the appeal mechanism
- Describe the characteristics of students admitted. Provide a copy of any technical standards that have been deployed for the admission of students with special needs
- Indicate the forecast student intake for the next five years. (Refer also to item 13 of Part B). Describe how the size of student intake is determined in relation to the capacity of the department and explain the mechanisms that exist for adjustments, taking into account the admission of visiting, exchange and transfer students

COPPA - Information on Area 4.1 to 4.3:

Student Selection - 3/6

4.1 Admission and Selection

Information on Benchmarked Standards (continue)

- Describe how the selection methods comply with the HEP's social responsibilities, human resource requirements and needs for further studies and lifelong learning
- If a selection interview is utilised, describe it
- State what special programmes are provided for those who are selected but need additional remedial assistance
- How does the department continuously monitor and periodically review student selection processes

COPPA - Information on Area 4.1 to 4.3: Student Selection - 4/6

4.1 Admission and Selection Information on Enhanced Standards

- Describe how the department engages the relevant stakeholders in the review of its admission policy and processes
- Show the relationship between student selection, the programme, and the learning outcomes

COPPA - Information on Area 4.1 to 4.3:

Student Selection - 5/6

4.2 Articulation Regulation, Credit Transfer and Credit Exemption

Information on Benchmarked Standard

- Describe the policies, regulations and processes of credit transfer, credit exemption and articulation practices, and how these are disseminated.

Information on Enhanced Standard

- Describe how the department keeps abreast of latest development with regards to articulation, credit transfer and cross-border provisions.

Admission Criteria and guidelines for Engineering Degree Studies 1/3

Admission Criteria for Engineering Degree Studies in Malaysia

Minimum entry qualifications approved by MOHE

-2 Principal Passes at STPM

including Mathematics and 1 Science subject

or

-equivalent

Admission Criteria and guidelines for Engineering Degree Studies 2/3

Admission Criteria for Engineering Degree Studies in Malaysia - continue

Equivalent qualifications approved by BEM:

- 2 Passes at A-Level including Mathematics and any Physical Science
- Diploma in Engineering approved by MOHE
- Foundation Studies in the relevant field approved by MOHE
- Or equivalent qualifications approved by MOHE

Admission Criteria and guidelines for Engineering Degree Studies 3/3

Admission Criteria for Engineering Degree Studies in Malaysia For International Applicants :

- Follow guidelines provided by MQA
 - Provide the matching of overseas qualifications to Malaysia SPM & STPM qualifications
- Entry requirements developed by Institution's Academic Committee
 - Establish the levels of achievement for overseas qualifications equivalent to approved local qualifications

Conclusion

In this lecture we have covered the following areas:

- Admission policies and student selection
- MQA-Student Selection and Support Services (AREA 4 COPIA & COPPA)
- COPIA Standards for Area 4.1 Admission and Selection
- COPIA Standards for Area 4.2 Articulation Regulations, Credit Transfer and Credit Exemption
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Thank you

Q & A

ASST Workshop on Engineering Education

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Overview of Knowledge Required in Far East Degree Programmes

20th July 2010

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Content

- Introduction
- Programme Rationale
- Programme Objectives
- Programme Outcomes
- Programme Structure/Specification
- Knowledge Required- examples applicable across engineering disciplines

Introduction – 1/5

- Engineering Education has a critical role in the development and continuous advancement of a nation and the world
 - different primary roles for underdeveloped, developing and developed countries

Therefore

- different challenges in training qualified and internationally recognised engineers

Introduction – 2/5

- Engineering Education must prepare engineering graduates:
 - to be able to solve problems in society which is constantly advancing with new technologies, expanding globally, increasingly diverse cultures, growing environmental concerns
 - to consider the moral and ethical implications of any solutions they provide
 - to be resourceful, innovative, creative and cooperative in seeking solutions to engineering problems which are increasingly multidisciplinary and complex

Introduction – 3/5

- Engineering graduates need to be able to
 - communicate and work effectively in any team
 - utilise systems approach to design and evaluate operational performance
 - use modern engineering tools
 - undertake lifelong learning

Introduction – 4/5

- Engineering programmes need to meet specified criteria and learning outcomes stated by regulatory/accrediting bodies such as:
 - UK QAA & EC – UK-Spec
 - US ABET-EC2000
 - MAL MQA & BEM-EAC2007

Introduction – 5/5

- The first step begins with establishing clear statements for the roles of Engineering Education
 - what is the position of the institution/nation
 - what the institution/nation wants to achieve
 - the constraints and limitations to overcome
 - what policies, systems, resources needed to be in place

Programme Rationale

- Statements describing the reasons driving the development of the programme establishing the needs for the programme
- There may be obvious learning gaps of entrants for the programme which are necessary to be stated including actions taken to address it (such as bridging courses/ remedial courses)
- It can also include any distinct features/medium to long term plan of the programme

Programme Objectives

- Statements of specific goals consistent with the vision and mission of the IHL and expressed interest of stakeholders
- Describe the expected achievements of graduates a few years after graduation
- Lead to the stated programme outcomes

Programme Outcomes

Statement of expected achievement of engineering graduates by regulatory/professional/accrediting bodies such as:

- UK-SPEC (QAA & EC, UK)
- EC2000 (ABET, US)
- EAC2007 (MQA & BEM, Malaysia)

Programme Structure – 1/5

EAC2007 (MQA & BEM, Malaysia)

- Entry level into programme
- Bridging/remedial courses
- Duration of the programme
- Credits of programme
- Component courses per level

Programme Structure – 2/5

EAC2007 (MQA & BEM, Malaysia)-continue

- Entry level into programme
 - STPM/A-level/Foundation/Equivalent
- Bridging/remedial courses
 - English/Mathematics/Sciences
- Duration of the programme
 - 4 years

Programme Structure – 3/5

EAC2007 (MQA & BEM, Malaysia)-continue

Credits of programme

- 120 credits hour (or 30 credit hours per level/year)
- 80 credit hours for engineering sciences and engineering design/projects
- 40 credit hours for general education component (e.g. mathematics, computing, languages, general studies, Co-curriculum, management, law, accountancy, economics, social sciences etc)

Programme Structure – 4/5

EAC2007 (MQA & BEM, Malaysia)-continue

Credit hours is based on a 14 week of study per semester, one credit hour is defined as:

- 1 hour per week of lecture
- 2 hours per week of laboratory or workshop
- 2 hours per week of supervised and compulsory tutorial session, subject to a maximum of one credit hour per course
- 3 hours per week of activities involving other modes of delivery such as PBL, e-learning, discovery learning, projects

Programme Structure – 5/5

EAC2007 (MQA & BEM, Malaysia)

Credit hours is based on a 14 week of study per semester, one credit hour is defined as: (continue)

- 3 hr per week of activities for final year project up to a maximum of 12 credit hours and a minimum of 6 credit hours
- 2 weeks of industrial training up to a maximum of 6 credit hours and with a minimum of 2 months continuous training

Examples of Knowledge/Skills Required across many of the engineering disciplines (extracted from UM, NTU, INTI Engineering websites)

Communication Skills-learning outcomes: to be able to

- Ask & answer questions in correct manner
- Hold a meeting and discussion
- Write technical reports and formal letters
- Give presentations using visual equipment
- Use correct language in writing and speaking

CREATIVE THINKING e.g. Applied to DESIGN

- Creative and visual thinking. Generation of ideas. Problem solving. Introduction to design process. Design elements and principles. Product presentation. Development of design. Visual design. Virtues of design. Related projects.

ENGINEERING INNOVATION AND DESIGN (EID)

- Practical-based:
Computer Aided Design (CAD) and Engineering (CAE). Design for assembly. Innovative product design. Design modeling. Engineering analysis. Appreciation of manufacturing processes. Pneumatic control. Mechatronics design and assembly. Material selection. Introduction to Business planning, and Project management. Practice in generating creative products, technology and innovative engineering solutions.

Engineering Drawing and CAD-knowledge required

- Graphics communication, sketching and text, engineering geometry, multiviews and visualization, auxiliary views, pictorial projections, section views, dimensioning and tolerancing practices; introduction to AutoCAD, 2-D and 3-D solid modelling using AutoCAD, multiview drawing using AutoCAD, pictorial and section drawings in AutoCAD, dimensioning in AutoCAD, creating 2-D drawings from 3-D models.

Engineering Drawing & CAD-learning outcomes- to be able to:

1. Clearly identify and control mental images.
2. Graphically identify technical designs, using accepted standard practices.
3. Apply plane and solid geometric forms to create and communicate design solutions.
4. Analyze graphics models, using descriptive and spatial geometry.
5. Solve technical design problems, using traditional tools or CAD.
6. Describe graphically, using sketches, traditional tools and CAD.
7. Apply technical graphics principles to engineering problems

Computer Programming- knowledge required

- Basic computer organization and the process of computer programming via a language such as C-language. Basic programming tools and programming procedures. Development of computer code to solve mathematical, science and engineering problems.
Running of complete code on PC; including use of a program to solve a particular problem.

Computer Programming-learning outcomes

1. Recognize the basic computer organisation and the needs for programming in engineering work.
2. Translate problem in science and/or engineering into program-development process.
3. Built up program codes using e.g. C-language, repetition, decision, array and demonstrate good programming practices.
4. Demonstrate and test running program with meaningful input and output.
5. Develop running program solving engineering problems incorporating subprograms and
6. Define parameters related to problems and to analyse systematically by parametric variation
7. Discuss computer programming method in problem-solving and relate to other known methods; e.g. comparison with analytical methods.

Applied Engineering Mathematics- knowledge required

- Ordinary differential equation, various types of equations, order and applications. Partial differential equation and its applications. Laplace transformation and its applications. Fourier series and transformation. Applications of Fourier series. Statistics and applications.

Applied Engineering Mathematics-learning outcomes

1. Apply ordinary differential equations (ODE) in engineering problems.
2. Solve first order and higher order ODE in homogenous and linear equations.
3. Explain the concept of ODE in integral & Euler equations.
4. Apply Partial Differential Equation (PDE) in engineering problems.
5. Apply PDE for partial derivatives and linear equation.
6. Recognize and apply Laplace transformation for engineering problems
7. Explain the applications of Fourier series and transformation for engineering problems.
8. Apply statistical concepts in engineering applications

Strength of Materials- knowledge required

- Stress, strain and elasticity. Bending and shearing stresses in beams. Torsion. Combined bending and direct stresses. Columns and struts. Slope and deflection of beams.

Strength of Materials-learning outcomes

1. Determine the stresses, strains and deformation of members in simple one-dimensional elastic system.
2. Evaluate the values and distribution of bending and shear stresses in beam section.
3. Calculate the torsional stresses that results from the action of torsional or twisting moment acting about a shaft.
4. Determine the stresses in a section due to direct axial loads and externally applied moments acting together.
5. Deduce the buckling load of columns and struts with different end conditions.
6. Employ the Macaulay or moment area methods to determine the deflection and rotation of loaded beams.

Applied Mechanics-knowledge required

- Applied mechanics using vector analysis approach; presentation of statics and dynamics. This approach leads to more concise derivations of the fundamental principles of mechanics. It also results in simpler solutions of three-dimensional problems in statics, and makes it possible to analyze many advanced problem in kinematics and kinetics, which could not be solved by scalar methods. The correct derivation and application of the principles of mechanics to the solution of engineering problems.

Applied Mechanics-learning outcomes

1. Explain fundamental concepts and principles of parallelogram law, transmissibility and Newton's law.
2. Apply vector mechanics in forces such as: forces in plane, forces in space and rigid bodies
3. Evaluate forces in rigid bodies and equilibrium of rigid bodies in two dimensions and three dimensions.
4. Evaluate distributed forces for centroids and centers of gravity.
5. Analyze structures, forces in beam and cables, friction and distributed forces: moment of inertia
6. Evaluate kinematics of particles and systems of particles
7. Investigate Kinetics of Particles: Newton's Second Law, Energy and Momentum and Rigid bodies

Thermodynamics of materials-knowledge required

- Fundamental concepts of thermodynamics of materials. Treatment of the laws of thermodynamics and their applications to equilibrium and the properties of materials.

Thermodynamics of materials-learning outcomes

1. Relate thermodynamic activity to engineering practice
2. Discuss chemical equilibrium including Ellingham diagram
3. Calculate cell voltage and electrochemical potential using Nernst equation
4. Determine direction of reaction of electrochemical cell
5. Define partial molar quantity and application to entropy and enthalpy
6. Apply Gibbs-Duhem Relation to the regular and polymer solution
7. Apply knowledge of Gibbs phase rule for condensed systems
8. Illustrate cooling curves and ternary diagrams
9. Compute average velocity of gas molecules and Boltzmann distribution
10. Compute partition function and the entropy of mixing of polymer solutions

Fluid Mechanics-knowledge required

Fluid mechanics and applications in engineering. Definition of fluid properties, Pascal law and pressure measurement. The underlying concept of fluid statics and dynamics, the relevant equations and their application. Analysis of flowing fluids in pipelines.

Fluid Mechanics-learning outcomes: to be able to:

1. Define fluid properties, Pascal law, and able to describe relationship between pressure and elevation.
2. Compute hydrostatic pressure and forces on submerged surfaces/objects.
3. Describe the fundamental equations of fluid dynamics, i.e. the continuity, Bernoulli, and momentum equations, and able to apply the Bernoulli and momentum equations for various problems.
4. Recognize the type of flow in pipelines, compute the energy grade line, and calculate the friction losses and minor losses, energy added and extracted.
5. Analyse and solve problems in simple and compound pipelines.

Electronic Materials and Devices- knowledge required

- Quantum mechanics and crystal structure of semiconductor, semiconductor materials and their properties, energy bands, carrier transport phenomena, p-n junction, diodes p-n junction, optoelectronics devices, transistors, the magnetic properties of the semiconductor materials and its advanced fabrication technology. All these components are vital to the understanding of both the operation of present day devices and any future development in the field. Knowledge includes many mathematical calculations which are needed in designing the electronic component for low cost and low energy expenditure.

Electronics materials and Devices- learning outcomes

1. Recognize and explain of the basic properties of semiconductors and their conduction process.
2. Relate the theoretical and practical aspects of the major steps in electronic devices fabrication.
3. Explain the physics and characteristics of all major semiconductor devices such as p-n junction, diodes, transistors, integrated devices, laser, solar cell etc.
4. Conduct experiments to study the physical properties of semiconductor materials.
5. Produce and present an engineering report on the technology trends of the semiconductor device-based electronic industry and their future demand.
6. Evaluate all identified case studies to cope with the rapidly growing of the electronic technology.
7. Employ and derive the basic governing mathematics equations for semiconductor device operation, and solve the engineering problems.

Environmental Engineering-knowledge required

- Introduction to environment, standards and legislations and related issues in Malaysia.

Water quality parameters, sources and characteristics of water and wastewater. Fundamentals of water and wastewater treatment processes, disposal of wastewater, sludge treatment, solid waste and hazardous waste management. Air pollution characteristics and control. Noise pollution characteristics and control.

Environment Engineering-learning outcomes

- Identify environmental standards and legislations and related issues in Malaysia.
- Describe important water quality parameters and their significance in terms of water treatment and river/stream analysis.
- Estimate the characteristics and effects of wastewater on the river system.
- Evaluate wastewater and water treatment units.
- Determine the characteristics, sources and management of solid and hazardous waste.
- Evaluate the types, sources, impacts and control strategies for air and noise pollution.

Safety Standards and Ethics- example of Biomedical engineering

- Medical safety standard or safety codes to guide equipment manufacturers in the production of safe equipment, electrical hazards of medical instruments e.g macroshock and microshock hazards, physiological effects of electricity on human body, leakage current, devices to protect against electrical hazards, and an equipment safety programme. The various ethical problems and concerns, which arise in health care, medicine and biomedical engineering. Ethical issues related to biomedical engineering e.g. genetic engineering.

Safety Standards and Ethics-example of biomedical engineering; learning outcomes

1. Explain the medical safety standard (Malaysia, IEC, BS)
2. Evaluate the electrical hazard for medical equipment.
3. Determine the devices available to protect against electrical hazards.
4. Determine the regulatory requirement related to biocompatibility of implant/medical device.
5. Explain the ethical issues related to biomedical engineering.

Feedback from industry during a recent round-table discussion regarding additional knowledge/ skills that industries look for in engineers

- In manufacturing, not so much the physical manufacturing level, but more of the value-added and higher end aspects like design, consultancy and project management
- Construction Industry would like Integrated Project & Contract Management courses in both Project Development & Construction and facilities management.
- Knowledge of law e.g. Malaysian Labour Law or basic principles of law.
- Marketing & communication skills; desirability of foreign languages.
- Financial management
- Occupational Safety & environmental and Health Management
- From this point of view it would be likely that a double major be preferred e.g. Engineering & law or engineering & marketing.
- Industry also emphasize the importance of additional practical training and industrial experience.
- Further emphases on soft skills, entrepreneurial skills, innovation and critical thinking.

INTI International University

Degree Programme Structure

Compulsory

- 3 Soft Skills Courses:-

- Entrepreneurship, Employability Skills, Personal Financial Planning Skills, Communicative Skills, Learning Skills and Critical Thinking Skills

- Communicative Foreign Languages

- Japanese, Mandarin, French and German

- University Courses where possible

- Technical Writing, Public Speaking, General Psychology, Introduction to Sociology, Art Appreciation and Introduction to Computing

Summary

In this lecture an overview of Engineering Programme and knowledge required is presented with the following content:

- Programme rationale
- Programme objectives
- Programme outcomes
- Programme structure
- Knowledge Required- examples applicable across engineering disciplines

ASST Workshop on Engineering Education

Present Status, Trends and Challenges

19-21 July, Damascus, Syria

Overview of Knowledge Required in Far East Degree Programmes

20th July 2010

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THANK YOU



ASST Workshop on Engineering Education

Present Status, Trends and Challenges 19 – 21 July 2010 Damascus, Syria

Quality Assurance in Engineering Education in Far East

21 July 2010

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Content

- ❑ Introduction
- ❑ Quality Assurance in Higher Education in Malaysia
- ❑ Quality Assurance by MQA
- ❑ 9 Areas of Quality Assurance Evaluation
- ❑ MQF – Learning Outcomes Descriptors – 8 Domains
- ❑ Achieving Quality Assurance
- ❑ An Example – Programme Review Procedure
- ❑ Establish Benchmark Standards
- ❑ Conclusion

Introduction – 1/4

What is Quality Assurance?

- Quality Assurance is a Systematic Process whereby the provider/supplier of goods/services guarantees to the receiver/consumer that the goods/services received will meet the Standards **consistently**

Introduction – 2/4

Engineering Education

- ❑ Quality Assurance is a systematic process to ensure appropriate and effective teaching, support, assessment and learning opportunities are provided to students to help them achieve the levels described by the academic standards of the awards and the standards set by the quality assurance, engineering accrediting bodies and related statutory and regulatory bodies.

Introduction – 3/4

- ❑ It provides confidence to potential students of the quality of education they can expect to receive from the Higher Education Institutions
- ❑ It provides confidence to employers on the quality of graduates they can expect to employ
- ❑ All Higher Education Institutions are expected to have a quality assurance system in place
- ❑ It covers both internal processes as well as external processes

Introduction – 4/4

Some key areas in Quality Assurance are:

- ☐ Design, approval, monitoring and review of programme (regular and periodic)
- ☐ Rigorous and fair assessment of students
- ☐ External examiners to ensure
 - Standards of awards, students performance and comparability of the standards,
 - Fair and sound processes of assessment , examinations and determination of awards

Quality Assurance in Higher Education in Malaysia (1/5)

☐ MOHE –

Ministry of Higher Education

- [http:// www.mohe.gov.my](http://www.mohe.gov.my)

☐ MQF –

Malaysian Qualifications Framework

- [Features](#)
- [Benefits](#)

☐ MQA –

- <http://www.mqa.gov.my/eng/mqf.cfm>

Malaysian Qualifications Agency

- Established by MQA Act 2007 (MQAA 2007)
- [Responsibilities](#)
- <http://www.mqa.gov.my/eng/index.cfm>

Quality Assurance in Higher Education in Malaysia (2/5)

❑ BEM –

Board of Engineering Malaysia

- registers graduates and professional engineers under the Registration of Engineers Act 1967 (Revised 2002)

❑ EAC –

Engineering Accreditation Council

- delegated by BEM for accreditation engineering degrees
- consists of representatives from BEM, IEM (Institute of Engineers, Malaysia), MQA, and PSD (Public Services Department, Malaysia).

Quality Assurance in Higher Education in Malaysia (3/5)

MQF - Features

- ❑ Classifies qualifications
 - benchmarked international practices.
- ❑ Specifies levels of qualifications, learning outcomes, credit system, and student academic load.
- ❑ Provides systematic links and pathways for qualifications
- ❑ Allows transfer of credits, recognition of prior learning

Quality Assurance in Higher Education in Malaysia (4/5)

MQF - Benefits

- ☐ Promotes international recognition
- ☐ Supports open access to education
- ☐ Promotes lifelong learning

Quality Assurance in Higher Education in Malaysia (5/5)

MQA - Responsibilities

- ☐ To implement MQF
- ☐ To establish and maintain MQR
 - [http:// www.mqa.gov.my/mqr/english/eperutusan.cfm](http://www.mqa.gov.my/mqr/english/eperutusan.cfm)
- ☐ To Quality Assure higher education:
 - ☐ Programme accreditation
 - ☐ Institutional Audit
 - ☐ Self accreditation status



Quality Assurance by MQA (1/3)

- ❑ Guided by 5 Quality Assurance Documents;
 1. The Malaysian Qualifications Framework (MQF);
 2. The Code of Practice for Programme Accreditation (COPPA);
 3. The Code of Practice for Institutional Audit (COPIA);
 4. Programme Discipline Standards; and
 5. Guide to Good Practices.

Quality Assurance by MQA (2/3)

- ❑ Covers 9 areas for COPPA and COPIA:
 1. Vision, mission, educational goals and learning outcomes
 2. Curriculum design and delivery
 3. Assessment of students
 4. Student selection and services
 5. Academic staff
 6. Educational resources
 7. Programme monitoring and review
 8. Leadership governance, admin situation and
 9. Continual quality improvement (CQI)

Quality Assurance by MQA (3/3)

- ❑ Consists of 2 level of standards:
 - Benchmarked standards (*HEP must comply*)
 - Enhanced standards (*HEP encouraged to fulfill them*)

9 Areas of Evaluation (1/10)

Area 1

Vision, Mission, Educational Goals and Learning Outcomes

- 1.1 Statement of vision, mission and educational goals
- 1.2 Participation in the formulation of vision, mission and educational goals
- 1.3 Academic autonomy
- 1.4 Learning outcomes

9 Areas of Evaluation (2/10)

Area 2

Curriculum Design and Delivery

- 2.1 Curriculum design and teaching-learning methods
- 2.2 Curriculum content and structure
- 2.3 Management of programmes
- 2.4 Linkages with external stakeholders

9 Areas of Evaluation (3/10)

Area 3

Assessment of Students

3.1 Relationship between assessment and learning

3.2 Assessment methods

3.3 Management of student assessment

9 Areas of Evaluation (4/10)

Area 4

Student selection and support services

4.1 Admission and selection

4.2 Articulation regulations, credit transfer and credit exemption

4.3 Transfer of students

4.4 Student support services and co-curricular activities

4.5 Student representation and participation

4.6 Alumni

9 Areas of Evaluation (5/10)

Area 5

Academic staff

5.1 Recruitment and management

5.2 Service and development

9 Areas of Evaluation (6/10)

Area 6

Educational resources

6.1 Physical facilities

6.2 Research and development

6.3 Educational expertise

6.4 Educational exchanges

6.5 Financial allocation

9 Areas of Evaluation (7/10)

Area 7

Programme monitoring and review

7.1 Mechanisms for programme monitoring and review

7.2 Involvement of stakeholders

9 Areas of Evaluation(8/10)

Area 8

Leadership, governance and administration

8.1 Governance

8.2 Institutional and academic leadership

8.3 Administrative and management staff

8.4 Academic records

8.5 Interaction with external sectors

9 Areas of Evaluation (9/10)

Area 9

Continual quality improvement

9.1 Quality improvement

9 Areas of Evaluation (10/10)

Areas	Number of Benchmarked Standards	Number of Enhanced Standards
Area 1	7	4
Area 2	19	11
Area 3	11	5
Area 4	21	13
Area 5	11	4
Area 6	12	10
Area 7	5	4
Area 8	11	6
Area 9	3	2

MQF – Learning Outcomes Descriptors – 8 domains

- ☐ Knowledge
- ☐ Practical skills
- ☐ Social responsibilities
- ☐ Values, attitudes and professionalism
- ☐ Communication, leadership and team skills
- ☐ Problem solving and scientific skills
- ☐ Information management and life-long skills and
- ☐ Managerial and entrepreneurial skills

Achieving Quality Assurance

With infrastructure and Teaching and Learning Resources in place and powered by:

⇒ Structure, Standards, System

⇒ People, Process, Procedures

Providing Quality Assurance

Structure – 1/2

- ❑ Institutional
 - Vision, Mission, Organisational Structure
 - Leadership, governance, policies
- ❑ National Agency/Frameworks
 - Quality assurance
 - Qualifications
- ❑ Legislations
 - Statutory
 - Regulatory

Structure – 2/2

□ Institution Level

- ❖ Constitution
- ❖ Board of Governors
- ❖ Academic Advisory Board
- ❖ Senate/Academic Board
- ❖ Senate Committees

□ Faculty/School/Centre

- ❖ Faculty/School/Centre Organisation Chart
- ❖ Faculty/School/Centre Management Team
- ❖ Faculty/School/Centre Committees

System

- ❑ Staff Recruitment, Development & Reward System
- ❑ Design, Development, Delivery, Monitoring and Review system
 - Curriculum
 - Programme
- ❑ Students support system – advising, acceptance, enrollment
- ❑ Students monitoring and evaluation system – attendance, assessment, feedback and progression
- ❑ Quality Management System
- ❑ Continuous Improvement System

Standards

- ☐ Legislation
- ☐ Qualifications Framework
- ☐ Code of Practice
- ☐ Subject benchmarks
- ☐ Accrediting bodies – core areas
- ☐ Professional bodies – programme outcomes
- ☐ Regulatory bodies
- ☐ Certification bodies

People

- ❑ Appropriate qualification/experience/skills (Competence)
- ❑ Job specifications – with clearly stated responsibilities and duties
- ❑ Key Responsibility Areas
- ❑ Key Result Areas
- ❑ Key Performance Index
- ❑ Induction, Staff Training and Development
- ❑ Continuous Professional Development
- ❑ Excellence Performance Recognition and Reward System
- ❑ Fair Remuneration Package



Process

Identify key processes

- ☐ Advising and Applications
- ☐ Orientation Programme
- ☐ Enrolment
- ☐ Programme Briefing & Advising
- ☐ Teaching and Learning
- ☐ Assessment
- ☐ Examinations
- ☐ Appeals
- ☐ Progression/Graduations
- ☐ Employability
- ☐ Alumni

Procedures

Establish clear procedures

- ☐ Purpose
- ☐ Scope
- ☐ Procedures
- ☐ Forms
- ☐ Attachments
- ☐ Related Procedures
- ☐ Records

An Example



Programme Review (AQP/CD/8) - Procedure (1/8)

- Purpose

- To provide information that the University can use to strengthen and improve the standards; quality and effectiveness of its academic programmes.

- Scope

- Faculties/ Schools/ Centres and Quality Assurance Committee.

Programme Review (AQP/CD/8) - Procedure (2/8)

- Criterion

- Each Faculty/ School/ Centre is required to conduct a review for each of the programmes offered by them.

- Frequency

- For **under-graduate programmes**, the review is to be carried out once every three years.
- For **certificate, diploma** and **post-graduate programmes**, the review is to be carried out once every two years.
- For **new programmes**, the review will be carried out not later than 1 year after the completion of studies of the first cohort of students from the programme.
- This is notwithstanding the regular reviews to update the changes of the programme.

Programme Review (AQP/CD/8) - Procedure (3/8)

- Procedure
 - Step 1 : Each Faculty/ School/ Centre should form their respective Programme Review Committees.

Programme Review (AQP/CD/8) - Procedure (4/8)

- o **Step 2** : The review shall include but not be limited to the following areas:
 - Programme Rationale
 - Programme Objective and Outcomes
 - Curriculum Enhancement
 - Teaching and Learning Resources
 - Student Enrolment
 - Student Performance
 - Student Progression
 - Student Completion and Withdrawal
 - Graduate Employment
 - Graduate pursuing higher studies
 - Feedback (from students, lecturers, parents, graduates, employers, industries, universities where graduate continue their higher studies, where applicable)
 - Performance of Subjects
 - Curriculum Review Committee Report
 - Annual Programme Monitoring Report
 - Recommendations

Programme Review (AQP/CD/8) - Procedure (5/8)

- **Step 3** : The Programme Review Committee shall submit the final report to their respective DOF/ DOS/ DOC/ HOS and the Chair of the Quality Assurance Committee of the Senate.
- **Step 4** : The respective DOF/ DOS/ DOC/ HOS shall evaluate the recommendation by the Programme Review Committee and take necessary and appropriate decisions/actions

Programme Review (AQP/CD/8) - Procedure (6/8)

- **Step 5** : The respective DOF/ DOS/ DOC/ HOS shall submit a report on the actions taken to the Quality Assurance Committee of the Senate.

Programme Review (AQP/CD/8) - Procedure (7/8)

- Related documents
 - Academic Programme Monitoring and Review Policy (POL/TL/02)
 - Programme Review Committee (AQP/CD/11)
 - Programme Monitoring (AQP/CD/10)
 - Performance of Subjects (AQP/CD/9)
 - Curriculum Review (AQP/CD/2)
 - Curriculum Review Committee (AQP/CD/12)
 - Quality Assurance Committee (SC/QAC/01)

Programme Review (AQP/CD/8) - Procedure (8/8)

- Records
 - The final reports of the programme review of each programmes and the DOF's/DOS's/DOC's/HOS's reports are kept at the relevant Faculty/ School/ Centre as permanent records.

Establishing the benchmark standards

- ❑ Regular, periodic & rigorous review process
- ❑ Self evaluation report
- ❑ Responsible actions for continuous improvement

Conclusion

In this lecture we covered:

- ❑ Quality Assurance in Higher Education in Malaysia
- ❑ Quality Assurance by MQA
- ❑ 9 Areas of Quality Assurance Evaluation
- ❑ MQF – Learning Outcomes Descriptors – 8 Domains
- ❑ Achieving Quality Assurance in Education
- ❑ An Example – Programme Review Procedure
- ❑ Establishing the Benchmark Standards

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21 July 2010

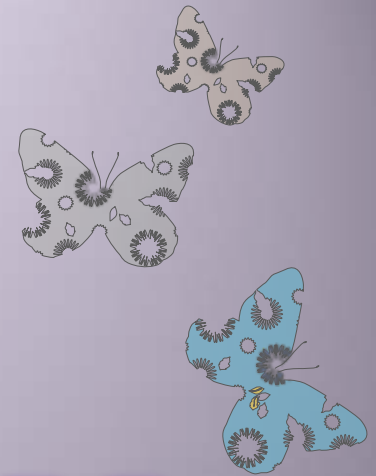
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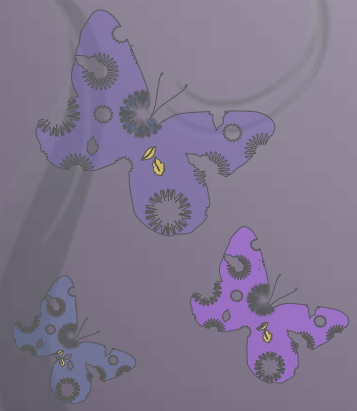
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Thank you



ASST Workshop on Engineering Education

Present Status, Trends and Challenges, 19-21 July 2010, Damascus, Syria

Future Role of Engineering Education in Promoting Sustainability

21st July 2010

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Content

➤ Sustainable Development

3 examples:

- 1. Sustainability in water usage-Australia
- 2. Product Design for Sustainable Development-Computers
- 3. Sustainable Development of Energy-Mankind's Giant Step Forward; The Future Role of Engineering Education in this Once-in-History Development

Sustainability

- Sustainability concerns responsible use of the world's finite natural resources in a manner which will not compromise the ability of future generations to meet their own needs.

Sustainable Development

- This requires optimum use of energy and materials to mitigate any associated environmental impacts
- Utilisation of concepts such as life cycle analysis, disassembly and recycling within design processes

Sustainable development:- areas:

- Water supply
- Food production
- Housing and shelter
- Sanitation and waste management
- Energy development
- Transportation
- Industrial processing
- Development of natural resources
- Cleaning up polluted waste sites
- Siting and planning projects to reduce environmental and social impacts
- Restoring natural environments such as forests, lakes, streams, and wetlands
- Improving industrial processes to eliminate waste and reduce consumption
- automation
- Recommending the appropriate and innovative use of technology

Engineering for Sustainable Development- Guiding Principles-Royal Academy of Engineering 2005

Fig 2a: Three dimensions of sustainability

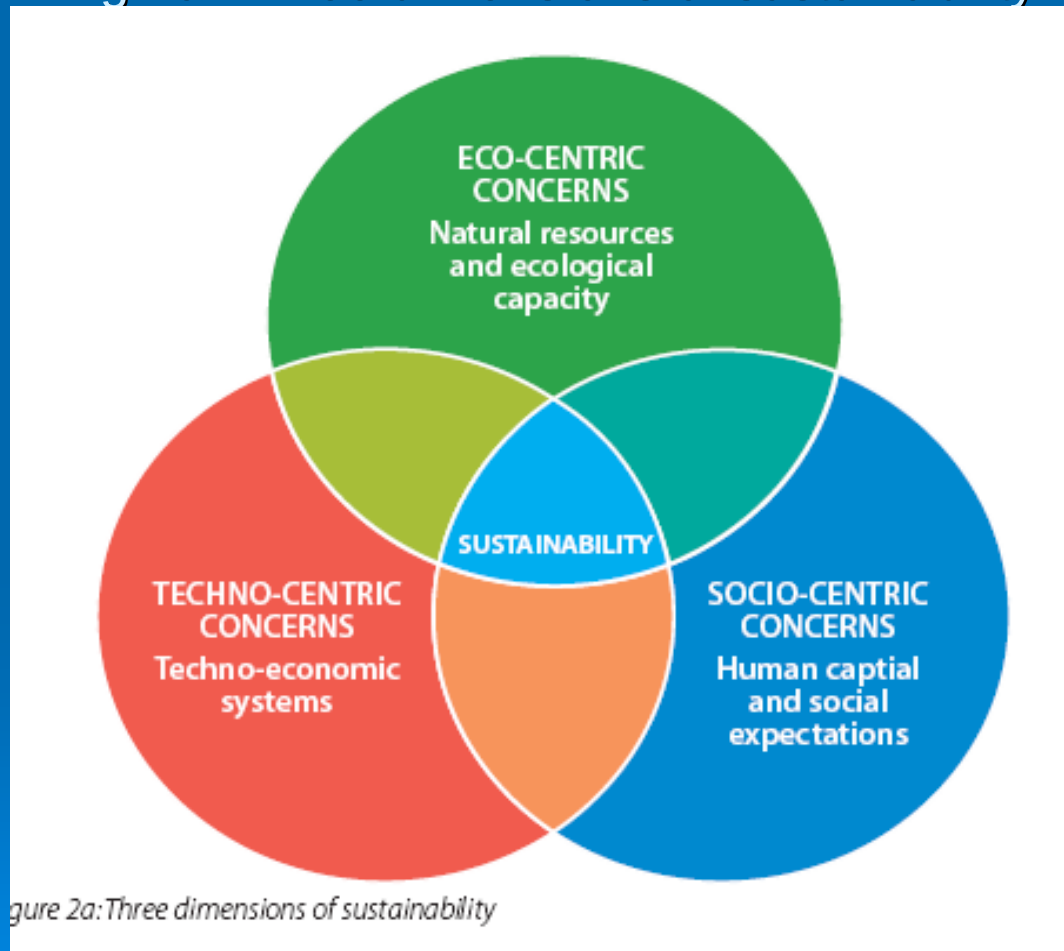
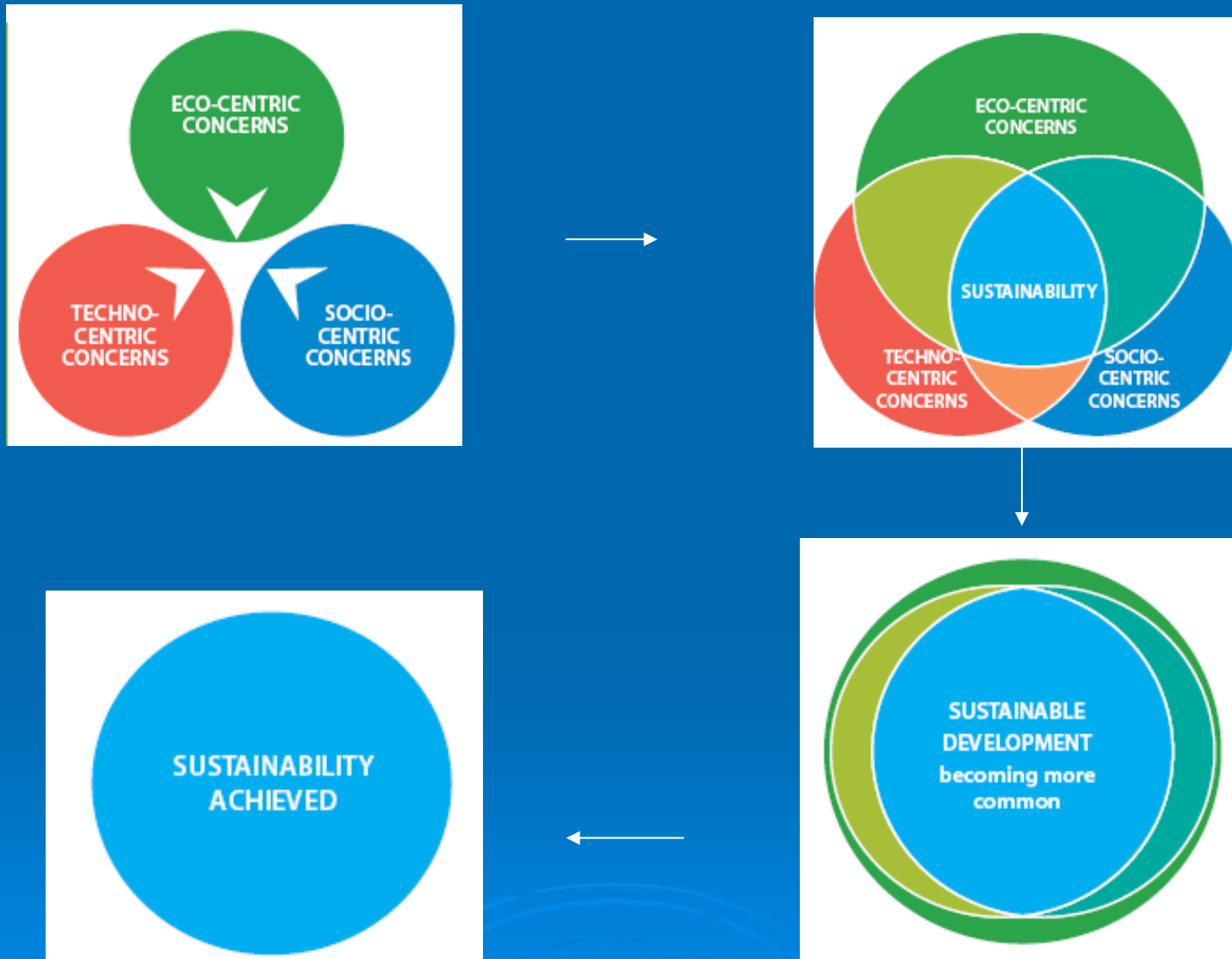


Fig 2b: Moving Towards Sustainable Development



Example 1: Sustainability in water usage- Australia's State of Environment Report and the Wentworth Group of Scientists

- Consequences of a range of critical global and national trends and their potential impact on current and future generations.
- Individual actions have implications well beyond their immediate surroundings.

Climate change, water shortages and biodiversity loss— can no longer be seen in isolation from each other or the wider context in which they occur. Threats to economic stability, environmental damage and social issues: current global trends are unsustainable.

Action is needed to create a sustainable future; to safeguard biodiversity and ecosystems.

- An holistic approach to decision-making:

Interconnected nature of social, economic and environmental issues needs to be recognised and acted upon.

Water for the Future is built on four key priorities

- Taking action on climate change
- Using water wisely
- Securing water supplies
- Supporting healthy rivers

Wentworth Group of Scientists-more details

- Every day, every Australian depends on water.
- Our health, food, and industries rely on healthy river systems, as does much of our leisure and cultural heritage.
- Yet in this dry continent, with its highly variable rainfall, Australians are now finding that water is no longer plentiful.
- The drought shows that urban and rural communities all face the same dilemma. We have had water restrictions in our cities, while unprecedented shortages of water for irrigation have caused huge losses to our economy.
- 300-year-old red gums die along the Murray floodplain due to extreme drought with severely depleted river flow; can Australia survive as a nation without changing the way water is used?
- We have made profound mistakes in many areas of Australia, Overallocation- too many licenses to take water from rivers & aquifers.
- Irrigation is critical to the wealth and social cohesion of rural Australia, generating half the profits of agriculture from 1% of land area. But we are degrading the rivers and natural systems that underpin this wealth.
- Urban Australians need to treat water as a scarce resource. With the projected growth of our major cities, there must be 25% reduction in per capita water use. We must reduce wastage; recycle treated effluents and retain stormwater for flushing toilets & watering lawns, sports fields & gardens.

Five Guiding Principles

Five guiding principles provide the foundation for a National Water Plan:

- 1. All Australians have a right to an adequate supply of safe water for domestic use;
- 2. We all have a responsibility to use water efficiently;
- 3. Our rivers, groundwater systems and landscapes must be managed to maintain the health of our ecosystems so they can provide for the variety of current and future human needs;
- 4. Those who use fresh water to create wealth need investment security and should take responsibility for their part in sustainable water management; and
- 5. Australians must become water literate and understand the effects that water use has on our environment and other people.

Forward into the future

- The way forward is to stop the profligate waste of water in our cities and irrigation areas, and establish institutional frameworks that encourage water to be put to its most valuable uses.

Australia needs to:

- protect river health by recovering environmental water in stressed rivers, and avoid the mistakes of the past in our undamaged rivers;
- promote opportunity by fully specifying water entitlements and responsibilities, and then removing impediments to water trading; and
- engage communities and ensure a fair transition, so no group is asked to bear an unreasonable burden.

Example 2: Product Design for Sustainable Development-Computers

Designing Computers-minimising resources

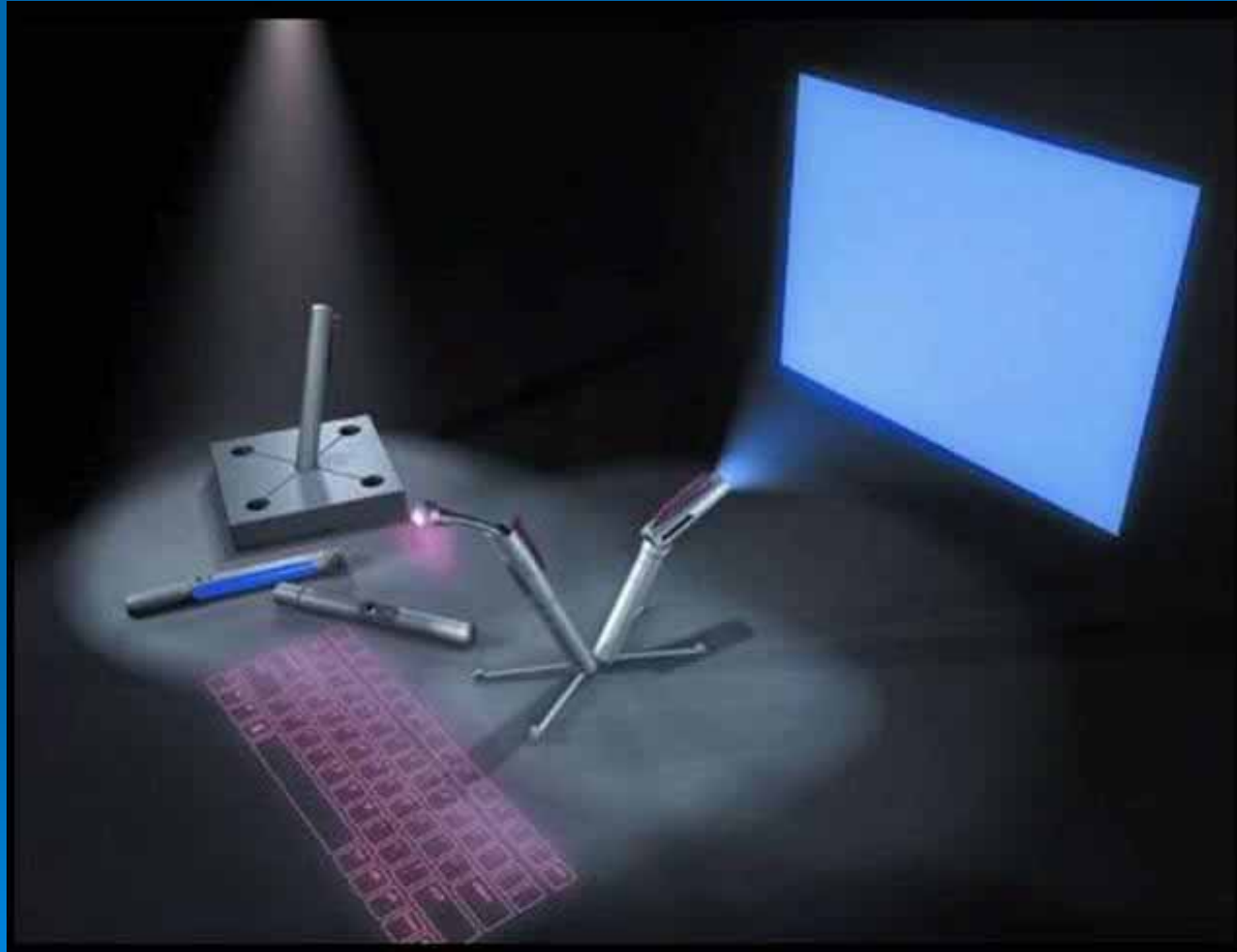
Computers:- represent highest levels of integrated hardware & software engineering

- continuous efforts at miniaturisation
- 1.3 billion computers;
1.8billion internet users

Progress in 60 years of Computer Engineering

- The incredible shrinking computer
"Computers in the future may weigh no more than 1.5 tons"
Popular Mechanics, forecasting the relentless march of science, 1949
- At the museum's PDP-1 restoration celebration in May 2006, Alan Kotok said his Mac G4 laptop:
 - was 1/2000 the size of the PDP-1
 - cost 1/100 as much
 - was 10,000 times faster,
 - had 100,000 times the RAM
 - and 500,000 times the storage

Another step forward?



The future in a Pen?

- *Connection between the latest technology and the human, in a form of a pen. P-ISM is a gadget package with five functions: a pen-style cellular phone with a handwriting data input function, virtual keyboard, a very small projector, camera scanner, and personal ID key with cashless pass function.*
- *P-ISMs are connected with one another through short-range wireless technology. The whole set is also connected to the Internet through the cellular phone function.*
- *This personal gadget in a minimalistic pen style enables the ultimate ubiquitous computing.*

Computers: engineering Issues

- Structure and material composition: thousands of components, large variety of materials, world-wide sources of materials
- Component miniaturisation, integration reduced size and amount of material
- Large number of stakeholders involved in life cycle
- Extraction and processing of raw materials, component manufacture, transport of components to assembly plant, assembly, transport of computers to distribution networks, use and end of life

Computers: engineering for sustainability

Phases: materials, manufacture, distribution, use

- Energy consumption during usage
- Transportation for distribution
- Printed circuit boards, IC, LCD displays are the components with highest environmental impacts
- Collection and management of computers and accessories at end of life-critical to problem of material and substance dispersal to environment, recovery of metals could contribute to positive impact
- Internet network systems and ISP
- Switches, routers, servers, workstations, also contribute to energy consumption

Environmental Impact Accounting

- CO₂ emission- energy usage over its lifetime
Typically a computer may use up energy equivalent to a car being driven over a distance of 500 km-50 liters of petrol. That is just the computer; networks, servers, routers, switches need to be accounted separately.
- Hazardous materials- inventory needed
- To reduce impact, sound management on product “Design for environment”, distribution network and end-of-life practice

Improvement options:

- Design improvements
- Optimisation of life spans
- Reduction of energy consumption
- Chemical usage during component manufacture
- Influencing buying, usage and disposal patterns of consumers
- End-of-life management of disposal
- Reduction of energy consumption of network infrastructures
- Development of suitable environmental assessment method

Standards

- Waste Electrical and electronic equipment (WEEE)
- Restriction on Use of Certain Hazardous Substances in Electrical and Electronic Equipment (RoHS)
- European Commission;s Integrated Product Policy (IPP):
- Reduce environmental impacts of products throughout their life cycle, harnessing market driven approach within which competitive concerns are integrated

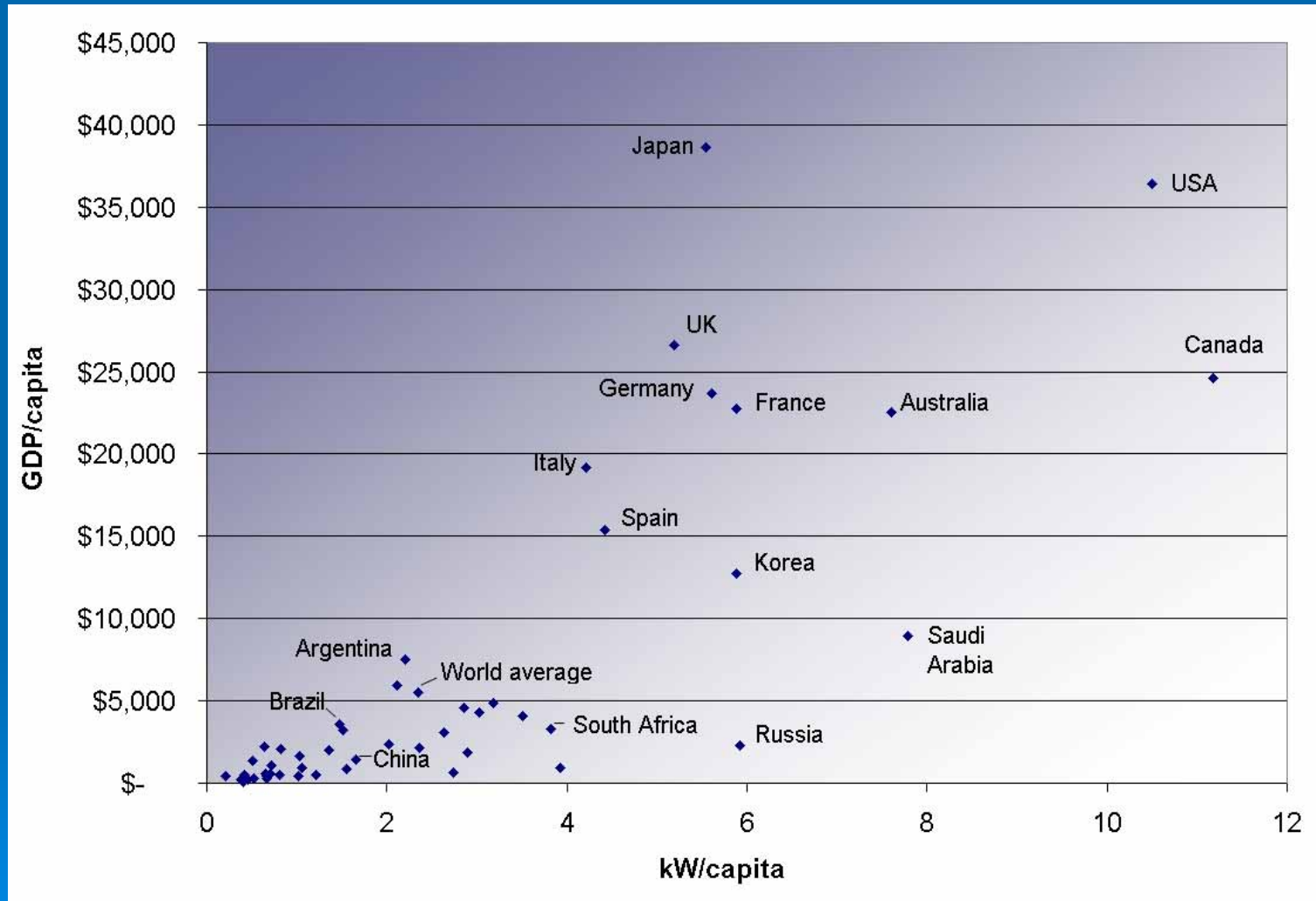
Example 3: Sustainability Development of Energy- Mankind needs a giant step forward

Energy is the most basic of resources

- With energy food can be grown, manufactured, processed
- With energy, materials can be mined, manufactured, processed
- With energy, infrastructure can be built to provide transport, communication, health and education
- With energy, Man can explore further afield, to the far corners of the Earth even to extra-terrestrial destinations
- With energy, life can be more comfortable, more in conformity with Man as a civilised creature
- Not surprisingly, there is close correlation between the wealth of a country and its energy consumption per capita.

Ibrahim Dincer (NURER2010): ENERGY is EVERYTHING!!!

Close correlation between economic wealth and energy consumption per capita-2006[2]



Trends in population growth and energy per capita consumption growth

- AD 1500 World population (estimated) 450 million
- AD 1900 1.6 billion [1]
- End 2009 6.8 billion.
- Past 100 years world population grew 4 times, whilst energy consumption grew 10 times.
- Thus energy consumption grew faster than population growth;
- in other words energy consumption per head grew more than 2 times.
- This trend of per capita energy consumption growth will continue as the rest of the world attempts to catch up with the best standard of living in the world.

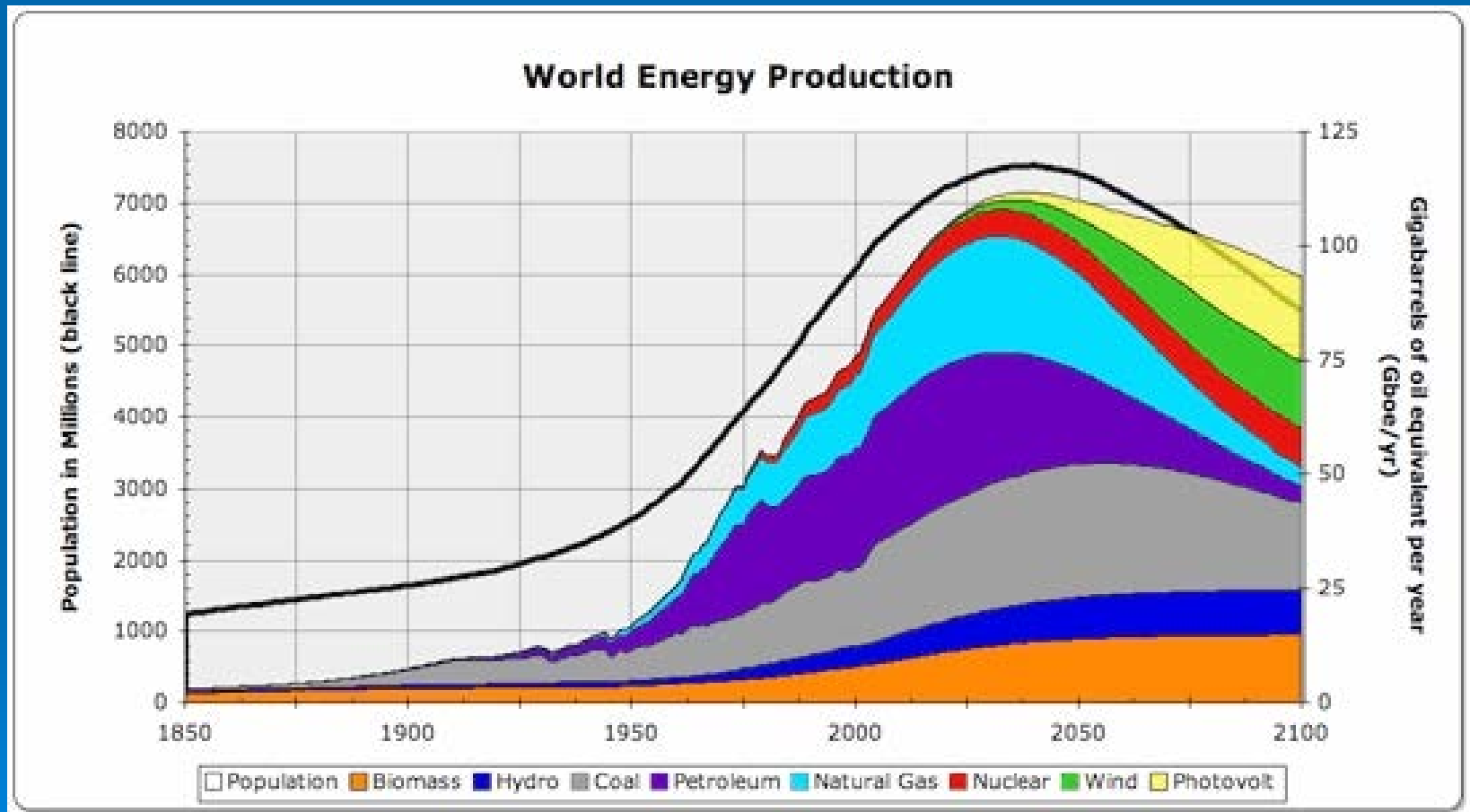
World Population and Energy Consumption

- Past 100 years the doubling time of world population was 50 years
- whilst the doubling time of energy consumption was 30 years.
- **Were this trend to continue**, world population would reach 27 billion in another hundred years
- whilst energy consumption would increase another 10 times.

Unsustainable

- Unsustainable; as the world is already near the critical point when supply of energy barely meets the demand.
- Energy resources are limited and supply trends are estimated to peak in a few short decades from now.
- This realisation underlies demographers' projections that world population growth must slow down in the near future.

Estimates of world energy production projected into the future in a scenario [3]



Unsustainability debate-missing the main point

- The current debates on the unsustainability of population growth energy consumption trends and degradation of the environment are important in raising public awareness but
- Does not address the fundamental problem.

Incrementals are not sufficient for sustainability; What is needed is one giant step

- Renewables are too diffused; low energy collection density, poor net energy production characteristics; projection of renewables do not show the ability to sustain historical rates of human population growth combined with growth of per capita consumption
- To safeguard Mankind's unimpeded progress incremental moves are not enough
- We need one giant bold step –
- Development of a new limitless source of energy
- Clean non-polluting energy which will not further aggravate the environment.

Once-in-History Development

- This is not a pipe dream
- 5 decades of research has gone into the processes of controlling nuclear fusion on earth
- The stage is set for this energy revolution- this once in history development

STARS

Nature's Plasma Fusion Reactors



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S Lee & S H Saw



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IPFS Institute For Plasma Focus Studies
• Knowledge Should Be Freely Accessible To All •

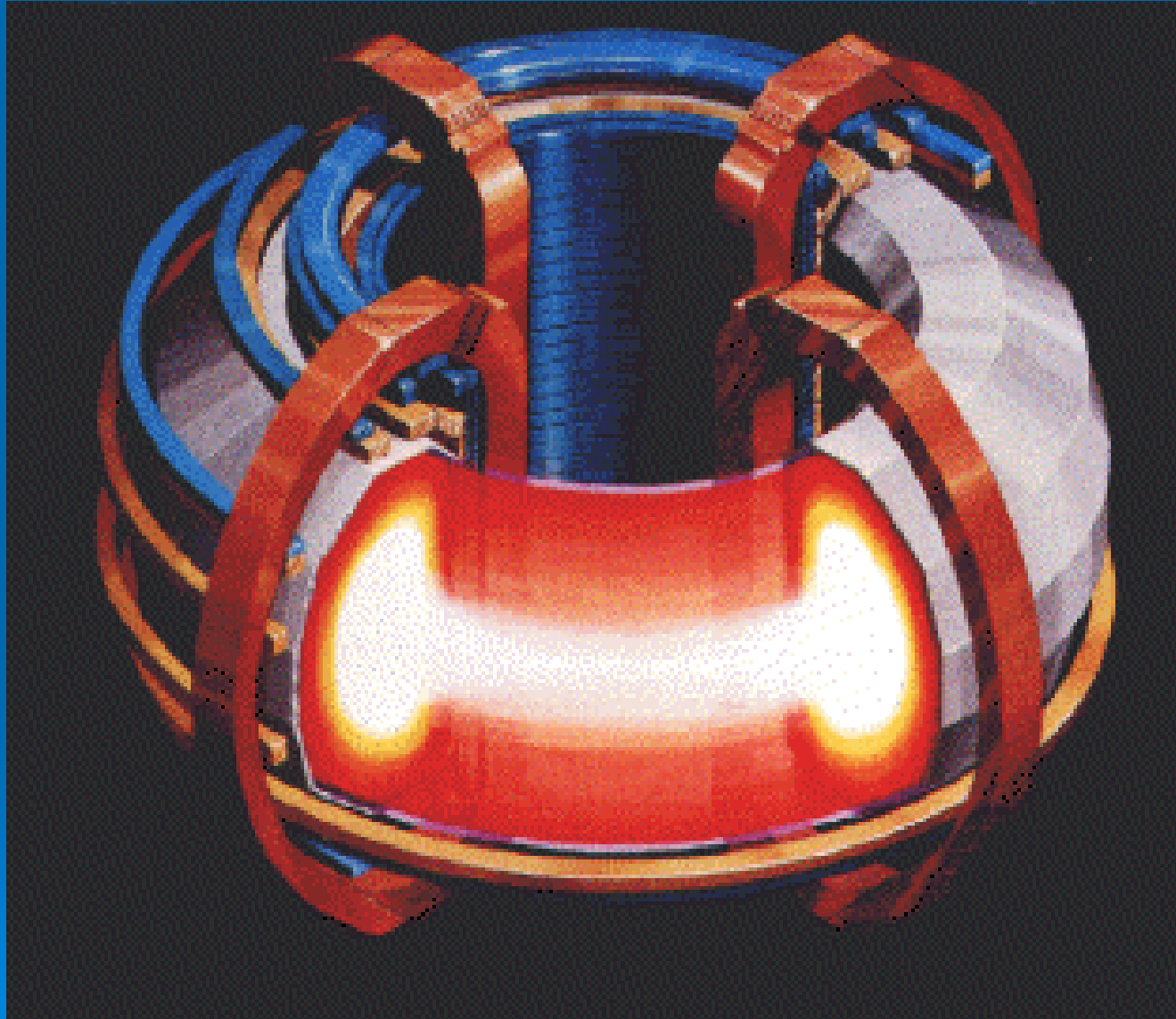
Stars- Nature's Nuclear Fusion Reactors

**And a dream lies on the river
And a glamour veils the night
Whilst above the white stars quiver
With Fusion Burning-bright**

(adapted from Ciribiribin)



Tokamak-planned nuclear fusion reactor



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• Knowledge Should Be Freely Accessible To All •

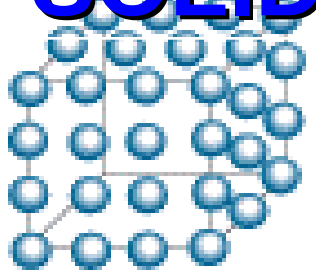
Introductory: What is a Plasma?

Matter heated to high temperatures becomes a Plasma



Four States of Matter

SOLID



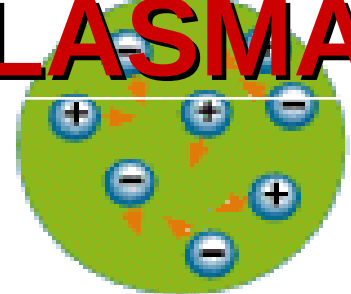
LIQUID



GAS



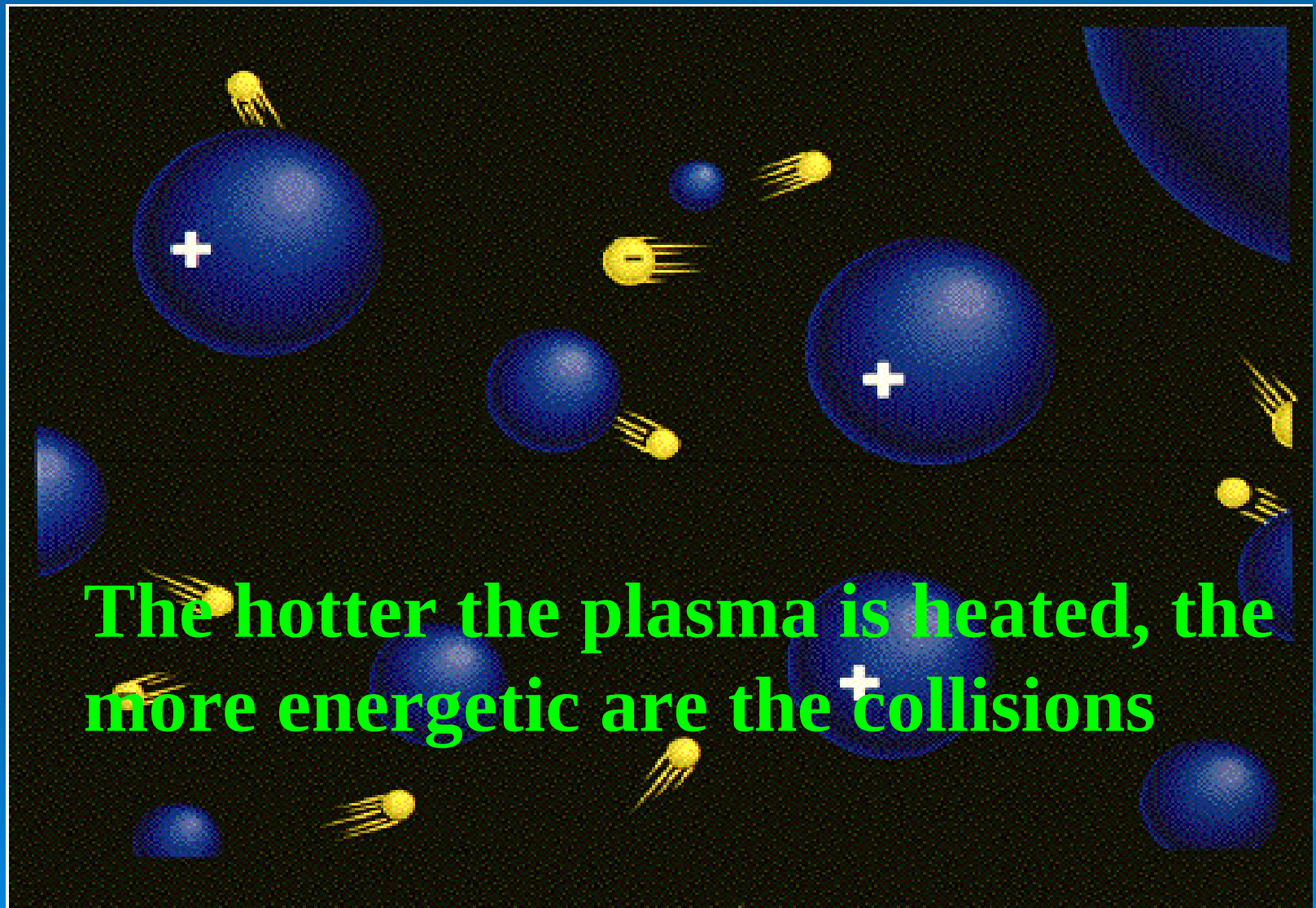
PLASMA



Characteristics of Plasma State

- Electrons and ions
- Electrically conducting
- Interaction with electric & magnetic fields
- Control of ions & electrons: applications
- High energy densities/selective particle energies
 - Hot plasmas: keV's ; (1keV~ 10^7 K)
- Most of matter in Universe is in the Plasma State (e.g. the STARS)

Collisions in a Plasma

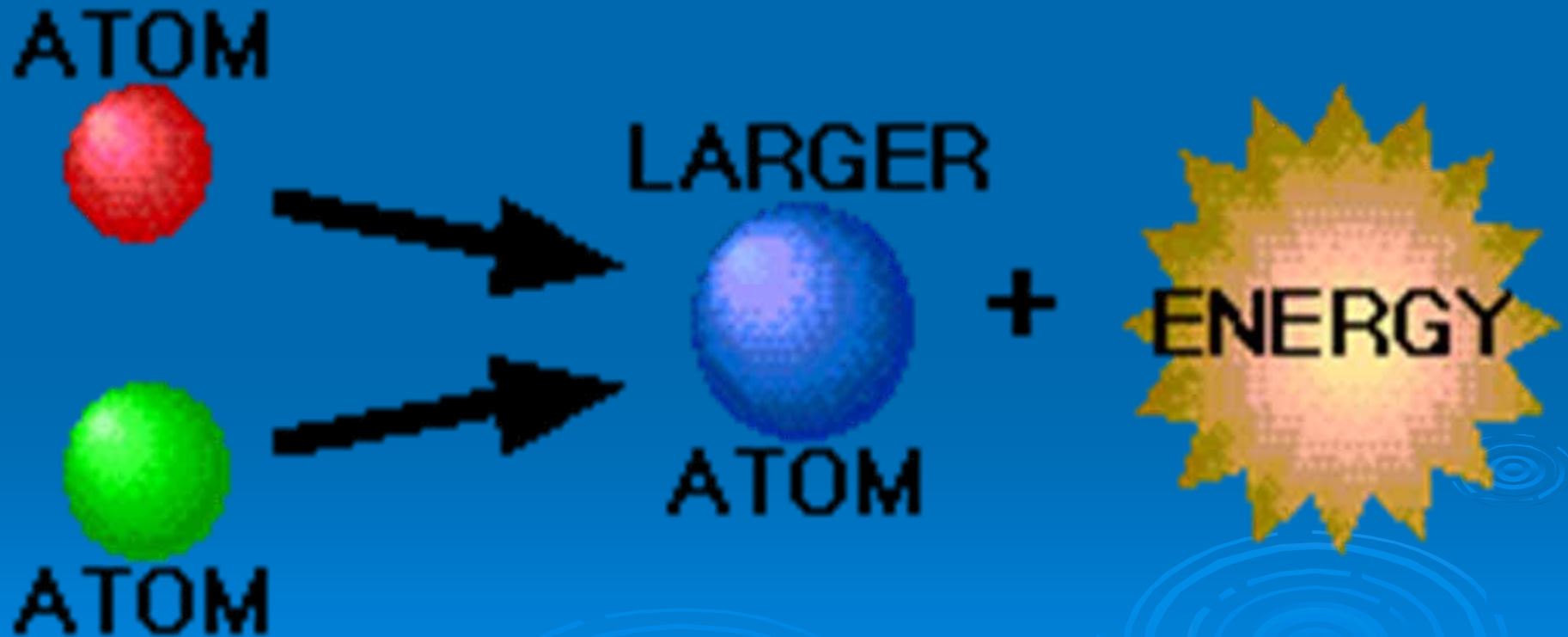


The hotter the plasma is heated, the more energetic are the collisions

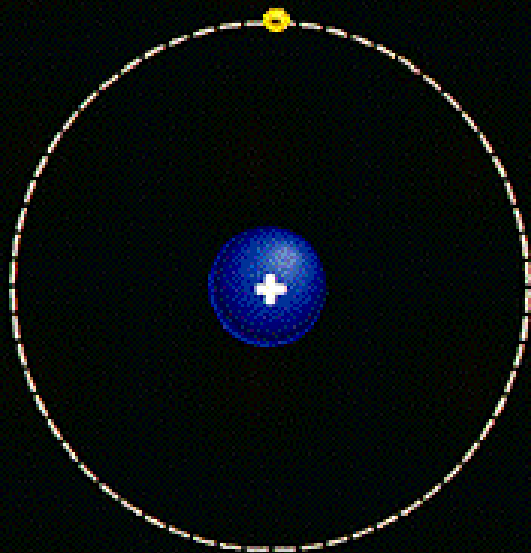


Nuclear Fusion

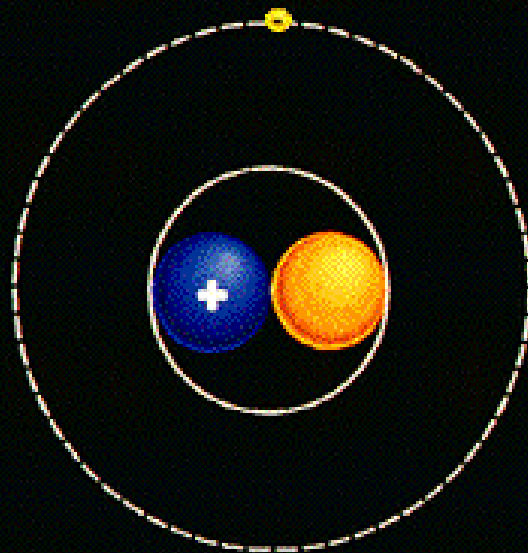
If a Collision is sufficiently energetic, nuclear fusion will occur



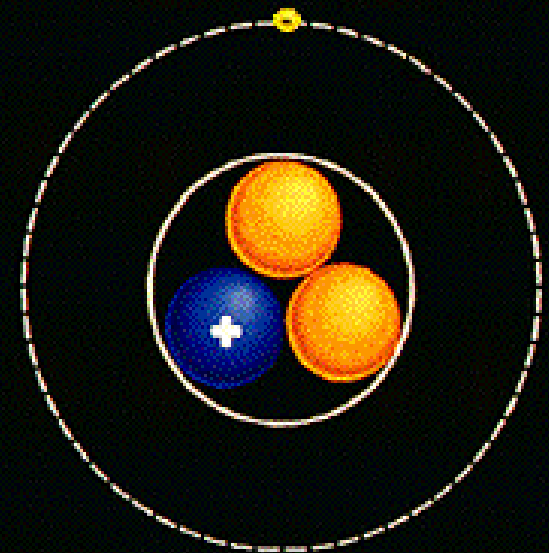
Isotopes of hydrogen- Fuel of Fusion



Hydrogen
 ${}_1\text{H}^1$



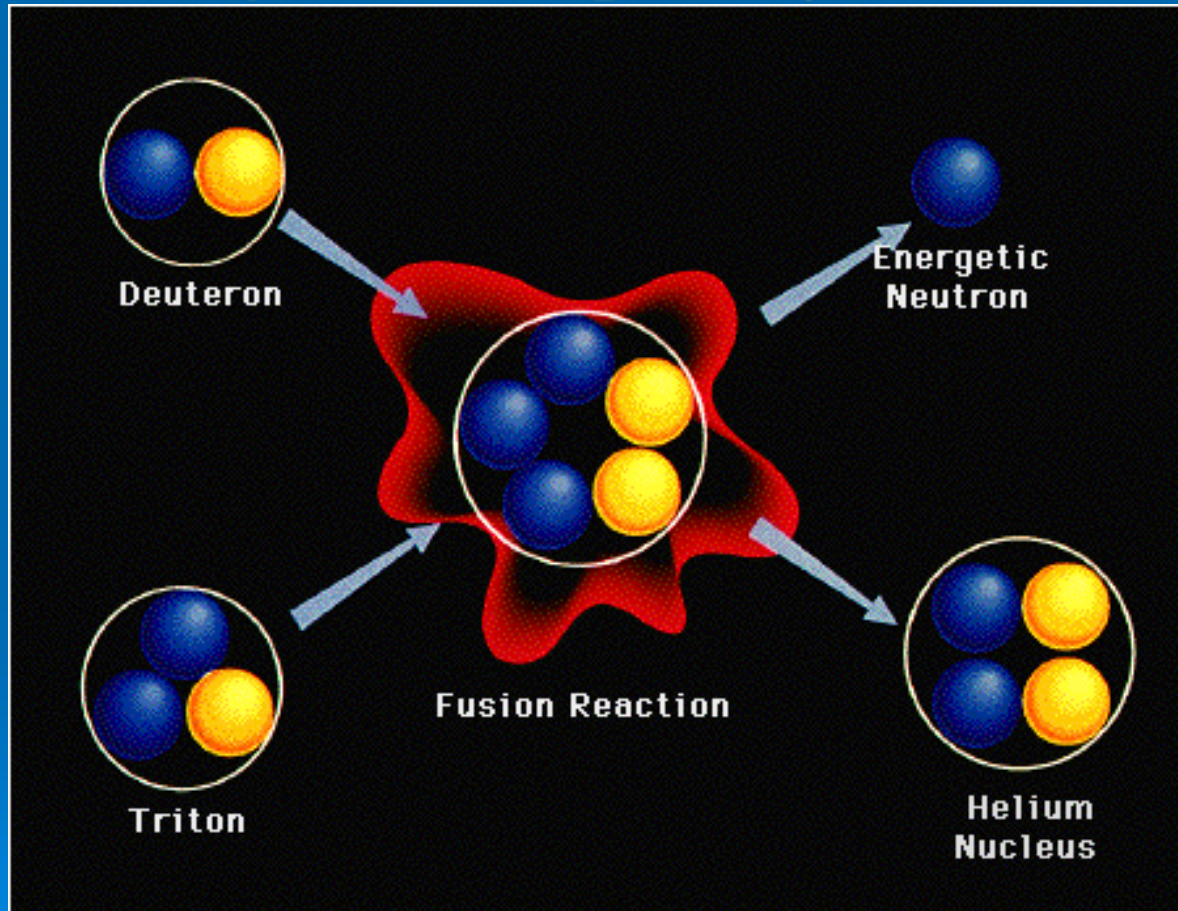
Deuterium
 ${}_1\text{H}^2$



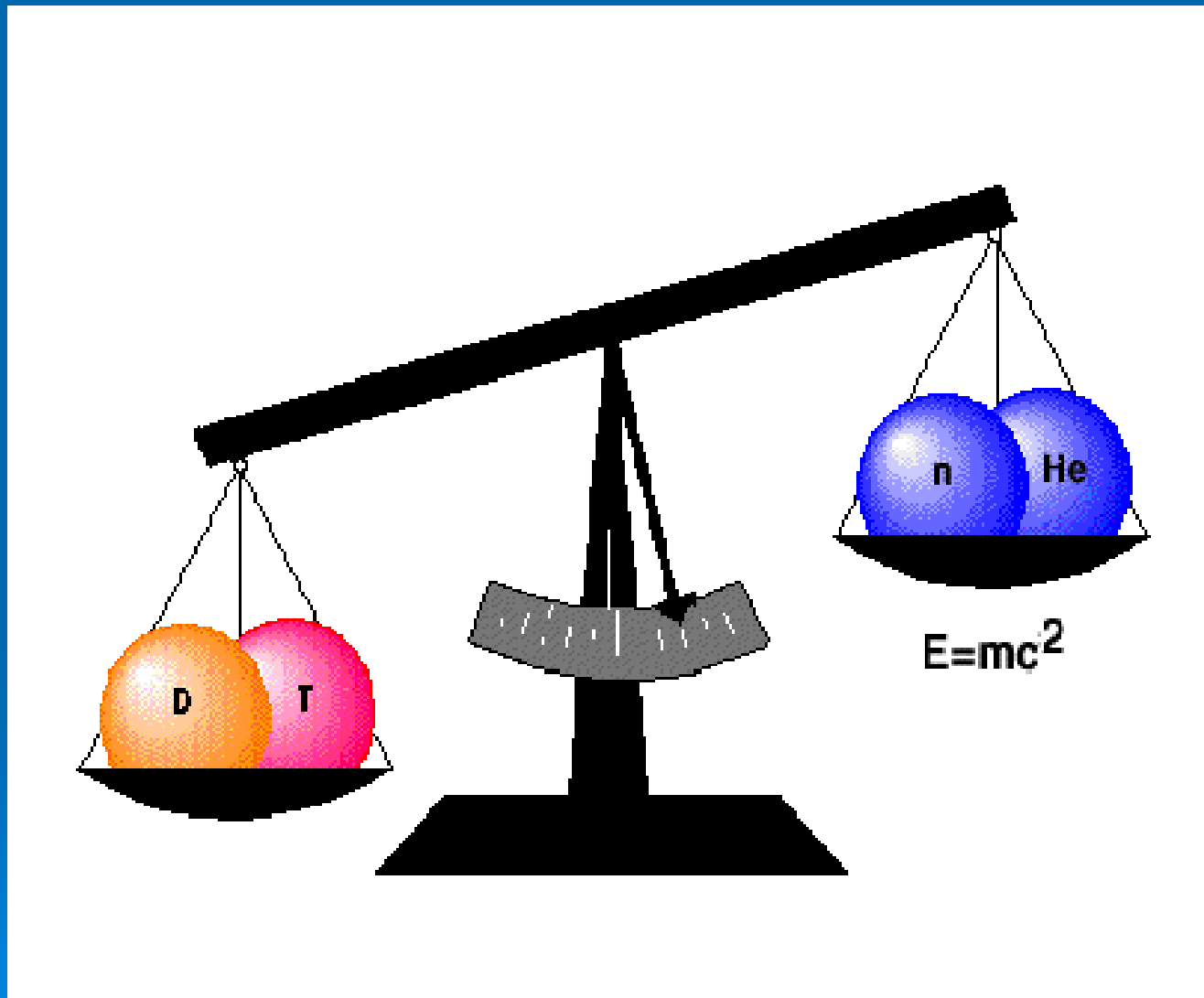
Tritium
 ${}_1\text{H}^3$



Release of energy in Fusion

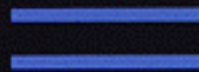


Conversion of mass into Energy

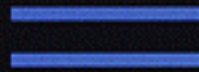


Fusion Energy Equivalent

50 cups water



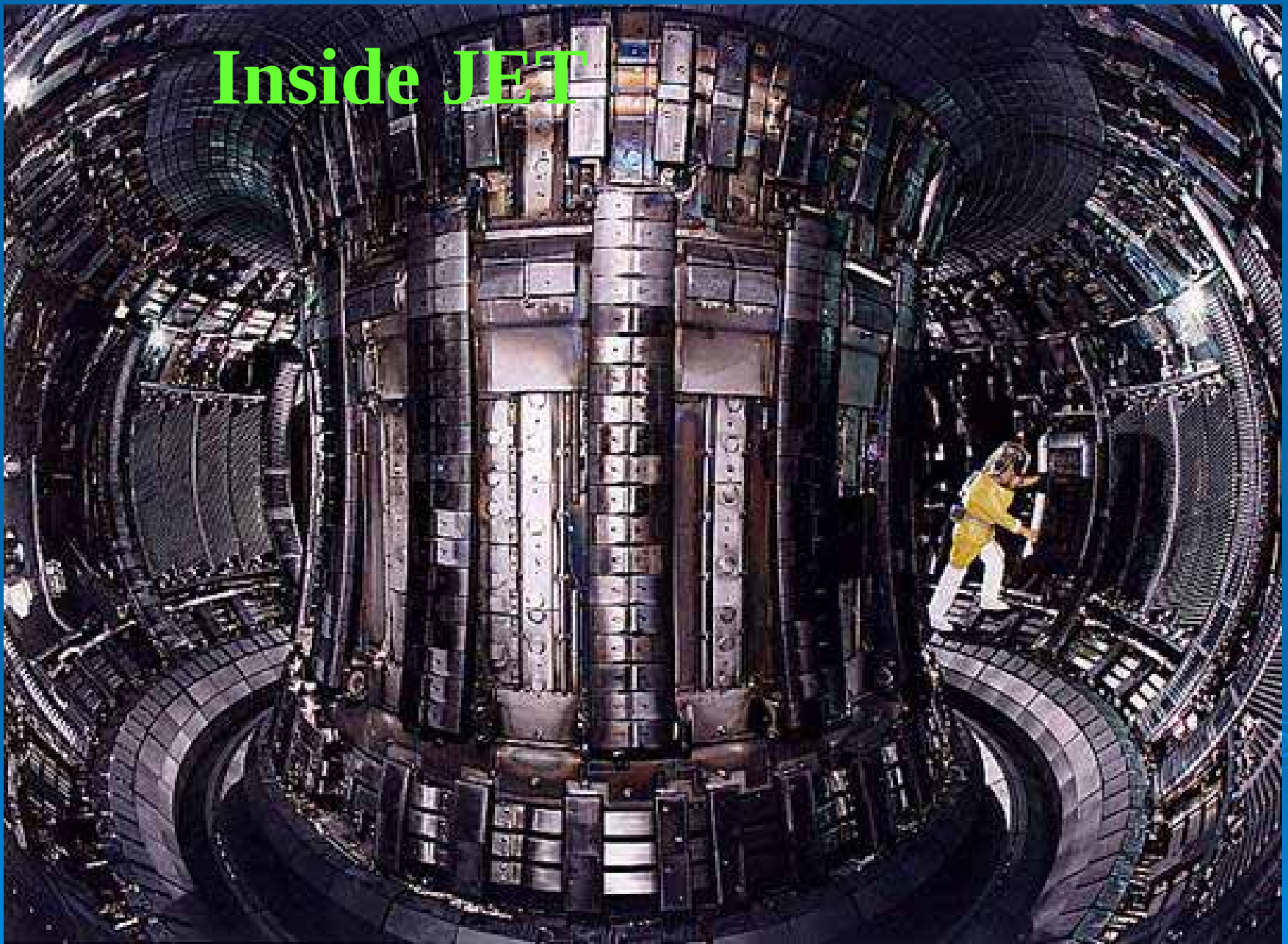
1 thimble heavy water,
extracted from 50 cups of
water



Natural Fusion Reactors vs Fusion Experiments on Earth



Inside JET



ASST Workshop on Engineering Education
Present Status, Trends and Challenges 19-21 July 2010 , Damascus, Syria
Future Role of Engineering Education in Promoting Sustainability
S Lee & S H Saw



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• Knowledge Should Be Freely Accessible To All •

Summary of Conditions

Technological Targets:

- $T > 100 \text{ million K (10keV)}$
- $nt > 10^{21} \text{ m}^{-3}\text{-sec}$

$n = 10^{20} \text{ m}^{-3}, \text{ confined } t = 10\text{s}$

Energy confinement time t scales as some functions of:

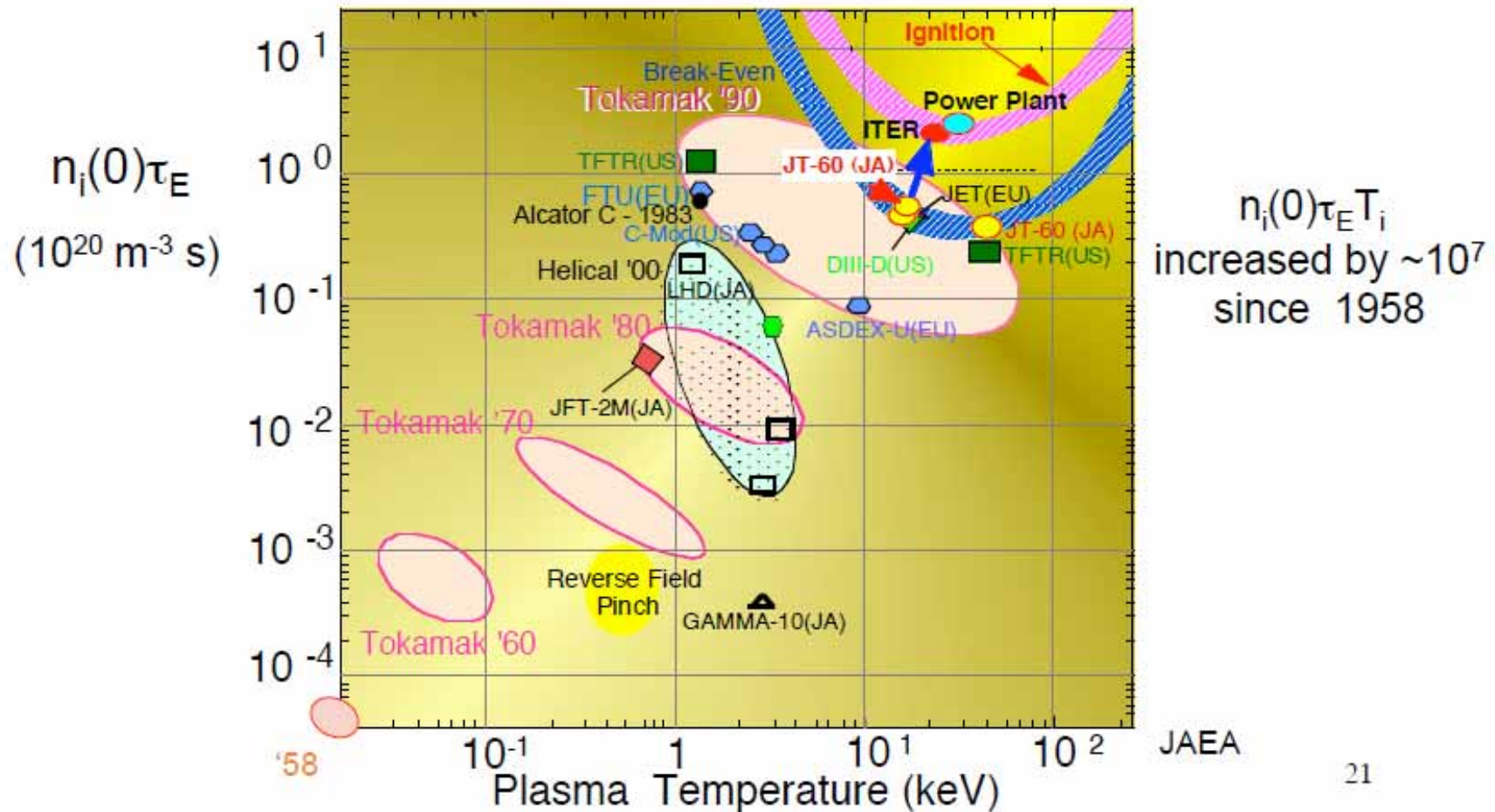
- Plasma current I
- Major Radius R
- Minor radius ' a '
- Toroidal Magnetic Field B

To achieve sufficient value of ntT requires:
scaling of present generation of Tokamaks
upwards in terms of:

I , R , ' a ' and B

Fusion Temperature attained

Fusion confinement one step away



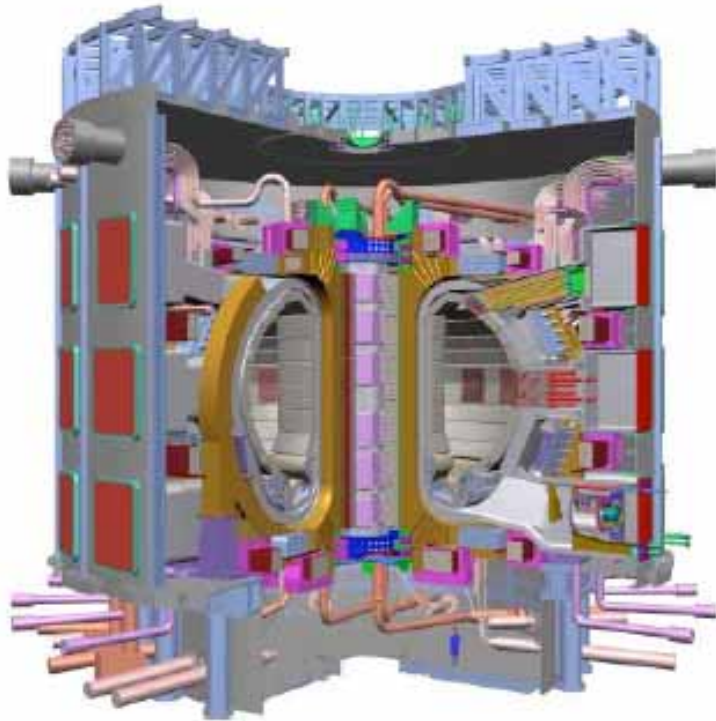
21



International Collaboration to develop Nuclear Fusion Energy-ITER

- 1985- Geneva Superpower Summit:
- Reagan (US) & Gorbachev (Soviet Union) agreed on project to develop new cleaner, sustainable source of energy- Fusion energy
- ITER project was born
- Initial signatories: former Soviet Union, USA, European Union (via EURATOM) & Japan
- Joined by P R China & R Korea in 2003 & India 2005
- ITER Agreement- signed in November 2006

ITER Construction has now started in Cadarache, France



Reactor scale

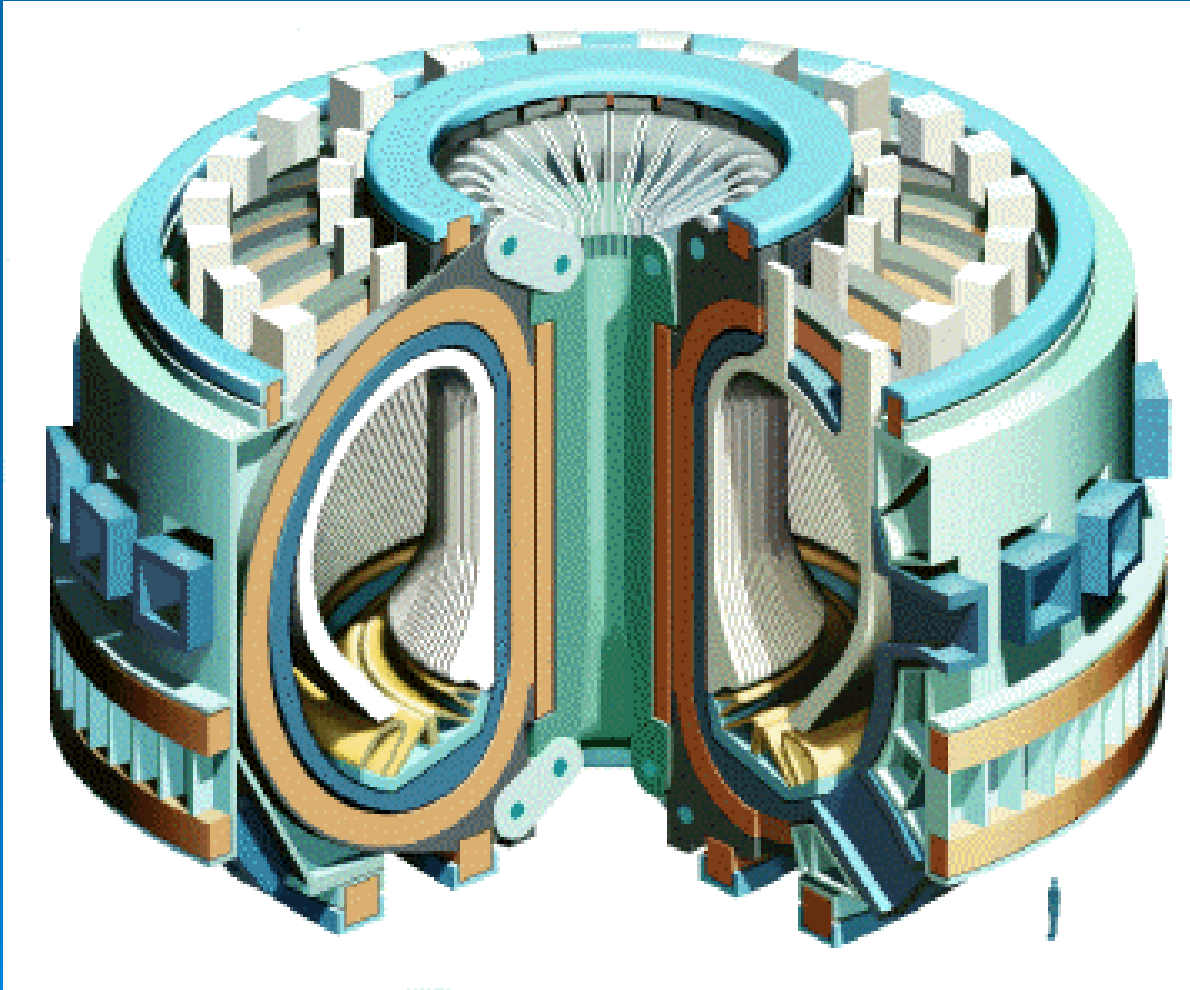


ITER Site Under Construction

First plasma planned 2018
First D-T planned 2022

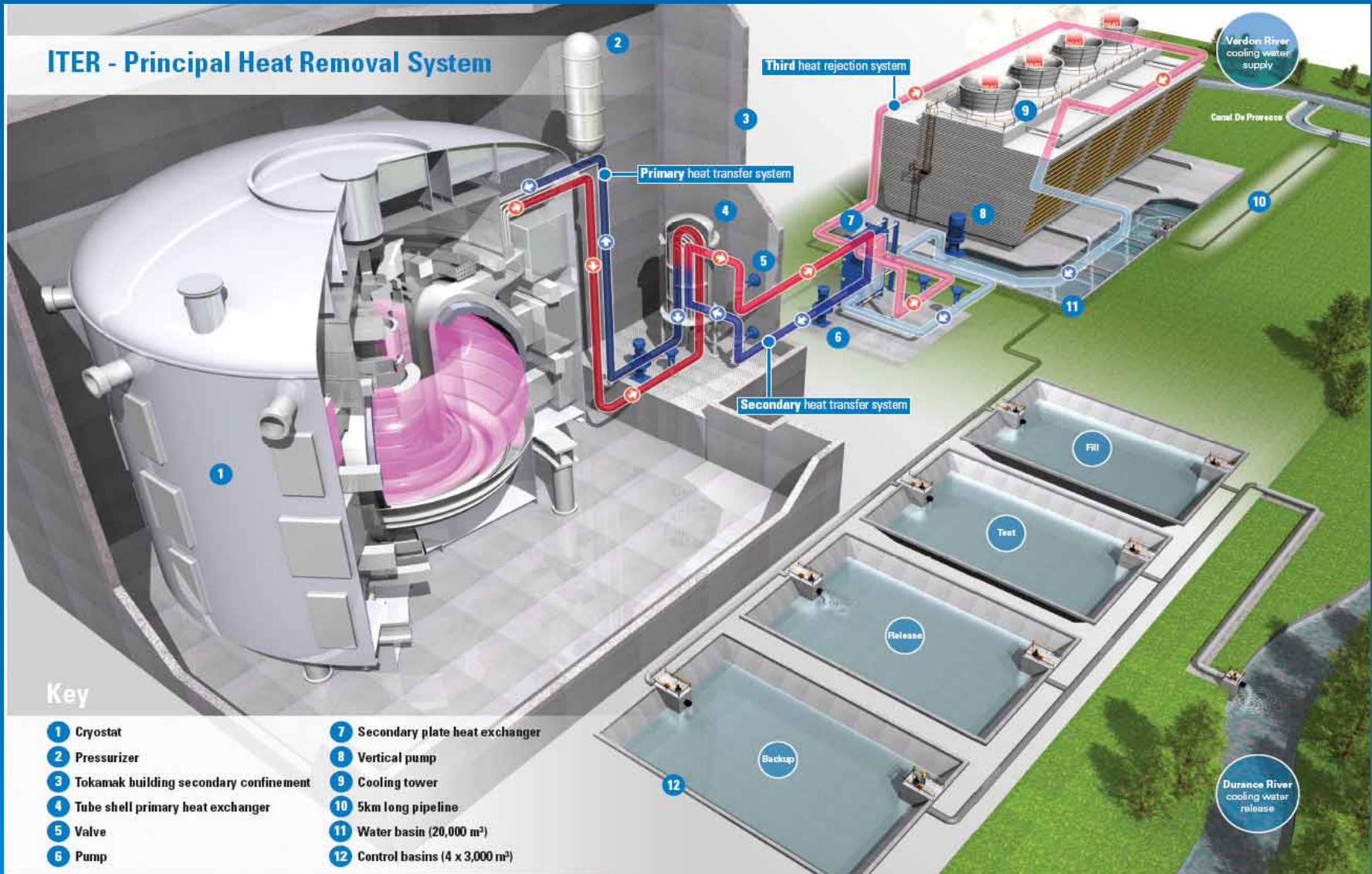


ITER (International Thermonuclear Experimental Reactor)



Cooling and heat Transfer

ITER - Principal Heat Removal System



$Q > 10$ and Beyond

ITER : to demonstrate: possible to produce commercial energy from fusion.

Q = ratio of fusion power to input power.

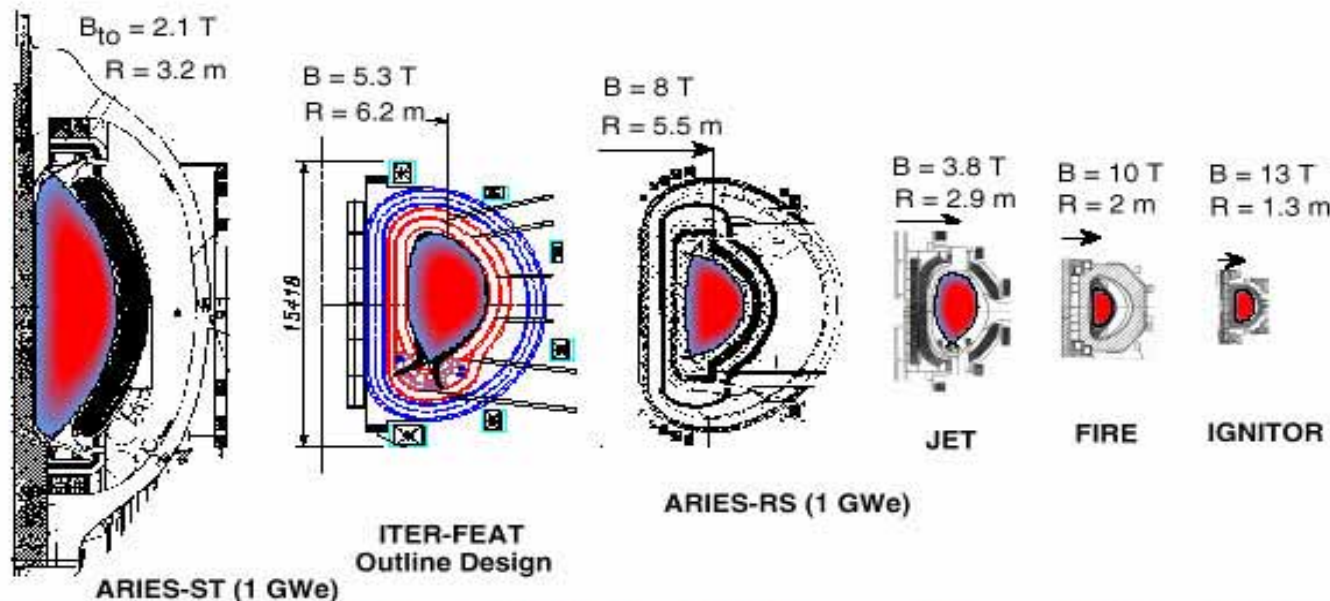
$Q \geq 10$ represents the scientific goal of ITER : to deliver 10x the power it consumes.

From 50 MW input power to 500 MW of fusion power - first fusion experiment to produce net energy.

Beyond ITER will be DEMO (early 2030's), demonstration fusion power plant which will put fusion power into the grid as early as 2040

Potential Next Step Fusion Burning Experiments

Potential Next Step Burning Plasma Experiments and Demonstrations in MFE



Cost Drivers	ARIES-ST	ITER-FEAT	ARIES-RS	JET	FIRE	IGNITOR
Plasma Volume (m^3)	810	837	350	95	18	11
Plasma Surface (m^2)	580	678	440	150	60	36
Plasma Current (MA)	28	15	11	4	6.5	12
Magnet Energy (GJ)	29	50	85	2	5	5
Fusion Power (MW)	3000	500	2200	16	200	100
Burn Time (s), inductive	steady	300	steady*	1	20	5

* assumes non-inductive current drive



Engineering skills required

- Energy economics
- Vacuum Engineering
- Electrical Power engineering
- Large structure engineering
- Cryogenics engineering
- High temperature radiation-resistant materials
- Heat flows and heat exchangers
- Special instrumentations and control systems
- Neutronics
- Safety issues-large structures, radiation, commissioning and eventual de-commissioning

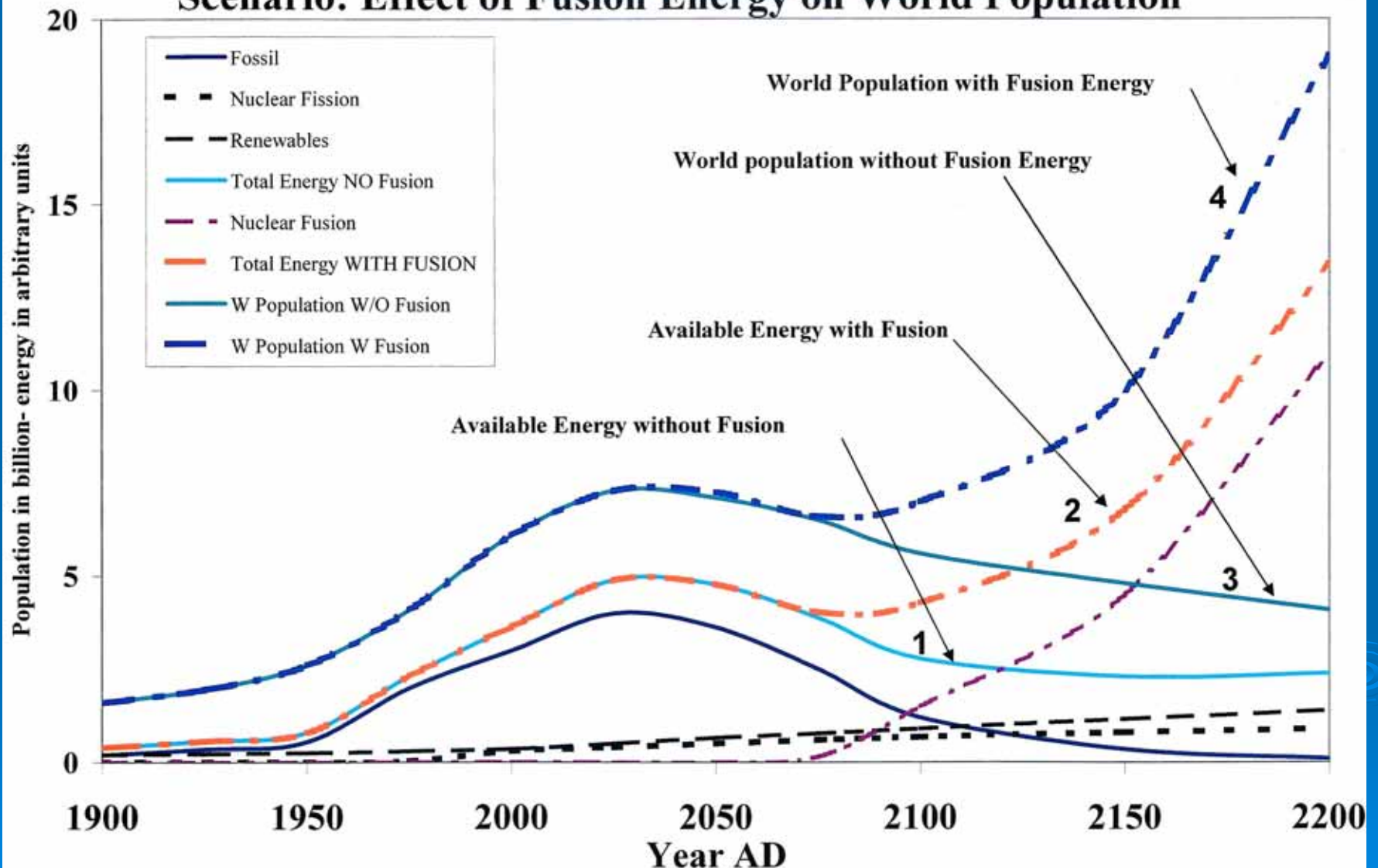
Unlimited clean energy supply

- no need to restrict the growth of energy consumption, a better standard of living or the growth of population.
- ❑ Need not imply unbridled wasteful consumption.
- ❑ The best practice of environmental conservatism could be incorporated into growth, so that
- ❑ efficient and 'green' habits become part of the sustained culture of the human race.

Sustainable Development with Advent of Fusion Energy

- Development of nuclear fusion energy is coming not a moment too soon.
- The critical point when total available energy starts to decline is seen to be reached just before the middle of this century;
- thereafter the consumption curve has to drop and
- Man will have to cope with a decreasing supply
- unless the increasing shortfall is made up by nuclear fusion energy

Scenario: Effect of Fusion Energy on World Population



How fast can we expect the take-up of nuclear fusion energy?

- Example: History of the uptake of nuclear fission energy.
- The first commercial fission power station installed at Calder Hall, UK, 1956 with 50 MW.
- By 1960 1 GW was installed.
- By the late 70's the installed nuclear fission power world-wide had reached 100 GW.
- From 1 GW to 100 GW in some 18 years (10-fold increase every 9 years)

Growth of nuclear fission industry as example

- 70's and 80's, pressure-group litigation
- rising costs of fission reactors due to regulatory changes
- and falling fossil fuel prices made fission plants less attractive
- The industry slowed down so that by the late 80's installed fission power had only grown to 300 GW
- and by 2005 only to 370 GW, supplying some 15% of the world's electricity consumption and 2.1% of total power consumption.

Following the same rate of growth as achieved by the nuclear fission industry

- Scenario assumes an installed 1 GW of commercial nuclear fusion power plant in 2050
- growing 10-fold every 9 years for 27 years to 1000 GW by 2077
- and then slower growth (10-fold in 23 years) to 10000 GW by 2100;
- thereafter tripling every 50 years.
- This is not unreasonable as by 2050 the world would know with a certainty the desperation of its energy position

Fusion Energy taking up the Slack

- Just into the next century fusion energy will take up the slack and allow Mankind to continue its progress and growth (brown dot-dash curve 2, available energy, with fusion energy).
- The scenario shows that there will be several decades when the constraints of energy shortage will be severely felt
- See the flattening of the energy consumption from around 2040 to 2100 Figure 5 (brown dot-dash curve 2, available energy, with fusion energy) .
- Such a period of stagnation in energy and corresponding stagnation in world population (dark blue dot-dash curve 4 showing world population scenario, with fusion energy) seems unavoidable even with the development and rapid adoption of fusion energy.
- Without nuclear fusion energy the scenario is as shown by the light blue curve 1, depicting a severe downturn in the fortunes of Mankind with world population dropping below 5 billion (green curve 3, world population scenario, without fusion energy).

An Unlimited Sustainable Future

- The human spirit,
its will to explore,
to always seek new frontiers,
the next Everest,
deeper ocean floors,
the inner secrets of the atom:
these are iconised into human consciousness
by the deeds of Christopher Columbus, Edmund
Hillary, Jacques Costeau, and Albert Einstein.

Man's Destiny- To Soar

- ❑ In the background of the ever-expanding universe,
- ❑ Mankind's boundless spirit will be curbed by a limitation to growth.
- ❑ That was never meant to be.
- ❑ That should never be so.

Mankind's Giant Step- Dawning of the Fusion age

- Man should have an unlimited destiny
- To reach for the moon, as he already has
- then to colonize it for its resources
- Likewise to reach for the planets
- Ultimately-the stars
- Man's spirit must and will remain indomitable

Fusion Energy and Connection with Engineering Education

- In respect to engineering curricula, “Institutes for higher Learning shall have mechanisms for regularly identifying topics of contemporary importance at local, national and global levels and topics that may not be adequately addressed in the curriculum*.”

* Engineering Programme Accreditation Manual 2007, Malaysian Engineering Accreditation Council

Critical Role of Educationists

It is up to the educationists to work to publicise and bring into social consciousness this once-in history critical development- nuclear fusion energy

Engineering Education will play the biggest role and will be the biggest beneficiary in this effort

Energy Engineering Programs in the USA- Rationale

Energy Engineering is the application of a mix of engineering disciplines , such as mechanical and electrical engineering, to solving the problems of extracting, collecting and utilising energy resources to satisfy human needs without destroying the environment.

- **Energy has been called the Ultimate Resource for two reasons: Firstly, without energy all the other resources on the planet are unobtainable.** Secondly, unlike water or carbon, energy cannot itself be re-cycled. In the industrial countries we have come to depend heavily on large amounts of energy to support our way of life, and we usually expect that it will always be there at affordable prices.
- However two major energy problems are looming. **The first is depletion.** About 90% of the world's energy comes from the fossil fuels (coal, oil and gas) which are finite. It seems likely that oil and gas will be virtually exhausted within one human lifetime from now. Coal could last much longer, perhaps several hundred years, but that brings us to **the second major problem. As a result of burning carbon based fossil fuels we are forcing global climatic changes at an unprecedented rate.** It is possible that we may have to leave most of the remaining fossil fuels in the ground.



Energy Education Courses in the US-Rationale

- The only alternatives to fossil fuels appear to be nuclear power and the renewable energy sources such as solar and wind power. Nuclear already supplies about 10% of world electricity but seems to have slowed down, with increasing concern about costs and environmental impact, in particular the question of how to deal with the growing accumulation of nuclear waste material. Renewables look promising and some are making rapid progress. For example, wind power is now the fastest growing energy technology in the world, with an annual growth rate of about 30%. However they still have a long way to go to match the contribution of the fossil fuels.

Another growing area of employment for energy engineering graduates is in the environmental aspects of energy utilisation. The collection and utilisation of energy can have a huge environmental impact and both industrialists and environmental regulators have a growing need for professionals who understand and can deal with these problems.

- This sets the stage for a new growth area for Energy and Environmental Engineering; for the emerging and rapidly expanding renewable energy industries in America. As mentioned before, wind power is leading the race to expand but some of the others, for example solar electricity from photovoltaics, are not far behind..



Energy Engineering Courses in the US

- 36 universities and colleges in USA offer energy engineering courses mainly M Eng, M S and PhD- mostly in renewables such as PV, wind, hydro, geothermal, solar heating; some nuclear (nuclear is mainly fission),
- University of California at Berkeley-has Masters degree in Nuclear Engineering on Generation 4 fission reactor systems, waste disposals etc and also on the DOE fusion roadmap and inertial confinement fusion systems. (Tokamaks)

Fusion Energy Courses

- Department of Physics at the University of York in the UK has launched a Masters (MSc) degree in fusion energy. The department has also developed a Doctoral Training Network (DTN) in fusion energy, in collaboration with the universities of Durham, Liverpool and Manchester, and the Culham Centre for Fusion Energy (CCFE), the Rutherford Appleton Laboratory (RAL) and AWE Aldermaston. Students on the DTN, part-funded by the UK's Engineering and Physical Sciences Research Council, focus during their first six months on the fundamentals of fusion physics with the Masters students before spending the rest of their three-year course doing research. A total of 13 students are taking the MSc course this year, with a further eight on the DTN.

Engineering Education in Fusion Energy- Interest is increasing

- In 2006 the Faculty of Nuclear Engineering and Physical Sciences at the Czech Technical University in Prague introduced a new nuclear-fusion module into its BSc and MSc programmes
- Bilgi University in Istanbul is planning an Energy Engineering programme to include nuclear-fusion modules
- Kansas State University has just introduced nuclear fusion experiments (small fusion machines) into their Department of Mechanical and Nuclear Engineering. We just ran a 2-week numerical experiments workshop to complement their experiments. We also just completed a 2-part Fusion Energy Workshop at Gazi University for NURER 2010 at Ankara, Turkey.

ENGINEERING

➤ “Application of Science to needs of humanity”

Conclusion

In this paper we covered the following topics:

- Sustainable Development
- Sustainability in water usage- Australia
- Product Design for Sustainable Development-Computers
- Sustainable Development of Energy- Mankind's Giant Step Forward; The Future Role of Engineering Education in this Once-in-History Development

ASST Workshop on Engineering Education

Present Status, Trends and Challenges, Damascus, 19-21 July 2010

Future Role of Engineering Education in Promoting Sustainability

21st July 2010

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³Nanyang Technology University, National Institute of Education, Singapore 637616

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A detailed landscape painting featuring a sunset over a body of water. In the foreground, a winding path leads through a lush garden filled with various flowers, including purple and yellow blooms. A hammock is strung between two large, leafy trees. A white swan is visible in the lower left corner, near the water's edge. The sky is filled with soft, warm colors from the setting sun, and a flock of birds is seen flying in the distance. The overall mood is peaceful and idyllic.

THANK YOU



A Course On Plasma Focus Numerical Experiments S Lee and S H Saw



Plasma Focus Numerical Experiments

Presented by S Lee & S H Saw

At the joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and Plasma Diagnostics
Trieste 15-26 November 2010

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1.4 Using the Lee model as reference for diagnostics	11
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2.3 Firing a shot in NX2	23
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	Supplementary notes	73
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Course materials

I: An e-manual containing: (This document)

- 7 modules of step-by-step instructions;
the first four modules for the basic course and
the next three modules for the advanced course.
- A section containing 3 supplementary notes
- A section with 11 papers about the various machines discussed in
this course and also with reference to various points made in this
manual.

II: An e-folder (named “codes and data” containing the Lee model
code and 8 data files needed for the exercises

10 October 2010

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• Knowledge Should Be Freely Accessible To All •



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The Abdus Salam
International Centre for Theoretical Physics



Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics Plasma Focus Numerical Experiments

Part 1 Basic Course

Module 1: Introduction - The Plasma Focus and the Lee model

Summary

- 1.1 Description of the plasma focus. How it works, dimensions and lifetimes of the hot dense plasma
- 1.2 Scaling properties of the plasma focus
- 1.3 The radiative Lee model: the 5 phases
- 1.4 Using the Lee model as reference for diagnostics
- 1.5 Insights on plasma focus from numerical experiments using Lee model code

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Description of the plasma focus. How it works, dimensions and lifetimes of the hot dense plasmas

1.1.1 Introduction

The Plasma Focus is a compact powerful pulsed source of **multi-radiation** [1]. Even a small table-top sized 3 kJ plasma focus produces an intense burst of radiation with extremely high powers. For example when operated in neon, the x-ray emission power peaks at 10^9 W over a period of nanoseconds. When operated in deuterium the fusion neutron burst produces rates of neutron typically 10^{15} neutrons per second over burst durations of tens of nanoseconds. The emission comes from a point source making these devices among the most powerful laboratory pulsed radiation sources in the world. These sources are plasma-based.

When matter is heated to a high enough temperature, it ionizes and becomes plasma. It emits electromagnetic radiation. The spectrum depends on the temperature and the material. The higher the temperature and the denser the matter, the more intense is the radiation. Beams of electrons and ions may also be emitted. If the material is deuterium, nuclear fusion may take place if the density and temperature are high enough. In that case neutrons are also emitted. Typically the temperatures are above several million K and compressed densities above atmospheric density starting with a gas a hundredth of an atmospheric density.

One way of achieving such highly heated material is by means of an electrical discharge through gases. As the gas is heated, it expands, lowering the density and making it difficult to heat further. Thus it is necessary to compress the gas whilst heating it, in order to achieve sufficiently intense conditions. An electrical discharge between two electrodes produces an azimuthal magnetic field which interacts with the column of current, giving rise to a self-compression force which tends to constrict (or pinch) the column. In order to 'pinch', or hold together, a column of gas at about atmospheric density at a temperature of 1 million K, a rather large pressure has to be exerted by the pinching magnetic field. Thus an electric current of at least hundreds of kA are required even for a column of small radius of say 1 mm. Moreover the dynamic process requires that the current rises very rapidly, typically in under 0.1 μ s in order to have a sufficiently hot and dense pinch. Such a pinch is known as a super-fast super-dense pinch; and requires special MA fast-rise (ns) pulsed-lines. These lines may be powered by capacitor banks, and suffer the disadvantage of conversion losses and high cost due to the cost of the high technology pulse-shaping line, in addition to the capacitor banks.

A superior method of producing the super-dense and super-hot pinch is to use the plasma focus. Not only does this device produce superior densities and temperatures, moreover its method of operation does away with the extra layer of technology required by the expensive and inefficient pulse-shaping line. A simple capacitor discharge is sufficient to power the plasma focus.

1.1.2 The plasma focus

The plasma focus is divided into two sections. The first is a pre-pinch (axial) section. The function of this section is primarily to delay the pinch until the capacitor discharge (rising in a sinusoidal fashion) approaches its maximum current. This is done by driving a current sheet down an axial (acceleration) section until the capacitor current approaches its peak. Then the current sheet is allowed to undergo transition into a radial compression phase. Thus the pinch starts and occurs at the top of the current pulse. This is equivalent to driving the pinch with a super-fast rising current; without necessitating the fast line technology. Moreover the intensity which is achieved is superior to the line driven pinch.

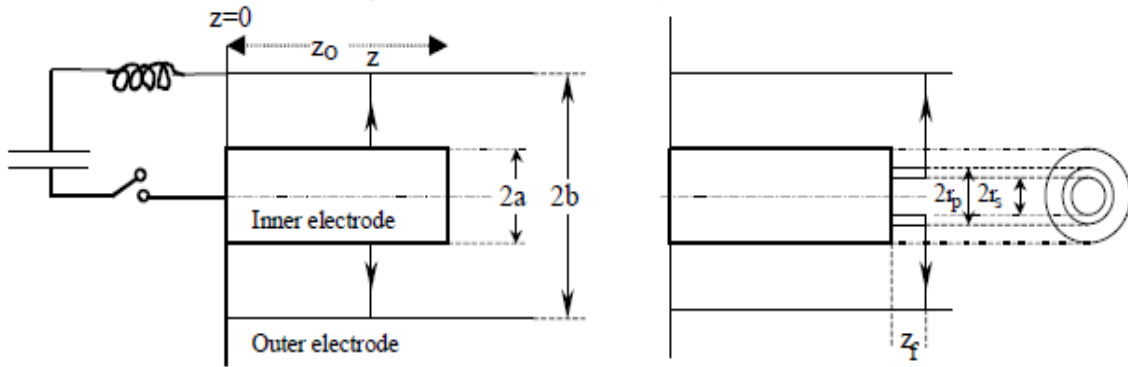


Figure 1. Schematic of the axial and radial phases. The left section depicts the axial phase, the right section the radial phase. In the left section, z is the effective position of the current sheath-shock front structure. In the right section r_s is the position of the inward moving shock front driven by the piston at position r_p . Between r_s and r_p is the radially imploding slug, elongating with a length z_f . The capacitor, static inductance and switch powering the plasma focus is shown for the axial phase schematic only.

The two-phase mechanism of the plasma focus [1] is shown in figure 1. The inner electrode (anode) is separated from the outer concentric cathode by an insulating backwall. The electrodes are enclosed in a chamber, evacuated and typically filled with gas at about 1/100 of atmospheric pressure. When the capacitor voltage is switched onto the focus tube, breakdown occurs axisymmetrically between the anode and cathode across the backwall. The ‘sheet’ of current lifts off the backwall as the magnetic field (B_θ) and it’s inducing current (J_r) rises to a sufficient value.

Axial phase: The $J_r \times B_\theta$ force then pushes the current sheet, accelerating it supersonically down the tube. This is very similar to the mechanism of a linear motor. The speed of the current sheet, the length of the tube and the rise time of the capacitor discharge are matched so that the current sheet reaches the end of the axial section just as the discharge reaches its quarter cycle. This phase typically lasts 1-3 μs for a plasma focus of several kJ .

Radial Phase: The part of the current sheet in sliding contact with the anode then ‘slips’ off the end ‘face’ of the anode forming a cylinder of current, which is then pinched inwards. The wall of the imploding plasma cylinder has two boundaries (see figure 1 radial phase). The inner face of the wall, of radius r_s is an imploding shock front. The outer side of the wall, of radius r_p is the imploding current sheet, or magnetic piston. Between the shock front and the magnetic piston is the annular layer of plasma. Imploding inwards at higher and higher speeds, the shock front coalesces on-axis and a super-dense, super-hot plasma column is pinched onto the axis (see figure 2 [2]). This column stays super-hot and super-

dense for typically ten *ns* for a small focus. The column then breaks up and explodes. For a small plasma focus of several *kJ*, the most intense emission phase lasts for the order of several *ns*. The radiation source is spot-like (1mm diameter) when viewed end-on.

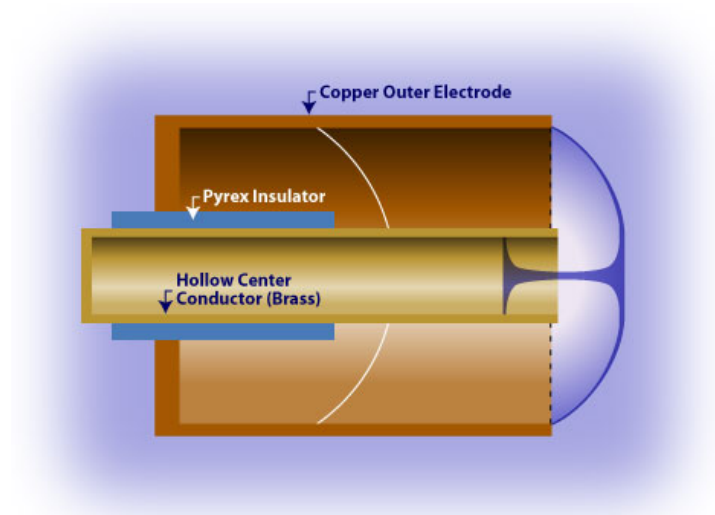


Figure 2. Dense plasma focus device. Image from Glenn Millam. Source: [Focus Fusion Society](#) For an animation of the plasma focus [click here](#).

1.1.3 Radial dynamics of the plasma focus

Figure 3 shows a drawing of a typical plasma focus, powered by a single capacitor, switched by a simple parallel-plate spark-gap. The anode may be a hollow copper tube so that during the radial pinching phase the plasma not only elongates away from the anode face but also extends and elongates into the hollow anode (see figure 2). In figure 3 is shown the section where the current sheet is accelerated axially and also the radial section. Also shown in the same figure are shadowgraphs [3] taken of the actual radially imploding current sheet-shock front structure. The shadowgraphs are taken in a sequence, at different times. The times indicated on the shadowgraphs are relative to the moment judged to be the moment of maximum compression.

That moment is taken as $t=0$. The quality of the plasma compression can be seen to be very good, with excellent axisymmetry, and a very well compressed dense phase. In the lower left of figure 3 are shown the current and voltage signatures of the radial implosion [4], occurring at peak current. The implosion speeds are measured and has a peak value approaching $30 \text{ cm}/\mu\text{s}$.

This agrees with modelling, and by considering shock wave theory together with modelling [5] of subsequent reflected shock wave and compressive effects, a temperature of 6 million *K* (0.5 *keV*) is estimated for the column at peak compression, with a density of 2×10^{19} ions per cm^3 . The values quoted here are for the UNU/ICTP PFF 3 *kJ* device.

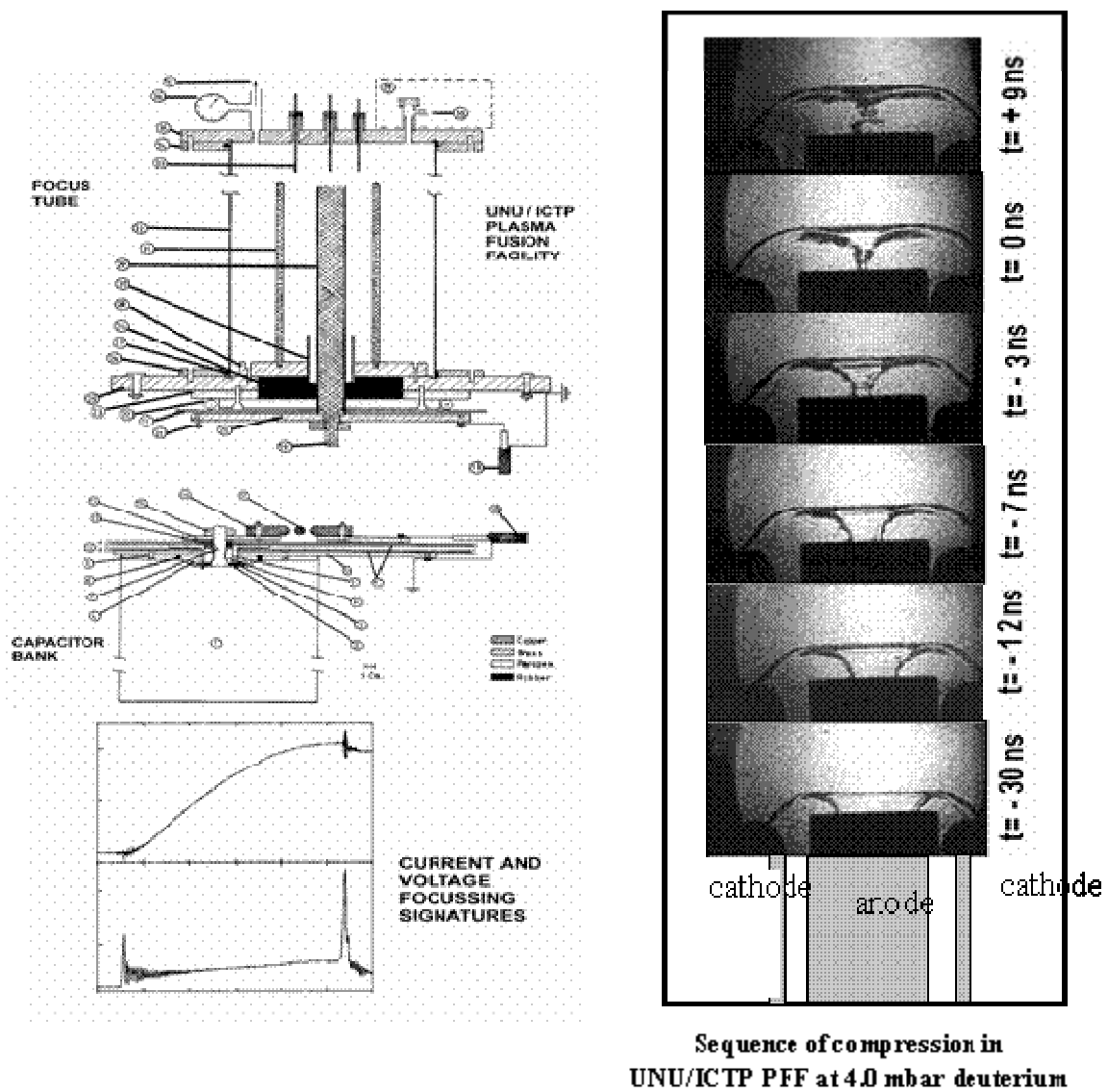


Figure 3. UNU/ICTP PFF - Design, Signatures and Dynamics

1.2 Scaling properties of the plasma focus

1.2.1 Various plasma focus devices

In figure 4a is shown the UNU ICTP PFF 3 kJ device [4-6] mounted on a 1 m by 1 m by 0.5 m trolley, which was wheeled around the International Centre for Theoretical Physics (ICTP) for the 1991 and 1993 Plasma Physics Colleges during the experimental sessions. The single capacitor is seen in the picture mounted on the trolley. In contrast, figure 4b shows the PF1000, the 1 MJ device [7] at the International Centre for Dense Magnetised Plasmas (ICDMP) in Warsaw, Poland. Only the chamber and the cables connecting the plasma focus to the capacitors are shown. The capacitor bank with its 288 capacitors, switches and chargers are located in a separate hall.

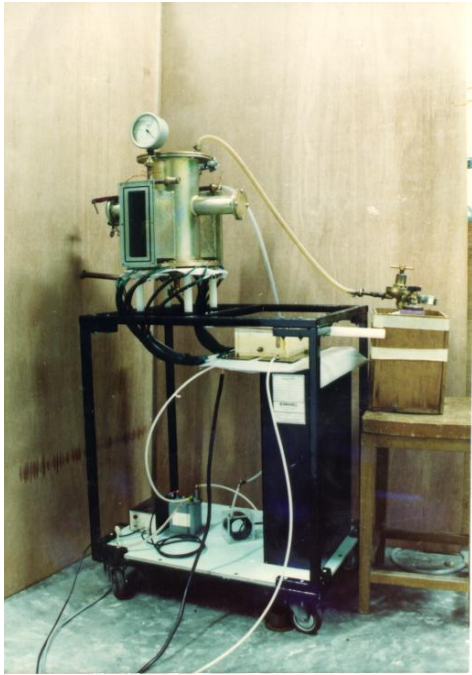


Figure 4a. 3kJ UNU ICTP PFF



Figure 4b. 1 MJ PF1000 plasma focus

We show here the characteristics of several plasma focus devices [7].

Table 1 Characteristics of three plasma focus devices

Plasma Focus Devices	E_0 (kJ)	a (cm)	Z_0 (cm)	V_0 (kV)	P_0 (Torr)	I_{peak} (kA)	v_a (cm/us)	ID (kA/cm)	SF [(kAcm ⁻¹) Torr ^{0.5}]	Y_n (10 ⁻⁸)
PF1000	486	11.6	60	27	4	1850	11	160	85	1100
UNU ICTP PFF	2.7	1.0	15.5	14	3	164	9	173	100	0.20
PF400J	0.4	0.6	1.7	28	7	126	9	210	82	0.01

In table 1 we look at the PF1000 and study its properties at typical operation with device storage at 500 kJ level. We compare this big focus with two small devices at the kJ level.

From table 1 we note:

Voltage and pressure do not have any particular relationship to E_0 .

Peak current I_{peak} increases with E_0 .

Anode radius ' a ' increases with E_0 .

ID (current per cm of anode radius) I_{peak}/a is in a narrow range from 160 to 210 kA/cm

SF (speed or drive factor) $(I_{peak}/a)/P_0^{0.5}$ is 82 to 100 $kAcm^{-1}/Torr^{0.5}$ deuterium gas [8].

Peak axial speed v_a is in the narrow range 9 to 11 cm/us .

Fusion neutron yield Y_n ranges from 10^6 for the smallest device to 10^{11} for the PF1000.

We stress that whereas the ID and SF are practically constant at around 180 kA/cm and (90 $kA/cm)/Torr^{0.5}$ deuterium gas throughout the range of small to big devices, Y_n changes over 5 orders of magnitude.

The data of table 1 is generated from numerical experiments [5,9] and most of the data has been confirmed by actual experimental measurements and observations.

Table 2 Properties of three plasma focus devices

Plasma focus Devices	$c = b/a$	a (cm)	T_{pinch} ($10^6/K$)	v_p (cm/ μs)	r_{min} (cm)	z_{max} (cm)	Pinch Duration (ns)	r_{min}/a	z_{max}/a	Pinch duration/a (ns/cm)
PF1000	1.4	11.6	2	13	2.2	19	165	0.17	1.6	14
UNU ICTP PFF	3.4	1.0	8	26	0.13	1.4	7.3	0.14	1.4	8
PF400J	2.6	0.6	6	23	0.09	0.8	5.2	0.14	1.4	9

Table 2 compares the properties of a range of plasma focus devices. The properties being compared in this table are mainly related to the radial phase.

From table 2 we note:

- The pinch temperature T_{pinch} is strongly correlated to the square of the radial pinch speed v_p .
- The radial pinch speed v_p itself is closely correlated to the value of v_a and $c = b/a$; so that for a constant v_a , v_p is almost proportional to the value of c .
- The dimensions and lifetime of the focus pinch scale as the anode radius ' a '.
 r_{min}/a (almost constant at 0.14-0.17)
 z_{max}/a (almost constant at 1.5)
- Pinch duration has a relatively narrow range of 8-14 ns per cm of anode radius.
- The pinch duration per unit anode radius is correlated to the inverse of T_{pinch} .

T_{pinch} itself is a measure of the energy per unit mass. It is quite remarkable that this energy density at the focus pinch varies so little (factor of 5) over a range of device energy of more than 3 orders of magnitude.

This practically constant pinch energy density (per unit mass) is related to the constancy of the axial speed moderated by the effect of the values of c on the radial speed.

The constancy of r_{min}/a suggests that the devices also produce the same compression of ambient density to maximum pinch density; with the ratio (maximum pinch density)/(ambient density) being proportional to $(a/r_{min})^2$. So for two devices of different sizes starting with the same ambient fill density, the maximum pinch density would be the same.

From the above discussion, we may put down as rule-of-thumb the following scaling relationships, subject to minor variations caused primarily by the variation in c .

i. Axial phase energy density (per unit mass)	constant
ii. Radial phase energy density (per unit mass)	constant
iii. Pinch radius ratio	constant
iv. Pinch length ratio	constant
v. Pinch duration per unit anode radius	constant

1.2.2 Summarising

- i. The dense hot plasma pinch of a small E_0 plasma focus and that of a big E_0 plasma focus have essentially the same energy density, and the same mass density.
- ii. The big E_0 plasma focus has a bigger physical size and a bigger discharge current. The size of the plasma pinch scales proportionately to the current and to the anode radius, as does the duration of the plasma pinch.
- iii. The bigger E_0 , the bigger ‘ a ’, the bigger I_{peak} , the larger the plasma pinch and the longer the duration of the plasma pinch. The larger size and longer duration of the big E_0 plasma pinch are essentially the properties leading to the bigger neutron yield compared to the yield of the small E_0 plasma focus.

The above description of the plasma focus combines data from numerical experiments, consistent with laboratory observations.

The next section describes the Lee model code.

1.3 The radiative Lee model: the 5 phases

The Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling a realistic simulation of all gross focus properties. The basic model, described in 1984 [1], was successfully used to assist several projects [4-6]. Radiation-coupled dynamics was included in the five-phase code, leading to numerical experiments on radiation cooling [5]. The vital role of a finite small disturbance speed discussed by Potter in a Z-pinch situation [10] was incorporated together with real gas thermodynamics and radiation-yield terms. This version of the code assisted other research projects [4,8,11,12] and was web published in 2000 [13] and 2005 [14]. Plasma self-absorption was included in 2007 [13], improving the SXR yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [3,8,11,12], NX2 [12,15-17], and NX1 [15,18] and has been adapted for the Filippov-type plasma focus DENA [19]. A recent development is the inclusion of the neutron yield Y_n using a beam–target mechanism [20-24], incorporated in recent versions [5] of the code (versions later than RADPFV5.13), resulting in realistic Y_n scaling with I_{pinch} [20,21]. The versatility and utility of the model are demonstrated in its clear distinction of I_{pinch} from I_{peak} [25] and the recent uncovering of a plasma focus pinch current limitation effect [22,23], as static inductance is reduced towards

zero. Extensive numerical experiments had been carried out systematically resulting in the uncovering of neutron [20,21] and SXR [26-33] scaling laws over a wider range of energies and currents than attempted before. The numerical experiments also gave insight into the nature and cause of ‘neutron saturation’ [9,30,34]. The description, theory, code, and a broad range of results of this “Universal Plasma Focus Laboratory Facility” are available for download from [5].

A brief description of the 5-phase Lee model is given in the following.

1.3.1 The 5 phases

The five phases (a-e) are summarised [5,13,14,27, 31,33,35] as follows:

a. Axial Phase (see figure 1 left part)

Described by a snowplow model with an equation of motion which is coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c . The mass swept-up factor f_m accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheet-shock front structure, boundary layer effects, and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor f_c accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall, and current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.

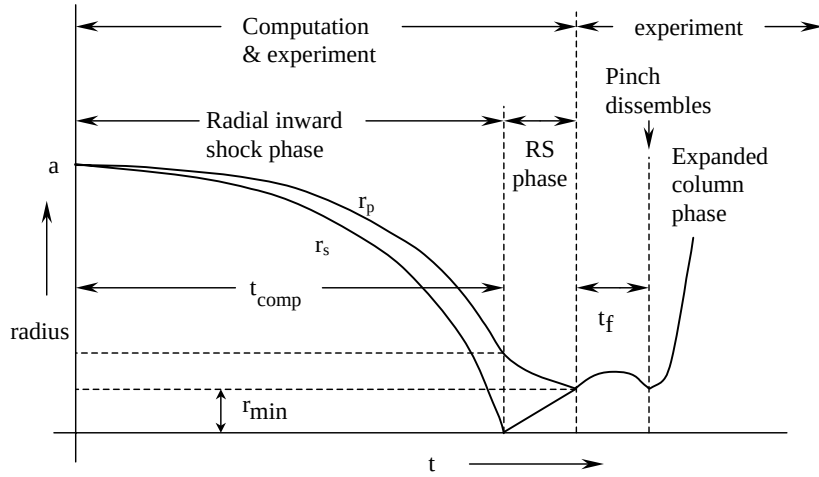


Figure 5. Schematic of radius vs time trajectories to illustrate the radial inward shock phase when r_s moves radially inwards, the reflected shock (RS) phase when the reflected shock moves radially outwards, until it hits the incoming piston r_p leading to the start of the pinch phase (t_f) and finally the expanded column phase.

b. Radial Inward Shock Phase (see figure 1 right part, also figure 2)

Described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, (magnetic piston), allowing the current sheath to separate from the shock front by applying an adiabatic approximation [5,7]. The fourth is the circuit

equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being particularly important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase using shock-jump equations. A communication delay between shock front and current sheath due to the finite small disturbance speed [10,35] is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors f_{mr} and f_{cr} are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor f_{cr} accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

c. Radial Reflected Shock (RS) Phase (See figure 5)

When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops which moves radially outwards, whilst the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the reflected shock moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the reflected shock undergoes a jump by a factor close to 2. Number densities are also computed using the reflected shock jump equations.

d. Slow Compression (Quiescent) or Pinch Phase (See figure 5)

When the out-going reflected shock hits the inward moving piston, the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.

e. Expanded Column Phase

To simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase the snow plow model is used, and two coupled equations are used similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

We note [31] that in radial phases *b*, *c* and *d*, axial acceleration and ejection of mass caused by necking curvatures of the pinching current sheath result in time-dependent strongly center-peaked density distributions. Moreover the transition from phase *d* to phase *e* is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These centre-peaking density effects and localized regions are not modeled in the code, which consequently computes only an average uniform density, and an average uniform temperature which are considerably lower than measured peak density and temperature.

However, because the four model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense, to all the processes which are not even specifically modeled. Hence the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

1.4 Using the Lee model as reference for diagnostics

1.4.1 From measured current waveform to modelling for diagnostics

The Lee model code [5,13,14] is configured [9] to work as any plasma focus by inputting:

Bank parameters, L_0 , C_0 and stray circuit resistance r_0 ;
 Tube parameters b , a and z_0 ;
 Operational parameters V_0 and P_0 and the fill gas.

The computed total current waveform is fitted to the measured waveform by varying model parameters f_m , f_c , f_{mr} and f_{cr} one by one, until the computed waveform agrees with the measured waveform.

First, the axial model factors f_m , f_c are adjusted (fitted) until the features in figure 6: ‘1’ computed rising slope of the total current trace; ‘2’ the rounding off of the peak current as well as ‘3’ the peak current itself are in reasonable (typically very good) fit with the measured total current trace (see figure 6, measured trace fitted with computed trace).

Then we proceed to adjust (fit) the radial phase model factors f_{mr} and f_{cr} until features ‘4’ the computed slope and ‘5’ the depth of the dip agree with the measured values. Note that the fitting of the computed trace with the measured current trace is done up to the end of the radial phase which is typically at the bottom of the current dip. Fitting of the computed and measured current traces beyond this point is not done. If there is significant divergence of the computed with the measured trace beyond the end of the radial phase, this divergence is not considered important.

In this case, after fitting the five features ‘1’ to ‘5’ above, the following fitted model parameters are obtained: $f_m=0.1$, $f_c=0.7$, $f_{mr}=0.12$, $f_{cr}=0.68$.

From experience it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace [27-29].

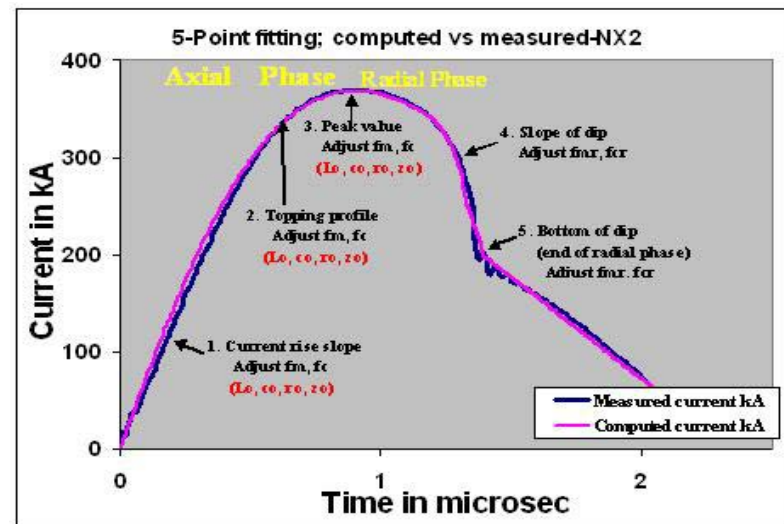


Figure 6. The 5-point fitting of computed current trace to measured (reference) current trace. **Point 1** is the current rise slope. **Point 2** is the topping profile. **Point 3** is the peak value of the current. **Point 4** is the slope of the current dip. **Point 5** is the bottom of the current dip. Fitting is done up to point 5 only. Further agreement or divergence of the computed trace with/from the measured trace is only incidental and not considered to be important.

The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current.

There are many underlying mechanisms in the axial phase such as shock front and current sheet structure, porosity and inclination, boundary layer effects and current shunting and fragmenting which are not simply modeled; likewise in the radial phase mechanisms such as current sheet curvatures and necking leading to axial acceleration and ejection of mass, and plasma/current disruptions. These effects may give rise to localized regions of high density and temperatures. The detailed profile of the discharge current is influenced by these effects and during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed total current trace to the measured total current trace in the procedure adopted by the Lee model code. Once matched, the fitted model parameters assure that the computation proceeds with all physical mechanisms accounted for, at least in the gross energy and mass balance sense.

1.4.2 Diagnostics-time histories of dynamics, energies and plasma properties computed from the measured total current waveform by the code

During every adjustment of each of the model parameters the code goes through the whole cycle of computation. In the last adjustment, when the computed total current trace is judged to be reasonably well fitted in all 5 waveform features, computed time histories are presented, in figure 7a-7o as an example, as follows: for the NX2 operated at 11 kV, 2.6 Torr neon [9,33].

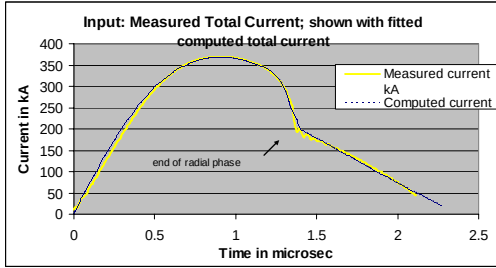


Figure 7a. Fitted computed I_{total}

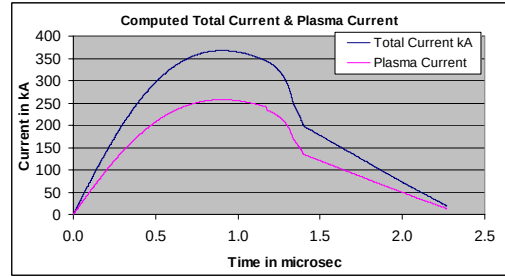


Figure 7b. Computed I_{total} & I_{plasma}

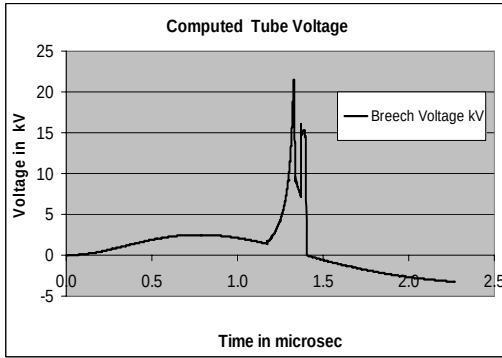


Figure 7c. Tube voltage

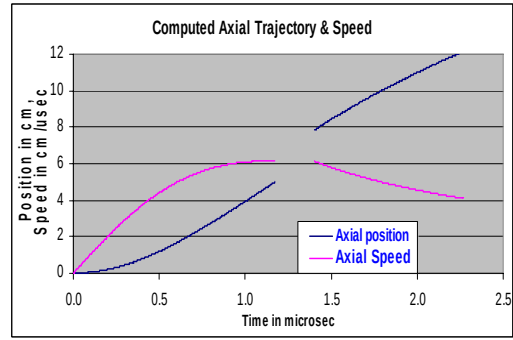


Figure 7d. Axial trajectory and speed

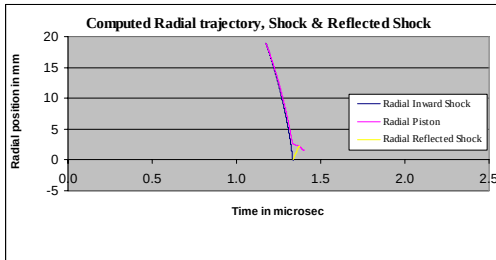


Figure 7e. Radial trajectories

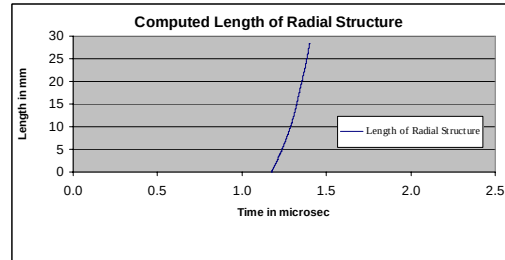


Figure 7f. Length of elongating structure

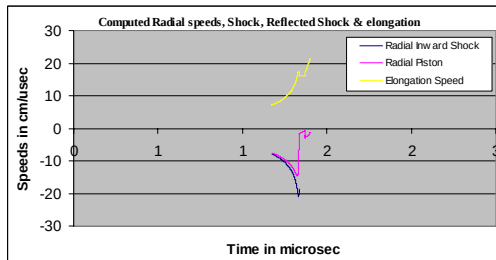


Figure 7g. Speeds in radial phases

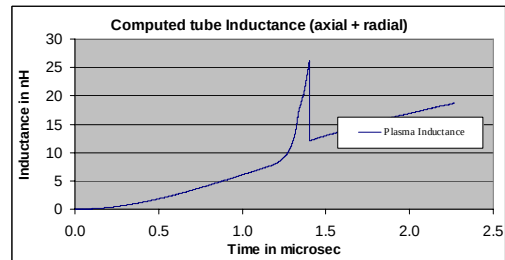


Figure 7h. Tube inductance-axial & radial phases

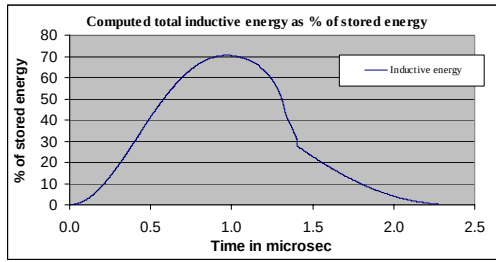


Figure 7i. Total inductive energy

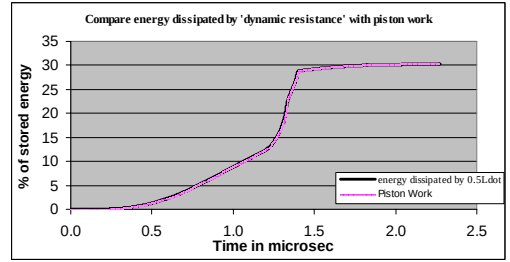


Figure 7j. Piston work and DR energy; both traces overlap

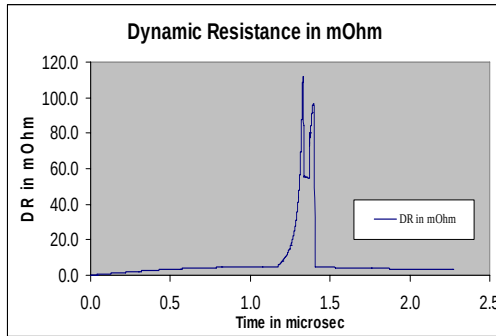


Figure 7k. DR axial and radial phases

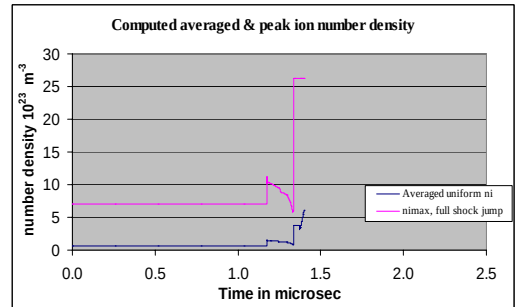


Figure 7l. Peak & averaged uniform n_i

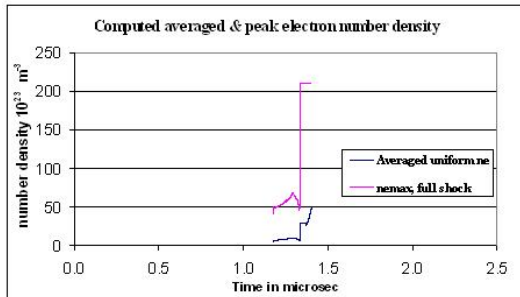


Figure 7m. Peak & averaged uniform n_e

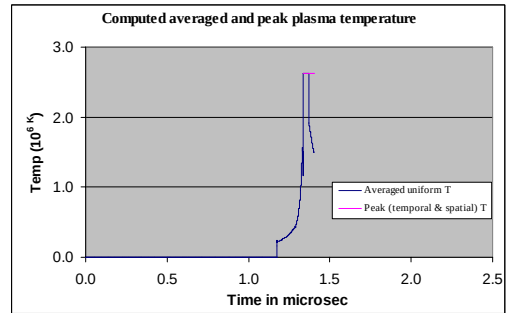


Figure 7n. Peak and averaged uniform T

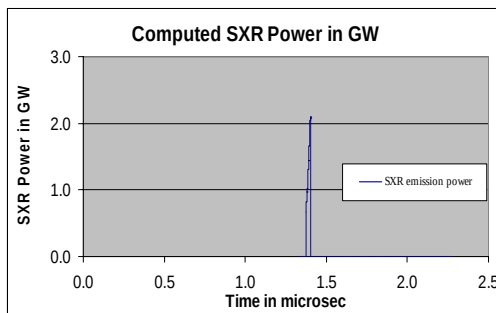


Figure 7o. Neon Soft x-ray power

1.4.3 Comments on computed quantities by Lee model code

- i. The computed total current trace typically agrees very well with the measured because of the fitting. The end of the radial phase is indicated in Figure 7a. Plasma currents are rarely measured. We had done a comparison of the computed plasma current with measured plasma current for the Stuttgart PF78 which shows good agreement of our computed to the measured plasma current [28]. The computed plasma current in this case of the NX2 is shown in figure 7b.
- ii. The computed tube voltage is difficult to compare with measured tube voltages in terms of peak values, typically because of poor response time of voltage dividers. However the computed waveform shape in figure 7c is general as expected.
- iii. The computed axial trajectory and speed, agree with experimental obtained time histories. Moreover, the behaviour with pressure, running the code at different pressures, agrees well with experimental results. The radial trajectories and speeds are difficult to measure. The computed trajectories figure 7e agrees with the scant experimental data available. The length of the radial structure is shown in figure 7f. Computed speeds radial shock front and piston speeds and speed of the elongation of the structure are shown in figure 7g.
- iv. The computed inductance (figure 7h) shows a steady increase of inductance in the axial phase, followed by a sharp increase (rising by more than a factor of 2 in a radial phase time interval about 1/10 the duration of the axial phase for the NX2).
- v. The inductive energy ($0.5LI^2$) peaks at 70% of initial stored energy, and then drops to 30% during the radial phase, as the sharp drop of current more than offsets the effect of sharply increased inductance (figure 7i).
- vi. In figure 7j is shown the work done by the magnetic piston, computed using force integrated over distance method. Also shown is the work dissipated by the dynamic resistance, computed using dynamic resistance power integrated over time. We see that the two quantities and profiles agree exactly. This validates the concept of half $L\dot{I}$ as a dynamic resistance.
- vii. Dynamic resistance, DR (DR will be discussed in Module 2, Section 2.3; Note 2). The piston work deposited in the plasma increases steadily to some 12% at the end of the axial phase and then rises sharply to just below 30% in the radial phase. Dynamic resistance is shown in figure 7k. The values of the DR in the axial phase, together with the bank surge impedance, are the quantities that determine I_{peak} .
- viii. The ion number density has a maximum value derived from shock-jump considerations, and an averaged uniform value derived from overall energy and mass balance considerations. The time profiles of these are shown in the figure 7l. The electron number density (figure 7m) has similar profiles to the ion density profile, but is modified by the effective charge numbers due to ionization stages reached by the ions.
- ix. Plasma temperature too has a maximum value and an averaged uniform value derived in the same manner; are shown in figure 7n. Computed neon soft x-ray power profile is shown in figure 7o. The area of the curve is the soft x-ray yield in J . Pinch dimensions and lifetime may be estimated from figures 7e and 7f.
- x. The model also computes the neutron yield, for operation in deuterium, using a phenomenological beam-target mechanism [25-27]. The model does not compute a time history of the neutron emission, only a yield number Y_n .

Thus as is demonstrated above, the model code when properly fitted is able to realistically model any plasma focus and act as a guide to diagnostics of plasma dynamics, trajectories, energy distribution and gross plasma properties.

1.4.4 Scaling parameters of the plasma focus pinch

The gross dynamics of the plasma focus is discussed in terms of phases. The dynamics of the axial and radial phases is computed using respectively a snowplow and an elongating slug model. A reflected shock phase follows, giving the maximum compression configuration of the plasma focus pinch. An expanded column phase is used to complete the post-focus electric current computation. Parameters of the gross focus pinch obtained from the computation, supplemented by experiments may also be summarised as follows:

Table 3

Plasma Focus Pinch Parameters		Deuterium	Neon (for SXR)
minimum radius	r_{min}	$0.15a$	$0.05a$
max length (hollow anode)	z	$1.5a$	$1.6a$
radial shock transit	t_{comp}	$5 \times 10^{-6}a$	$4 \times 10^{-6}a$
pinch lifetime	t_p	$10^{-6}a$	$10^{-6}a$

where, for the times in s, the value of anode radius, a , is in m . For the neon calculations radiative terms are included; and the stronger compression (smaller radius) is due to thermodynamic effects.

1.5 Insights on plasma focus from numerical experiments using Lee model code

Using the Lee model code, series of experiments have been systematically carried out to look for behaviour patterns of the plasma focus.

Insights uncovered by the series of experiments include:

- i. pinch current limitation effect as static inductance is reduced;
- ii. neutron and SXR scaling laws;
- iii. a global scaling law for neutrons versus storage energy combining experimental and numerical experimental data; and
- iv. the nature and a fundamental cause of neutron saturation.

These significant achievements are accomplished within a period of twenty months of intensive numerical experimentation in 2008 to 2009. The numerical experimental research continues in 2010 with widening international collaboration.

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Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics

Plasma Focus Numerical Experiments

Part 1 Basic Course

Module 2: Universal Plasma Focus Laboratory-The Lee model code

Follow the instructions (written for EXCEL 2007 but easily adapted to EXCEL 2003) in the following notes. You may also wish to refer to the supplementary notes **SP1.doc** (scroll down to pg 77, after Module 7). Instructions are given in some details in order to accommodate participants who may not be familiar with EXCEL. Those who find the instructions unnecessarily detailed may wish to skip the unnecessary lines. The code seems to be unnecessarily cumbersome when used with EXCEL 2007. So EXCEL 2003 is preferred.

Summary

- 2.1 Introduction to the Worksheet
- 2.2 Configuring the Universal Plasma Focus Laboratory (UPFL)
- 2.3 Firing a shot in NX2
- 2.4 Studying the results
- 2.5 Exercise 1: Interpreting and recording data from the Worksheet
- 2.6 Conclusion

The material

You should have **RADPFV5.15de.xls** (contained in the e-folder “Code and Data” accompanying this file) on your Desktop for the next step. Please also ensure you have kept an identical original copy in a RESERVE folder. You are going to work with the desktop copy; and may be altering it. Each time you need an unaltered copy; you may copy from the reserve folder and paste it onto the desktop.

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2.1 Introduction to the Worksheet

2.1.1 Opening the worksheet

(Note: Click means the ordinary click on the left button of the mouse; as distinct from the term Right Click, which means the special click on the right button of the mouse.)

Double click on **RADPFV5.15de.xls**

Work sheet appears and should look like Figure 1 [shown only as an example]; for the following please refer not to Figure 1 but to your worksheet.

Security Warning “Macros have been disabled” appears at top left hand corner of Worksheet with side box “options”.

Click on “options”- select the button “Enable this content”- click OK

After this procedure, the worksheet is macro-enabled and is ready for firing.

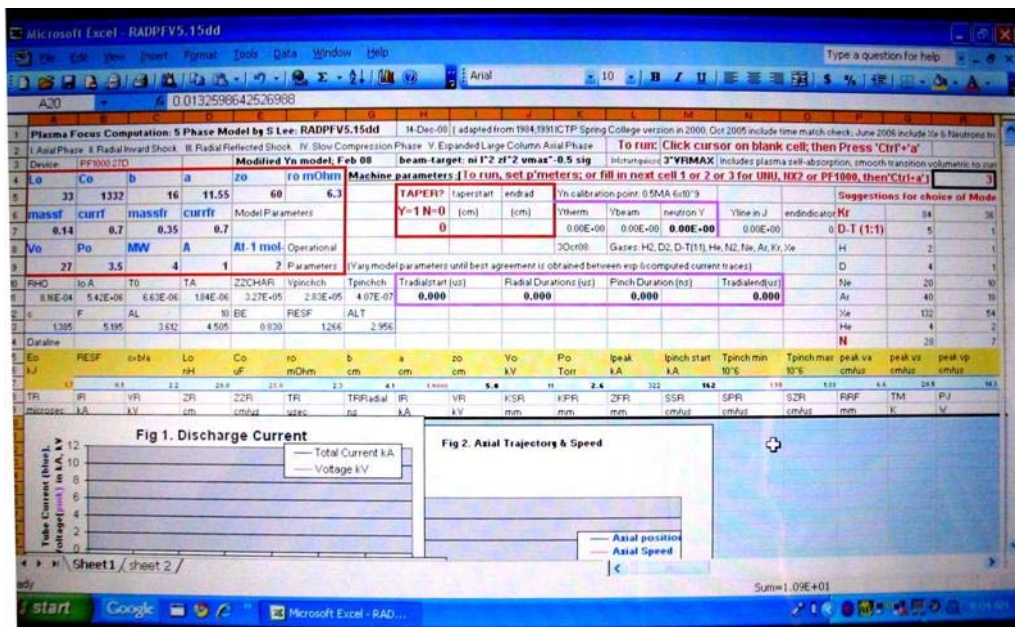


Figure 1. Appearance of worksheet-EXCEL 2003 version; EXCEL 2007 version should not look too different.

2.1.2 Preliminary orientation for setting controls

(For the following instructions, use your Excel Sheet; not the above image)

Device configuration:

(Note: Each Cell of the Excel Worksheet is defined by a Column alphabet A, B, or C..... and a Row number 1, 2 or 3 etc. The Column alphabets are shown along the top border of the worksheet. The Row numbers are shown along the left border of the Worksheet. For example, Cell A4 is located at column A row 4. Another example: A4-F9 refers to the block of cells within the rectangle bordered by row A4-F4, column A4-A9, row A9-F9 and

column F4-F9; the larger orange-red bordered rectangle, containing 6x6 cells, near the top left of figure1.)

Locate Cells A4 to F9. These cells are for setting bank parameters, tube parameters, operating parameters and model parameters.

Taper: Control Cells for anode taper are normally inactivated by typing 0 (number zero) in Cell H7. **Ensure that H7 is filled with 0** (number zero); unless anode taper feature is needed.

One Click Device: This control cell R4 allows choice of a specific plasma focus using numbers; currently 3 machines are available chosen with numbers 1, 2 or 3. **Ensure that R4 is filled with the number 0.** (Otherwise the code will keep defaulting to the selected machine '1' or '2' or '3'.)

2.1.3 Preliminary orientation for computed results

Cells A10-G13: computed characteristic quantities of the configured plasma focus.

Cells K6-M7: computed neutron yield, component & total; if operated in deuterium

Cell N6-N7: computed SXR line radiation

Cells H10-N11: computed durations of axial phase, radial phase and pinch phase and end time of radial phase.

Cells A15-AI17: dataline: contains data on row 17 with corresponding labels (and units) in rows 15 and 16. Data: E_0 , RESF, $c=b/a$, L_0 , C_0 , r_0 , b , a , z_0 , V_0 , P_0 , I_{peak} , $I_{pinchstart}$, $T_{pinchmin}$, $T_{pinchmax}$, peak v_a , peak v_s , peak v_p , a_{min} (which is r_{min}), z_{max} , pinch duration, V_{max} , $n_{ipinchmax}$, Y_n , Q_{sxr} , $Q_{sxr}\%$, f_m , f_c , f_{mr} , f_{cr} , $EINP$, $taxialend$, SF , ID and Q_{line} ; others may be added from time to time.

This is a recently introduced very useful feature; enables computed data for each shot to be copied and pasted onto another sheet; so different shots may be placed in sequence, and comparative charts may be made.

Columns A20 to AP20: computed point by point results (data are correspondingly labeled in row A18 with units in row 19) for the following quantities respectively:

Time in μs , total current, tube voltage, axial position, axial speed, time of radial phase in μs measured from the start of axial phase, time of radial phase in ns from the start of the radial phase, corresponding quantities of current, voltage, radial shock position, radial piston position, radial pinch length, radial shock, piston and pinch elongation speeds, reflected shock position, plasma temperature, Joule power, Bremsstrahlung, recombination, line emission powers, total radiation power, total power, Joule, Bremsstrahlung, recombination, line emission energies, total radiation energy, total energy, plasma self-absorption correction factor, black-body power, specific heat ratio and effective charge number, number thermonuclear neutrons, number beam target neutrons, number total neutrons, ion density, volume radiation power, surface radiation power, plasma self-absorption correction factor, radial phase piston work in % of E_0 , neon SXR energy emission.

Each computed quantity as a function of time (displayed in the relevant column) is displayed in a column. After a run each of these columns is typically filled to several thousand cells.

Computed results are also summarized in 8 figures:

Figure 1: (Top left) total discharge current and tube voltage

Figure 2: (Top right) axial trajectory and speed

Figure 3: radial trajectories

Figure 4: total tube voltage during radial phase

Figure 5: radial speeds

Figure 6: plasma temperature

Figure 7: Joule heat and radiation energies

Figure 8: Joule power and radiation powers

An additional figure 8a on the right displays the specific heat ratio and effective charge number during the radial phase.

2.2 Configuring the Universal Plasma Focus Laboratory (UPFL)

2.2.1 Configuring the worksheet for a specific machine

As a first exercise we configure the UPFL so it operates as the NX2, the High-repetition rate neon focus developed for SXR lithography in Singapore.

The parameters are:

Bank: $L_0=20\text{ nH}$, $C_0=28\text{ }\mu\text{F}$, $r_0=2.3\text{ m}\Omega$

Tube: $b=4.1\text{ cm}$, $a=1.9\text{ cm}$, $z_0=5\text{ cm}$

Operation: $V_0=11\text{ kV}$, $P_0=2.63\text{ Torr}$, $MW=20$, $A=10$, $At-Mol=1$ (these last 3 defines neon for the code i.e. molecular (atomic) weight, atomic number and whether atomic or molecular)

Model: $massf(f_m)=0.0635$, $currf(f_c)=0.7$, $massfr(f_{mr})=0.16$, $currfr(f_{cr})=0.7$; these are the mass and current factors for the axial and radial phases.

(Note: 1. *These model parameters had been fitted earlier by us so that the computed total current best fits a measured total current trace from the NX2.*

Note:2. *We will carry out exercises in fitting model parameters in Module 3)*

Configuring: Key in the following: (e.g. in Cell A5 key in 20 [for 20nH], Cell B5 key in 28 [for 28 μF] etc.

	A5	B5	C5	D5	E5	F5
	20	28	4.1	1.9	5	2.3
Then	A9	B9	C9	D9	E9	
	11	2.63	20	10	1	
Then	A7	B7	C7	D7		
	0.0635	0.7	0.16	0.7		

You may of course find it easier to follow the labels in A4-F4, to key in A5-F5 for the relevant parameters; i.e. A4 states L_0 nH; so fill in below it in A5 20 ; and so on. For identification purposes key in at B3 'NX2'

2.3 Firing a shot in NX2

Place the cursor in any blank non-active space, e.g. G8. (point the cursor at G8 and **click** the mouse). Press 'Ctrl' and 'A' (equivalent to firing a shot).

The programme runs and in less than a minute the run has completed and your worksheet will look something like figure 2 below:

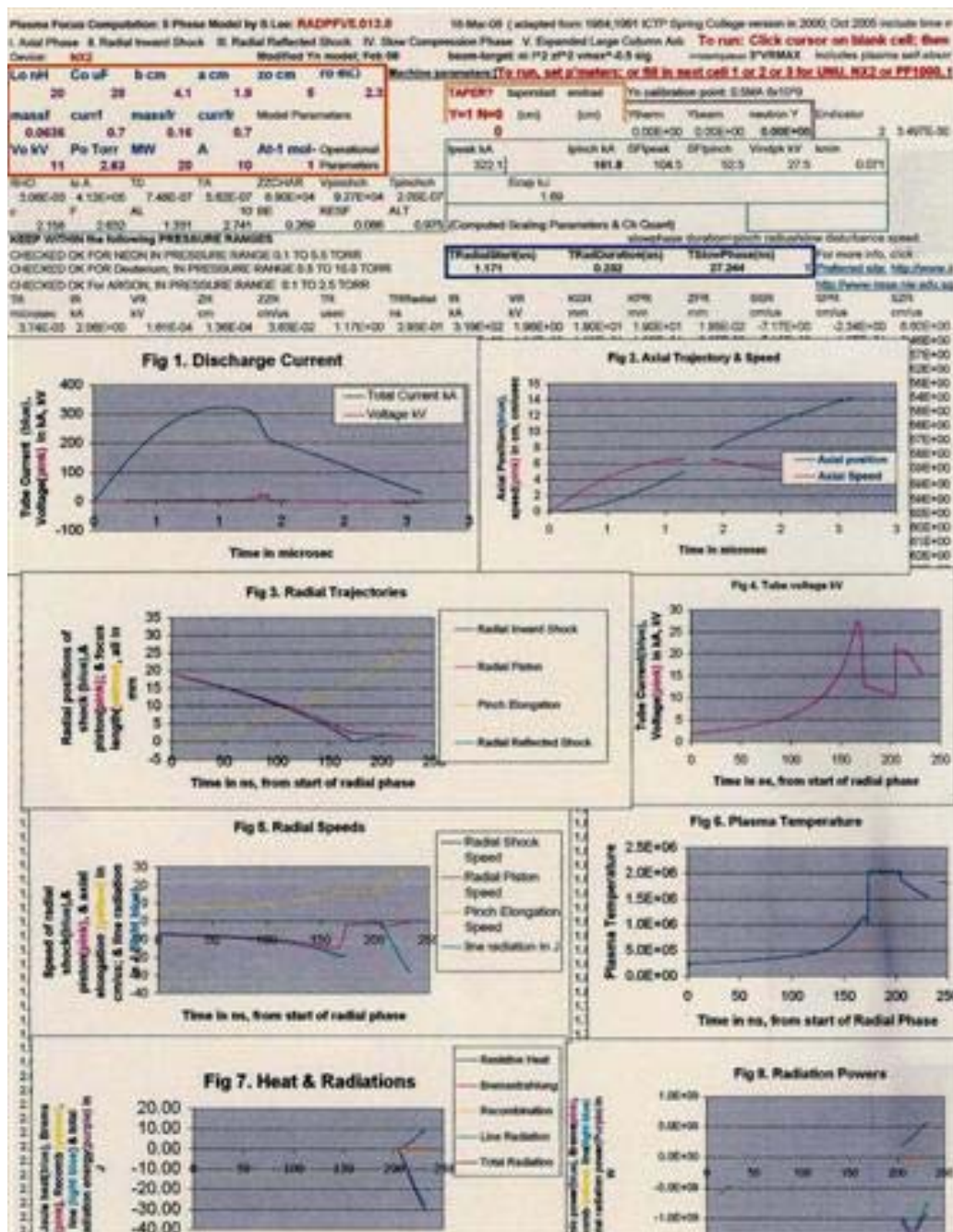


Figure 2. Appearance of Worksheet after a shot.

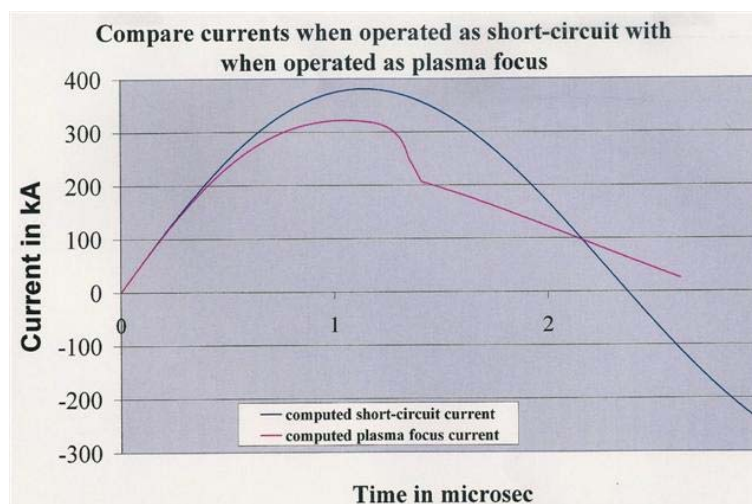


Figure 3. Plasma focus current is distorted from unloaded current waveform.

In figure 3 is superimposed a current waveform (in blue; *you do not have this waveform*) of the plasma focus short-circuited across its input end insulator; with the current waveform (pink) you have just computed [see your worksheet figure 2] (*In a later session you will learn how to do the short-circuit computation and superimposition*).

Notes:

Note 1

The first important point to stress (and one that should never be forgotten) is that the plasma focus current waveform is very much distorted from the damped sinusoid of the L - C - R discharge without the plasma focus load (figure 3). The ‘distortions’ are due to the electrodynamical effects of the plasma motion, including the axial and radial dynamics and the emission of SXR from the Neon plasma. The way we use the code is based on the premise that the features of these ‘distortions’ contain the information of the plasma electrodynamics.

The plasma focus loads the electrical circuit in the same manner as an electric motor loads its driving circuit. The loading may be expressed as a resistance. More specifically we may compute the loading or ‘dynamic’ resistance as follows in Note 2; which shows that the dynamic resistance due to the motion in the axial phase is more than the stray resistance of the capacitor bank in the case of the NX2. Note 3 shows further that the dynamic resistance due to the plasma motion in the radial phase is so large as to completely dominate the situation. This causes the large current dip as shown in figure 3.

Note 2

As an example we may estimate the effect of one of the electrodynamic effects. The quantity $(1/2)(dL/dt)$ is a dynamic resistance.

In the axial phase $L = (\mu/2\pi) \ln(b/a) \cdot z$ where μ is permeability and z is the position of the current sheath.

Differentiating, $0.5 \cdot dL/dt = 10^{-7} \cdot \ln(4.1/1.9) \cdot \text{axial speed} \sim 0.8 \text{ m}\Omega \text{ per } 10^4 \text{ m/s axial speed}$; or $0.8 \text{ m}\Omega \text{ per unit speed of cm}/\mu\text{s}$. At the peak axial speed of $6.6 \text{ cm}/\mu\text{s}$ (see figure 2 of worksheet), that gives us a circuit loading of $\sim 5 \text{ m}\Omega$; which is reduced to $3.5 \text{ m}\Omega$ when we consider the effect of the current factor. This is more than the loading of the stray resistance r_o of $2.3 \text{ m}\Omega$. So the axial motion of the current sheath is an important loading to the circuit.

Note 3

Continuing along this vein we may estimate the dynamic resistive loading of the current sheath motion in the radial phase when $L = (\mu/2\pi) \ln(b/r_p) \cdot z_f$, where r_p = radial piston position and z_f = length of the elongating column; both r_p & z_f changing with time.

$$\begin{aligned} \text{Thus } dL/dt &= (\mu/2\pi) \ln(b/r_p) \cdot dz_f/dt + (\mu/2\pi) \cdot z_f \cdot (dr_p/dt)/r_p \\ &= 2 \cdot 10^{-7} \cdot (\ln(b/r_p) \cdot dz_f/dt + z_f \cdot (dr_p/dt)/r_p) \quad [\text{both terms RHS are positive}] \end{aligned}$$

In the section (2.4) below we will get from the output figures of the worksheet the following values at around the time of peak piston speed:

$r_p \sim 2.4 \text{ mm}$, $z_f \sim 15 \text{ mm}$, $dr_p/dt \sim 13.5 \text{ cm}/\mu\text{s}$ [$1.35 \cdot 10^5 \text{ m/s}$]; $dz_f/dt \sim 1.7 \cdot 10^5 \text{ m/s}$; Substituting into expression above, we get at the time of peak piston speed $dL/dt \sim 190 \text{ m}\Omega$; giving us (after considering current factor of 0.7) still around $130 \text{ m}\Omega$ of dynamic resistive loading due to the current sheath motion. This dynamic resistance (compared to r_o of just $2.3 \text{ m}\Omega$) dominates the current profile at this stage.

Note 4

$d[LI]/dt$ generates an induced voltage; with one important component in this situation being $I \cdot (dL/dt)$. Since we have already estimated that $dL/dt \sim 0.19 \Omega$; multiplying this by $0.7 \times 200 \text{ kA}$ of current (which is the approx value of current at this time) gives us just under 30 kV . So we note that the dynamics at this time (just as the radial shock is going on axis) contributes a back voltage of $\sim 30 \text{ kV}$ through this term.

The other term $L \cdot (dI/dt)$ terms is negative; so the maximum induced voltage is considerably less than 30 kV , as you can see from figure 2.

Note 5

As a separate exercise which you may like to do one day: What is the basis for saying that $(1/2) \cdot (dL/dt)$ is a dynamic resistance? Can you show this by examining the power term in the situation when an inductance is changing? Compare the inductive power flow: $(d/dt)(0.5 \cdot L \cdot I^2)$ and the total power flow: $VI = I \cdot (d/dt)(L \cdot I)$.

What do you notice?

2.4 Studying the results

(The results are obtained from your **Excel Sheet**; not from the above images in figure 2)
Remember we are operating a neon plasma focus.

Here are some important quantities obtained from the data line in row 17.

Computed	I_{peak} :	L17	322 kA
	I_{pinch} :	M17	162 kA (pinch current at start of pinch phase)
	Peak tube voltage:	V17	26.1 kV
	k_{min} :	S17	0.075 (r_{min} or a_{min}/a) [you may also check this against figure 3 of the worksheet.]

Durations: H11-N11

Axial phase ends at 1.172 μ s

Radial phase ends at 1.407 μ s (add 1.172 to 0.235 μ s) of which the last 26.2 ns is the pinch phase.

Now we study the **various figures displayed on the worksheet**, Sheet1 (also shown in figure 2).

Fig 1

Computed current trace; One point of interest is to locate the ends of axial and radial phases on this trace; as well as the start and end of the pinch phase. To do this, select Fig 1 (by pointing cursor on figure 1 and clicking). Then point cursor arrow at trace near peak and move until point 1.17 μ s appears; that is the end of axial phase which is also the start of the radial phase.

Note: This point occurs not at the apparent start of current dip, but a little before that. There is no distinct indication on the trace that precisely marks this point. The term rollover may be a better term suggesting a smooth merging of the axial and radial phase. The apparent current dip occurs a little after the end of the axial phase.

Next locate point 1.41 μ s which is the end of the radial phase. Also locate the point 1.38 μ s which is the start of the pinch phase. There is no clear indication on the trace to mark this point either.

Fig 2

Select Fig 2 (with cursor) and read off the pink curve that the peak axial speed reached is 6.6 cm/ μ s. Confirm this on the data line; Cell P17. How many km per hour is this? And what is the Mach Number? 1 cm/ μ s=36,000 km/hr; so 6.6 cm/ μ s=237,600 km/hr

Expressing this speed in km/hr is to give an idea of how fast the speeds are in the plasma focus; it should give the idea also of temperature, since for strong shock waves (high Mach number motion) there is efficient conversion of energy from directed to thermal, i.e. from high kinetic energy to high temperature.

Mach number=speed/sound speed; finding this number is one of the questions for Exercise 1 (see below).

Fig 3

Select Fig 3. Read from dark blue curve that piston hits axis (radius=0) at 178ns after start of radial phase; and outgoing reflected shock (light blue) hits incoming piston (pink curve) at 210ns at radius of 2.1mm. The pinch phase starts at this 209ns and ends at 235 ns at a further compressed radius of 1.42mm.

Note the square of the ratio of pinching a/r_{min} is a measure of the how much the ambient density has been increased by the pinching effect.

Fig 4

Computed waveform of tube voltage during radial phase. Note the peak value of the tube voltage induced by the rapid plasma motion.

Fig 5

Select Fig 5. Note from the dark blue curve that peak radial shock speed is 20.4 cm/μs just before the radial shock hits the axis at 178 ns after start of radial phase. Also read from the pink curve that peak piston speed is 14.2 cm/μs reached just before the radial shock reaches its peak speed. Yellow curve shows column elongation speed. Note that these peak speeds are also recorded in the data line.

Other figures:

Select **Fig 6**: and read the peak temperature reached.

Select **Fig 7**: and read the various energies.

Select **Fig 8**: and read the various powers

Note that more charts are plotted on Sheet2 of **RADPFV5.15de.xls** These charts form a more complete picture of the plasma focus pinch, and may be used as starting guides for laboratory measurements of the various plasma properties.

2.5 Exercise 1: Interpreting and recording data from the worksheet.

Fill in the following blanks:

- Q0: Given the speed of sound in neon at room temperature is 450 m/s (1600 km/hr), the Mach number of the peak axial phase speed, and of the peak radial phase speed (radially inwards shock speed) are _____ and _____. (Note: *The peak axial speed can be found from Fig.2, and the peak radial speed can be found from Fig.5, for this particular plasma focus operation, also recorded in the dataline*)
- Q1: The peak temperature reached is _____ K.
- Q2: At that temperature the effective charge number (from small figure) is _____ and specific heat ratio has a range as follows _____.
- Q3: There is a moment in time when the temperature jumps by a factor of approximately 2. This is at _____ ns from start of radial phase (Note: *this happens at reflected shock according to the model*)
- Q4: Joule heating reached a maximum value of _____ J.
- Q5: Total radiation reached a maximum value of _____ J (Note: *the – sign for the radiation energy indicates energy taken out of the plasma by emission; ignore the – sign for this measurement*)
- Q6: Line Radiation reached a maximum value of _____ J.
- Q7: Peak radiation power reaches a value of _____ W.

2.6 Conclusion

We had an introduction to the Worksheet of **RADPFV5.15de.xls**

We configured the UPFL as the NX2 at 11 kV 2.6 Torr neon.

We used properly fitted model parameters. (*Note: Fitting model parameters will be covered in a future session*).

We noted that the current waveform is distorted from damped sinusoid-like waveform (damped sinusoid-like waveform is the current waveform when the plasma focus is short-circuited).

We studied the computed results, including total current, tube voltage, pinch current, radial and axial trajectories, radial and axial speeds, plasma temperature and plasma Joule heating and radiation energies.

We also located various points on the current trace including: end of axial phase/start of radial phase; end of radial phase; start and end of pinch phase.

Note: This particular numerical ‘shot’ used properly fitted model parameters. The results of dynamics, electrodynamics and radiation as seen above are, in our experience, comparable with the actual experiments conducted at NTU/NIE.

End of Module 2.

Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics

Plasma Focus Numerical Experiments

Part 1 Basic Course

Module 3: I. Configuring and fitting computed current to measured current II. Comparing a large PF with a small PF-neutron yield etc

(Follow the instructions in the following notes. You may also wish to refer to the supplementary notes **SP2.doc**)

Summary

For this module we fit model parameters so that computed current waveform matches measured current waveform.

First we configure the UPFL (**RADPFV5.15de.xls**) for PF1000 operating in deuterium; using trial model parameters. We fire a shot. **We do not know how good our results are without a reference point; i.e. some comparison with experimental results.**

A total current waveform of the PF1000 has been published; we have it in digitized form in a file **PF1000data.xls**. We also have the chart of this waveform displayed in this file. To ensure that our computed results are comparable to experimental results, the key step is to fit model parameters, by adjusting the model parameters until the computed total current trace matches the measured total current trace.

To do this, we add **PF1000data.xls** to our numerical focus laboratory **RADPFV5.15de.xls**. Next we plot the computed current waveform in the same chart. The model parameters are varied; at each variation the focus is fired, and the computed current waveform is compared with the measured waveform. The process is continued until the waveforms are best matched. A good match gives confidence that the computed results (trajectories, speeds, temperature, neutron and radiation yields etc) are comparable with actual experimental results.

After the guided fitting of the PF1000, we have a self exercise to fit the Chilean PF400J. We then tabulate important results of both machines, and do a side-by-side comparison of big versus a small plasma focus to obtain important insights into scaling laws/rules of the plasma focus family.

Configuring and fitting computed current to measured current

- 3.1 Configure the code for the PF1000 using trial model parameters
- 3.2 Place a measured (published) PF1000 current waveform on Sheet3
- 3.3 Place the computed current waveform onto the same chart as the measured current waveform in Sheet3
- 3.4 Vary the model parameters to obtain matching of computed versus measured current traces

II. Comparing a large PF with a small PF-neutron yield etc

3.5 Exercise 2: Tabulate results for PF1000 obtained in numerical experiments

3.6 Exercise 3: Fitting the PF400J and tabulate the results for PF400J side by side with the results for PF1000, for a comparative study

3.7 Conclusion

The material

You need **RADPFV5.15de.xls** for the following work. Copy and paste a copy on your Desktop. You also need the files **PF1000data.xls**, **PF400data.xls** and **compareblank.xls** for this session. Copy and paste a copy of each file onto your desktop.

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I. Configuring and fitting computed current to measured current (guided–4 hrs)

3.1 Configure the code for the PF1000 using trial model parameters

Double click on **RADPFV5.15de.xls** on your Desktop.

Security Warning “Macros have been disabled” appear at top left hand corner of Worksheet with side box “options”.

Click on “options”- select the button “Enable this content”- click OK

After this procedure, the worksheet is macro-enabled and is ready for firing.

Type in cell B3: PF1000; for identification purposes.

The PF1000, at 40 kV, 1.2 MJ full capacity, is one of the biggest plasma focus in the world. Its 288 capacitors have a weight exceeding 30 tonne occupying a huge hall. It is the flagship machine of the ICDMP, International Centre for Dense Magnetised Plasmas.

We use the following bank, tube and operating parameters for the PF1000:

Bank: $L_0=33.5\text{ nH}$, $C_0=1332\text{ }\mu\text{F}$, $r_0=6.3\text{ m}\Omega$
Tube: $b=16\text{ cm}$, $a=11.55\text{ cm}$, $z_0=60\text{ cm}$
Operation: $V_0=27\text{ kV}$, $P_0=3.5\text{ Torr}$, $MW=4$, $A=1$, $At\text{-Mol}=2$

For this exercise we do not know the model parameters. We will use the trial model parameters recommended in the code (See cells T9-V9)

Model: $\text{massf}(f_m)=0.073$, $\text{currf}(f_c)=0.7$, $\text{massfr}(f_{mr})=0.16$, $\text{currfr}(f_{cr})=0.7$; our first try.

Configuring: Key in the following:

	A5	B5	C5	D5	E5	F5
	33.5	1332	16	11.55	60	6.3
Then	A9	B9	C9	D9	E9	
	27	3.5	4	1	2	
Then	A7	B7	C7	D7		
	0.073	0.7	0.16	0.7	for first try	

Or follow the guide in A4-F4, to key in A5-F5 for the relevant parameters.

Fire a shot: Place the cursor in any blank non-active space, e.g. G8. (point the cursor at G8 and click the mouse). Press ‘Ctrl’ and ‘A’. (equivalent to firing a shot)

The program runs and results are displayed in columns and also in figures.

Is our simulation any good? Not if there is no reference point!!

To assess how good our simulation is, we need to compare our computed current trace with the measured current trace, which has been published.

Note that at this point: the configured **RADPFV5.15de.xls** contains computed data for PF1000 with the trial model parameters of: $\text{massf}(f_m)=0.073$, $\text{currf}(f_c)=0.7$, $\text{massfr}(f_{mr})=0.16$, $\text{currfr}(f_{cr})=0.7$

3.2 Place a measured (published) PF1000 current waveform on Sheet3

3.2.1 The PF1000 current waveform is in the file **PF1000data.xls**. You now want to place this data file as an additional sheet in our **RADPFV5.15de.xls** workbook.

RADPFV5.15de.xls is already open, minimize it by clicking on top right hand corner tab with the - sign.

Open **PF1000data.xls**.

Locate tab Sheet3 on lower left corner of worksheet.

Right click on tab Sheet3.

Select move or copy to book **RADPFV5.15de.xls**

Click on (move to end)

Tick Create a copy

Click OK

3.2.2 With this procedure you have copied **PF1000data.xls** as Sheet3 in **RADPFV5.15de.xls**. The chart has already been prepared. The measured current waveform appears in the chart.

3.3 Place the computed current waveform onto the same chart as the measured current waveform in Sheet3

In the next steps we will place the computed current data from Sheet1 into this same chart in Sheet 3, by the following procedure.

Position the cursor on the chart in Sheet3 containing the measured current waveform. Now **right click**. Pop-up appears. Click on Select data. Select the series “Computed current in *kA*”. Click on Edit. On the new pop-up for series the name “=”Computed current in *kA*” is already there.

For Series X values key in “=Sheet1!\$a\$20:\$a\$6000”

For Series Y values key in “=Sheet1!\$b\$20:\$b\$6000”

Click OK; and click OK.

The computed current waveform from Sheet1 is charted in the figure in Sheet3 with the same time scale and the same current scale.

You can now compare the computed current trace with the measured current trace.

You should see a pink trace which has just appeared on the chart. The pink trace (see figure 1 below) is the computed current trace transferred from Sheet1 (where the time data in μs is in column A, from A20-A several thousand; and corresponding computed current data in *kA* is in column B, from B20 to B several thousands). We are selecting the first 5980 points (if that many points have been calculated) of the computed data; which should be adequate and suitable.

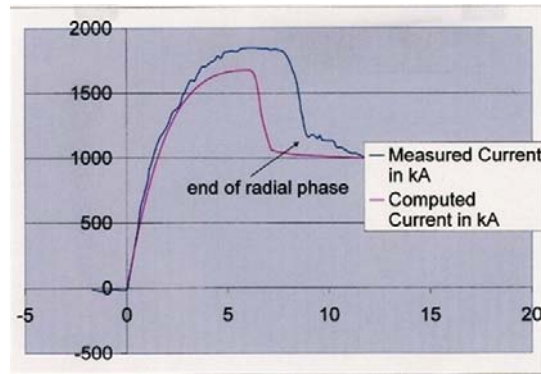


Figure 1. Comparison of traces: Note that there is very poor matching of the traces; using the first try model parameters.

3.4 Vary the model parameters to obtain matching of computed versus measured current traces

Note that bank, tube and operating parameters have all been given correctly.

3.4.1 First fit the axial phase

[suggestion: read SP2.doc pg 3 'First step is fitting the axial phase'.]

From the comparison chart on Sheet2,

We note: that the **computed current dip comes much too early**;
that the **computed current rise slope is only very slightly low**;
that the **computed current maximum is too low**.

All these 3 observations are consistent with a possibility that the axial speed is too fast; which would cause the radial phase to start too early. Too high an axial speed would also cause too much loading on the electrical circuit (similar to the well known motor effect) as the quantity $[0.5 \times dL/dt = 0.5 \times L' \times dz/dt]$ is a dynamic resistance loading the circuit during the axial phase; here the inductance per unit length $L' = (\mu/2\pi) \times \ln(b/a)$. This too high speed would also lower the peak current.

To reduce the axial speed, we could increase the axial mass factor. We note that the axial phase ends too early by some 20%; indicating the axial speed is too fast by 20%.

In the plasma focus (as in pinches, shocks tubes and other electromagnetically driven plasma devices) $\text{speed} \sim \text{density}^{0.5}$. So the correction we need is to increase the axial mass factor by 40%. So try an axial mass factor of $0.073 \times 1.4 \sim 0.1$.

We toggle to Sheet1 by clicking on 'Sheet1' (just below the worksheet).

Click on cell A7, and type in 0.1.

Fire the focus by pressing Ctrl+A.

Program runs until completed, and results are presented.

Note TRadialStart (H11) has increased some $0.6 \mu\text{s}$.

Toggle to Sheet3 (i.e. click on Sheet3 just below work sheet).

Note that the computed current dip is now closer to the measured current dip in time (still short by some 10%; reason being that increasing the axial mass factor reduces the speed which in turn causes a reduced loading. This increases the current which tends to increase the axial speed so that our mass compensation of 40% becomes insufficient). The value of the computed peak is also closer to the measured. So we are moving in the right direction! But still need to move more in the same direction. Next try axial mass factor of 0.12. Toggle to Sheet1, type 0.12 in A7. Fire. Back to Sheet3. Note improvement in all 3 features.

In similar fashion, gradually increase the axial mass factor. When you reach 0.14 you will notice that the computed current rise slope, the topping profile, the peak current and the top profile are all in good agreement with the measured. The computed trace agrees with the measured trace up to the start of the dip. Note that the axial model parameters at this stage of agreement are: 0.14 and 0.7. You may wish to try to improve further by making small adjustments to these parameters. Or else go on to fit the radial model parameters.

3.4.2 Next, fit Radial phase

Note that the computed current dip is too steep, and dips to too low a value. This suggests the computed radial phase has too high a speed. Try increasing the radial mass factor (cell C7), say to 0.2. Observe the improvement (dip slope becomes less steep) as the computed current dip moves towards the measured dip.

Continue making increments to $\text{mass}(f_r)$ (cell C7). When you have reached the $\text{mass}(f_r)$ value of 0.4; it is becoming obvious that further increase will not improve the matching; the computed dip slope has already gone from too steep to too shallow, whilst the depth of the dip is still excessive.

How to raise the bottom of the dip? Here we suppose the following scenario:

Imagine if very little of the current flows through the pinch, then most of the total current will flow unaffected by the pinch. And even if the pinch were a very severe one, the total current (which is what we are comparing here) would show hardly a dip. So reducing the radial current fraction, ie currfr (or f_{cr}) should reduce the size of the dip.

Let us try 0.68 in cell D7. Notice a reduction in the dip. By the time we go in this direction until $\text{currfr}(f_{cr})$ is 0.65, it becomes obvious that the dip slope is getting too shallow; and the computed dip comes too late.

One possibility is to decrease $\text{massfr}(f_{mr})$ (which we note from earlier will steepen the dip slope); which however will cause the dip to go lower; and it is already too low. Another possibility is to decrease the axial phase $\text{massf}(f_m)$, as that will also move the computed trace in the correct direction.

Try a slight decrease in axial $\text{massf}(f_m)$, say 0.13.

Note that this change aligns the dip better but the top portion of the waveform is now slightly low, because of the increased loading on the electrical circuit by the increase in axial speed. This suggests a slight decrease to circuit residual resistance r_0 (or changes to L_0 or C_0 ; fitting those could be tricky, and we try to avoid unless there are strong reasons to suspect these values). Easier to try lowering r_0 first. Try changing r_0 to $6.1 \text{ m}\Omega$.

The fit is quite good now except the current dip could be steepened slightly and brought slightly earlier in time. Decrease $\text{massfr}(f_{mr})$, say to 0.35. The fit has improved, and is now quite good, except that the dip still goes too low. At this stage we check where we are at.

Toggle to Sheet1. Note from Sheet1 that the radial phase ends at 9.12 μs . Back to Sheet3.

Observe (using cursor) that the point 9.12 is not at the point where the computed (pink curve) dip reaches its inflection point; but some 0.02 μs before that point (see figure below).

So we note that the computed curve agrees with the measured curve up to the end of the radial phase with a difference of less than 0.02 MA out of a dip of 0.66 MA (or 3%).

The fitting has already achieved good agreement in all the features (slopes and magnitudes) of the computed and measured total current traces **up to the end of the radial phase**.

Do not be influenced by agreement, or disagreement of the traces beyond this end point.

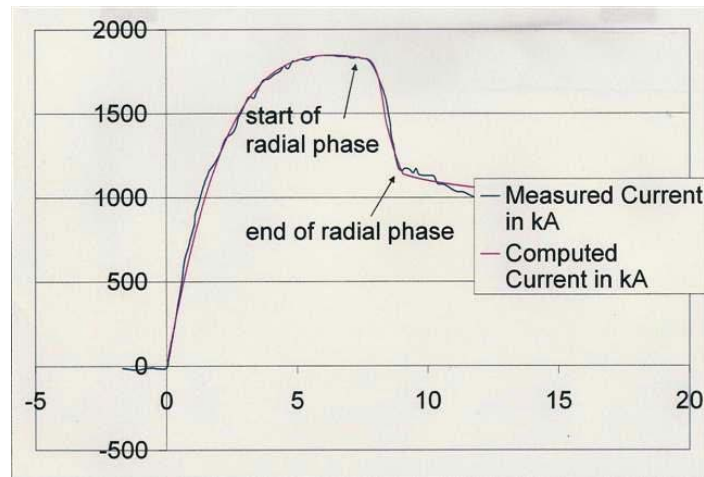


Figure 2. The best fit.

So we have confidence that the gross features of the PF1000 including axial and radial trajectories, axial and radial speeds, gross dimensions, densities and plasma temperatures, and neutron yields up to end of radial phase may be compared well with measured values. Moreover the code has been tested for neutron and SXR yields against a whole range of machines and once the computed total current curve is fitted to the measured total current curve, we have confidence that the neutron and SXR yields are also comparable with what would be actually measured.

Having said that, those of you who have some experience with the plasma focus would note that at the end of the radial phase, some very interesting effects occur leading to a highly turbulent situation with occurrence, for example, of high density hot spots. These effects are not as yet modeled in the code. Despite this drawback, the postulated beam-target neutron yield mechanism seems able to give estimates of neutron yield which broadly agree with the whole range of machines. For example, the neutron yield computed in this shot of 1.08×10^{11} is in agreement with the reported PF1000 experiments.

One further note: We have recently confirmed that the above discussion of fitting applies typically to machines with low L_0 , below perhaps 60 nH. For machines above 100 nH another strategy of fitting or even modelling may need to be adopted. This is related to the comment just above this note.

II. Comparing a large PF with a small PF - neutron yield etc

3.5 Exercise 2: Tabulate results for PF1000 obtained in numerical experiments

You have been following the guided steps in the above fitting:
Fill in the following:

Q1: My best fitted model parameters for PF1000, 27kV, 3.5 Torr deuterium are:

$f_m =$	$f_c =$	$f_{mr} =$	$f_{cr} =$

Q2: Insert an image of the discharge current comparison chart in Sheet3 here.

Q3: Fill up the following table. Use the file **compareblank.xls** for this purpose.

Parameter	PF 1000 (at 27 kV 3.5 Torr D ₂)
Stored Energy E_0 in kJ	
Pressure in Torr, P_0	
Anode radius a in cm	
$c=b/a$	
anode length z_0 in cm	
final pinch radius r_{min} in cm	
pinch length z_{max} in cm	
pinch duration in ns	
r_{min}/a (r_{min} is also called a_{min})	
z_{max}/a	
I_{peak} in kA	
I_{peak}/a in kA/cm	
$S=(I_{peak}/a)/(P_0^{1/2})$ (kA/cm)/Torr ^{1/2}	
I_{pinch} in kA	
I_{pinch}/I_{peak}	
Peak induced voltage in kV	
peak axial speed in cm/μs	
peak radial shock speed cm/μs	
peak radial piston speed cm/μs	
peak temperature in 10 ⁶ K	
neutron yield in units of 10 ⁶	

[After filling, save this Excel sheet you will use the same Excel sheet to fill in the results for PF400J which is the subject of the next exercise.]

3.6 Exercise 3: Fitting the PF400J and tabulate results for PF400J side by side with the results for PF1000 for a comparative study

Participants are to fit computed current to measured current waveform of PF400J (bank, tube and operating parameters all correctly given)

In Module 2, we worked with the Singaporean NX2; a 3kJ neon plasma focus designed for SXR lithography. For our first fitting exercise we worked with the Polish PF1000, one of the largest plasma focus (MJ) in the world. You are now given data for the PF400J, a small sub-kJ plasma focus operated in Chile at the Atomic Energy Commission, for the specific purpose of investigating small focus devices. [Note: The 1000 in PF1000 refers to kJ. The 400 in PF400J refers to J; so the energy ratio of the PF1000 to the PF400J is: $1,200,000/400 \sim 3000$]. The PF400J is a small table top device with all components fitting on a small table top. The PF1000 has a huge chamber and its capacitor bank fills a whole big hall.

Given: the current waveform data of the PF400J, digitized from a published waveform. The data is in the file **PF400data.xls**.

Your task: is to fit model parameters until the computed current waveform matches the measured waveform. Some guidance is given below.

Suggested steps to fit PF400J

- i. **Make a clean copy** of **RADPF05.15de.xls** from your Reserve folder to your Desktop. Open this file.
- ii. **Copy PF400data.xls** as Sheet3 of **RADPF05.15de.xls** using procedure as in section 3.2.1 above. **The measured waveform is already pre-charted.**
- iii. **Transfer computed current data from Sheet1 onto the measured current chart in Sheet3;** as in step in section 3.3 above using strings: “=sheet1!\$a\$20:\$a\$6000” [without the quotation marks] and “=sheet1!\$b\$20:\$b\$6000” [without the quotation marks]. No trace of computed current appears yet, since we have not yet ‘fired’ PF400J.

Write down the bank, tube and operating parameters (from the table in the lower part of Sheet3, NOT from the top line, which contains some nominal values). Toggle to Sheet1.

iv. **Configure the Universal Plasma Focus with the following bank, tube and operating parameters**

$L_0(nH)$	$C_0(\mu F)$	$b(cm)$	$a(cm)$	$z_0(cm)$	$r_0(m\Omega)$
40	0.95	1.55	0.6	1.7	10
massf	currf	massfr	currfr	model parameters	
$V_0(kV)$	$P_0(Torr)$	MW	At No.	At1;Mol2	Operation Parameters
28	6.6	4	1	2	

Key in the first try model parameters; [scroll a little to the right and use the suggested parameters for the UNU ICTP PFF, cells T9-V9].

v. **Fire PF400J;** and see the comparative results by toggling to Sheet3.

vi. **Fitting the computed current waveform to the measured waveform**

vii. **Suggested first steps:** Fit the axial region by small adjustments to f_m and f_c , where necessary. In fitting the axial phase, the more important region to work on is the **later part of the rising slope and the topping profile towards the end of the axial phase**. So each time you should note the position of the end of the axial phase from Sheet1 and locate that position on the chart in Sheet2, using the cursor.

viii. **Final steps:** When you have done the best for the axial phase up to the end of the axial phase, then proceed to fit the radial phase. Tip: The dip for the PF400J is not very dramatic. **Enlarge** the trace so the rollover and the dip can be more clearly compared.

Fill in the following questions, copy and paste and e-mail to me.

Questions

Q1: My best fitted model parameters for PF400J, 28 kV, 6.6 Torr deuterium are:

$f_m =$	$f_c =$	$f_{mr} =$	$f_{cr} =$

Q2: Insert an image of the discharge current comparison chart in Sheet3 here.

Q3: Complete the Excel Sheet which you started in the last Exercise; to compare a BIG (~500 kJ) plasma focus with a small one (~400J). As you fill up, note particularly each group of ratios (each group is denoted by a different colour). Note particularly the order of magnitude of the ratios. [use the Excel sheet, rather than this table].

The ratios below were calculated from the actual PF1000 and PF400J results; and left here as a check for you. Calculate your own ratios from your own results. At the end of the exercise save this Excel Sheet as **PFcomparison.xls**. It will be used again if eventually you go to the more advanced exercises of Modules 5.

Make up the following table comparing a BIG plasma focus with a small plasma focus.

Parameter	PF1000 (at 27kV 3.5 Torr D ₂)	Ratio PF1000/PF400J	PF400J (at 28kV 6.6 Torr D ₂)
Stored Energy E_0 in kJ	486	1313	0.37
Pressure in Torr, P_0	3.5	0.53	6.6
Anode radius a in cm	11.55	19.3	0.6
$c=b/a$	1.39	0.54	2.6
anode length z_0 in cm	60	35.2	1.7
final pinch radius r_{min} in cm		26.7	
pinch length z_{max} in cm		22.2	
pinch duration in ns		53	
r_{min}/a		1.4	
z_{max}/a		1.16	
I_{peak} in kA		14.6	
I_{peak}/a in kA/cm		0.76	
$S=(I_{peak}/a)/(P_0^{1/2})(kA/cm)/Torr^{1/2}$		1.05	
I_{pinch} in kA		9.64	
I_{pinch}/I_{peak}		0.65	
Peak induced voltage in kV		2.4	
peak axial speed in cm/ μ s		1.24	
peak radial shock speed cm/ μ s		0.48	
peak radial piston speed cm/ μ s		0.48	
peak temperature in 10 ⁶ K		0.19*	
neutron yield Y_n in 10 ⁶		81920	
Measured Y_n in 10 ⁶ : range	(2 - 7)E+03		0.9-1.2
Measured Y_n in 10 ⁶ : highest	2.0E+04		

We could then use the tabulation for several applications including the following:
Think of scaling rules, laws:

Q4: What is the significance of the Speed Factors S of PF1000 and PF400J? Which one's temperature should be higher?

***The ratio radial speed/axial speed is:

$$v_r / v_a = \left[\frac{(c^2 - 1)(\gamma + 1)}{4 \ln c} \right]^{1/2}$$

Note: <http://www.plasmafocus.net/>; download the [Theory of the model](#)

3.7 Conclusion

In this module we have learned how to fit a computed current trace with a measured current waveform, given all bank, tube and operational parameters. For the PF1000 we obtained a good fit of all features from the start of the axial phase up to the end of the radial phases; giving confidence that all the computed results including trajectories and

speeds, densities, temperatures and neutron and radiation yields are a fair simulation of the actual PF1000 experiment.

We also fitted the computed current to the measured current of the PF400J; thus computing its dynamics and plasma characteristics and neutron yield.

We tabulated important results of the two machines side by side.

We noted important physics:

Although the machines differ greatly in storage energy and hence in physical sizes, the **speed factor S is practically the same**. This has given rise to the now well-known observation that all plasma focus, big and small, all operate with essentially the same energy per unit mass when optimized for neutron yield. See e.g.: http://en.wikipedia.org/wiki/Dense_plasma_focus

The axial speed is also almost the same; in which case the radial speeds would have been almost the same, except they (the radial speeds) are influenced by a geometrical factor $[(c^2-1)/\ln c]^{0.5}$. For these 2 machines the factors differ by 1.5; hence explaining the **higher radial speeds in PF400J**; and also the **higher temperatures** in the smaller PF400J.

The **pinch dimensions scale with 'a'** the anode radius. The **pinch duration also scales with 'a', modified by the higher T** of the PF400J, which causes a higher small disturbance speed hence a smaller small disturbance transit time. In this model this transit time is used to limit the pinch duration.

Finally we may note that just by numerical experiments we are able to obtain extensive properties of two interesting plasma focus machines apparently so different from each other, one a huge machine filling a huge hall, the other a desk top device. Tabulation of the results reveals an all important characteristic of the plasma focus family. They have essentially the same energy per unit mass (S).

A final question arising from this constant energy/unit mass: Is this at once a **strength** as well as a **weakness** of the plasma focus?

End of Module 3.

Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics

Plasma Focus Numerical Experiments

Part 1 Basic Course

Module 4: PF1000 neutron yield versus pressure

(Follow the instructions in the following notes. You may also wish to refer to the supplementary notes [SP3.doc](#))

Summary

This module looks at variation of neutron yield with pressure; running PF1000 from short-circuit (very high pressure), through optimum pressure to low pressure. The very high pressure of the short-circuit shot stops all current sheath motion thus simulating a short circuit. The aim of this shot is just to obtain short-circuit current waveform for comparison with the focusing waveforms at different lower pressures. When the operating (ambient) pressure is low enough neutrons are emitted. The variation of the yield and other properties with pressure are compiled together, and presented on one chart in normalized form. In this way correlation of various quantities may be seen.

- 4.1 Configure the code for the PF1000 at 27 kV, 3.5 Torr D₂ using model parameters which we had fitted earlier
- 4.2 Fire the PF1000 at very high pressure, effectively a short circuit
- 4.3 Fire the PF1000 at lower pressures from 19 Torr down to 1 Torr; looking for optimum neutron yield
- 4.4 Exercise 4: Place the current waveforms (from section 4.3) at different pressures on the same chart for comparative study
- 4.5 Exercise 5: Tabulate results at different pressures for comparative study; including speeds, pinch dimensions, duration, temperature and neutron yield
- 4.6 General notes on fitting, yield scaling and applications of the Lee model code

The material

You need **RADPFV5.15de.xls** for the following work. Copy and Paste on your Desktop. You also need the files **PF1000pressureblank.xls**. This file contains also tabulation blanks for your convenience.

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4.1 Configure the code for PF1000 at 27 kV, 3.5 Torr deuterium using model parameters which we had fitted earlier

i. Preparing Sheet3

Open **RADPFV5.15de.xls**; copy **PF1000pressureblank.xls** as Sheet3; using procedure which we have practiced in Module 3.
Examine Sheet3

PF1000pressureblank.xls, copied as Sheet3 has the parameters of PF1000 recorded along the top rows. One set of measured time-current data is supplied at Columns A and B. To save participants some time, time-current data for several traces are already computed and filled in: 3.5 Torr at columns C and D; 19 Torr at columns G & H; 7.5 Torr at columns M & N and 1 Torr at columns S & T. These are already plotted in figure 1. You are required to fire shots at several other pressures and copy the time-current data for these shots onto the columns reserved for these shots. In this way you fill up the charts with sufficient traces to cover the optimum pressures for neutron yield. These pressures are 100,000, 14, 10, 6 and 2 Torr.

Scrolling to the right you see table 1 with plasma focus properties at various pressures. The properties corresponding to the shots at 19, 7.5, 3.5 and 1 Torr are already filled in. Data from those other shots to be fired by the participants are to be filled in to complete the table. Below table 1 is table 2 with data normalized from table 1 using the shot at 7.5 Torr as the reference shot for normalizing.

Figure 2 displays the normalized Y_n , I_{peak} , I_{pinch} and radial phase piston work EINP versus pressures from table 2.

[Note that the curves in figure 2 have places where they all come to zero. That is because the table has not been filled for those shots yet to be fired. For these shots the data points have been put to zero. The curves will take on their correct shapes once the data has been correctly filled in. This is the job for the participant]

ii. Configure the code for PF1000

Use the data in PF1000 pressureblank.xls to configure.

Bank:	$L_0=33.5\text{ nH}$, $C_0=1332\text{ }\mu\text{F}$, $r_0=6.1\text{ m}\Omega$
Tube:	$b=16\text{ cm}$, $a=11.55\text{ cm}$, $z_0=60\text{ cm}$
Operation:	$V_0=27\text{ kV}$, $P_0=?\text{ Torr}$, $MW=4$, $A=1$, $At\text{-Mol}=2$
Model:	$f_m=0.13$, $f_c=0.7$, $f_{mr}=0.35$, $f_{cr}=0.65$

4.2 Fire the PF1000 at very high pressure, effectively a short circuit

Key in 100,000 Torr at B9.

[Note: In the laboratory it is of course impossible to fire such a shot and a physical short-circuit may need to be used at the insulator end of the plasma focus; or fire at the highest safe pressure in argon. In the lab we have used 50 Torr argon, to obtain very approximate results.]

[In the numerical experiment at this high pressure the current sheath only moves a little down the tube, adding hardly any inductance or dynamic loading to the circuit. So it is equivalent to short-circuiting the plasma focus at its input end. In the code there is a loop during the axial phase, computing step-by-step the variables as time is incremented. The

loop is broken only when the end of the anode (non-dimensionalised $z=1$) is reached. In this case we do not reach the end of the anode. However there is an alternative stop placed in the loop that stops the run when time reaches nearly 1 full cycle is completed (non-dimensionalised time=6 ie nearly 1 full cycle time, 2π , of the short-circuited discharge).

At the start of the run, the code computes a quantity $ALT = \text{ratio of characteristic capacitor time to sum of characteristic axial \& radial times}$. Numerical tests have shown that when this quantity is less than 0.65, the total transit time is so large (compared to the available current drive time) that the radial phase may not be efficiently completed. Moreover because of the large deviation from normal focus behaviour, the numerical scheme and 'house keeping' details incorporated into the code may become subjected to numerical instabilities leading to error messages. To avoid these problems a time-match guard feature has been incorporated to stop the code from being run when $ALT < 0.65$. When this happens one can over-ride the stop; and continue running unless the run is then terminated by Excel for e.g. 'over-flow' problems. In that case one has to abandon the run and reset the code.]

Fire the high pressure shot. The pressure is too high for a normal run and we are automatically toggled over to the Macro; the Visual Basics Code appears at Statement 430 Stop; with a warning message that pressure is too high. In this case we know what we are doing, and over-ride as follows: Click on 'Run' (above the code sheet), and 'continue'. Another 'Stop' appears just below line 485; with a warning about transit time. Click on 'Run' and 'continue'; another 'Stop' appears below Line 488. Click on 'Run' and 'Continue'.

In a little while the run has proceeded and finally the statement "If $T > 6$ Then Stop" appears; indicating we have completed nearly one cycle of the capacitor discharge; and reached the pre-set time limit.

Now, locate the 'x' at the extreme right hand corner of the screen. Click on this 'x'; pop-up appears with the message 'This command will stop the debugger'. Click on OK, which toggles us back to the worksheet, Sheet1.

Copy the data in columns A & B from A20 and B20 to the end of the computed current data (several thousand cells down); toggle to Sheet3 and 'paste' the copied time-current data onto Columns E & F (in the labeled space provided in Sheet3. Locate table 1 by scrolling to the right. Fill in the value of I_{peak} [read from figure 1 or from the relevant cell of the dataline] onto the table 1 against 100,000 Torr. Put zero against all the other quantities (I_{pinch} , peak v_a , S, peak v_s T and Y_n)

4.3 Fire the PF1000 at lower pressures from 19 Torr down to 1 Torr, looking for optimum neutron yield

Fire the next shot at 14 Torr. As the ALT value is over 0.65, the run proceeds as normal. Copy the time-current data from Columns A & B (from rows 20 down) to Sheet3 columns I & J. Fill in the table 1 [I_{peak} , I_{pinch} , peak v_a , S, peak v_s ... T... Y_n ... n_i & EINP taken from the data line) for the data from shot 14 Torr.

Repeat for pressures **10, 9, 8, 7.5, 7, 6, 3.5, 2** and 1 Torr; tabulating the data for all these shots onto table 1; but copy and paste the time-current data for only selected shots of 14, 10, 6 and 2 [in order for figure 1 not to become too crowded]. The list of pressures had been chosen as above in order to demonstrate the way the neutron yield varies with pressure. It is clear that Y_n increases rapidly from 14 Torr to 10 Torr. More points are chosen between 10 Torr and 6 Torr and it is obvious that the optimum pressure (for Y_n) is between 8 Torr and 7 Torr. The participant will notice this as the shots are fired and as the Y_n data is copied on to the table 1.

4.4 Exercise 4: Place the current waveforms (from section 4.3) at different pressures on the same chart for comparative study

Suggested procedure: To save you time, the comparison chart, figure 1 has already been created for you, and pre-filled with several waveforms namely 19, 7.5, 3.5 and 1 Torr. You only have to fill in the ones for 100,000 and 14, 10, 6 and 2 Torr in the correct columns indicated by the column headings already placed on Sheet3.

You will note that the computed current waveform for 3.5 Torr falls neatly on the measured current waveform (as you have seen during an earlier exercise precisely with this PF1000 27 kV, 3.5 Torr current waveform.) You will recognize that we are using the computed 3.5 Torr shot to fit our UPFL to the PF1000 to obtain the model parameters for the PF1000.

Thereafter the assumption is that the model parameters apply for all the other shots.

It would of course be better if for every pressure or every shot we have a measured current trace to fit the code. However despite this assumption about the model parameters, the numerical experiment does show some very interesting features as we proceed below.

4.5 Exercise 5: Tabulate results at different pressures for comparative study; including speeds, pinch dimensions, duration, temperature and neutron yield

This tabulation has already been done as step (4.3) proceeded above.

In order to chart some of the computed data on one comparative chart, as mentioned already, as you fill in table 1, table 2 is at the same time filled in with each data column normalized to the data at 7.5 Torr, which was found to be the pressure with the highest Y_n . Thus the values of all the data in the normalized table is in the region of 1.

Plot normalized Y_n , I_{peak} , I_{pinch} , and radial EINP against P_0 .

[As you fill in table 1, the normalized quantities are automatically computed, and the chart begins to take the correct shape. At the start the chart is in a jumble because many points have not been filled in, and thus there are erratic zero points all over the place.]

Discussion

Note 1

Look at the change of current waveforms from very high pressures to low pressures. At very high pressures the waveform is a damped sinusoid. At 19 Torr the characteristic flattening of the current waveform due to dynamics is already clearly evident. The current peak comes earlier and is lower than the unloaded (high pressure) case, the current then

droops until the rollover into the dip (due to the increased radial phase loading) at around $15 \mu\text{s}$. At lower pressures these characteristics remain the same except that the current trace is depressed more and more as speed increases. The peaking (reaching maximum current) also comes earlier and earlier, as does the radial phase rollover of the current trace. This is characteristic of an L-C-R circuit with increasing resistance R , as R increases from light towards critical damping.

At 2.6 Torr, there is hardly any droop, the current waveform showing a distinct flat top leading to the rollover. At 1 Torr the axial speed is now so high that the axial phase is completed in less than $5 \mu\text{s}$ and the current is still rising when it is forced down by the radial phase dynamics.

Note 2

A very important point to note in neutron scaling is that there exists some confusion and even misleading information in published literature because of sloppy practice with regards to I_{peak} and I_{pinch} . These quantities are sometimes treated as one and the same or when a distinction is attempted there is then confusion between the total current at the time of pinch and I_{pinch} . For example in the case of PF1000, there appears to be some disappointment (in their publications) that (at 35 kV) with the current at more than 2 MA, Y_n is still at best in the mid 10^{11} ; and not at least an order of magnitude higher that one might expect for currents around 2 MA. However if you numerically run PF1000 at 35 kV you will find that I_{pinch} is only 1 MA; so we are not surprised that the measured yield is at best an order of magnitude down from what you would expect thinking that your current is around 2 MA. (scaling at $Y_n \sim I^4$, a factor of 2 in current gives a factor of 16 in the yield; at $Y_n \sim I^3$, a factor of 8). So it is important that the thinking of yield should be in terms of I_{pinch} as the relevant scaling parameter. When using this model code, the distinction of I_{pinch} and I_{peak} is clear.

Next we look at the detailed tabulations: As P_0 decreases, I_{peak} decreases, and continues to decrease, because the increasing axial speed increases the circuit loading, throughout the whole range of pressures. However it is noticed that I_{pinch} increases from high pressures, peaking in a flat manner at 6 Torr and then decreases sharply as pressure is reduced towards 1 Torr. One factor contributing to the increase is the shift of the pinch time from very late in the discharge (when discharge current has dropped greatly) to earlier in the discharge (when current has dropped less). That is the main factor for I_{pinch} increasing despite a decreasing I_{peak} . At low pressures (e.g. 1 Torr), the radial phase now occurs so early that it is forcing the current down early in the discharge. That lowers both the I_{peak} as well as the I_{pinch} . These points are clear when you look at the comparative chart of current traces at various pressures.

The radial EINP follows the same pattern as I_{pinch} , and for the same reasons. The radial EINP computes the cumulative work done by the current sheath (piston) in the radial phases.

Looking at the other quantities, we note that the speeds (axial, radial shock and radial piston) and temperature all continue to rise as pressure lowers; similarly S and maximum induced voltage V also increase as pressure is decreased. Pinch length z_{max} is almost a constant. Minimum pinch radius and pinch duration continue to decrease; the former due to better compression at higher speeds and the latter due to the increased T . The number density progressively drops, due to the decreasing starting numbers, despite the increasing compression.

From the tabulations of the above numerical experiments, it might be useful to consider the beam-target mechanism which we are using to compute the neutron yield. This is summarized in the following note.

Note 3

(Taken from SP3.doc)

$$Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 (\ln(b/r_p)) \sigma V_{max}^{1/2}$$

where σ is the D-D fusion cross section. In the range we are considering we may take $\sigma \sim V_{max}^n$ where $n \sim 2-3$; say we take $n=2.5$; then we have

$$Y_{b-t} \sim n_i I_{pinch}^2 z_p^2 (\ln(b/r_p)) V_{max}^2$$

The factor $z_p^2 (\ln(b/r_p))$ is practically constant.

Thus we note that it is the behaviour of n_i , I_{pinch} and V_{max} as pressure changes that determines the way Y_n increases to a maximum and then drops as pressure is changed.

An additional experiment is suggested, in which you can see how numerical experiments on Y_n versus operating pressure compare with measured results in the case of PF400J. This is discussed in Module 7.

4.6 General notes on fitting, yield scaling and applications of the Lee model code

On fitting: In the numerical experiments we soon learn that one is not able to get a perfect fit; in the sense that you can defend it as absolutely the perfect fit. The way to treat it is that one has got a working fit; something to work with; which gives comparable results with experiments; rather than perfect agreement. There is no such thing anyway; experiments in Plasma Focus (i.e. on one PF under consistent conditions) give a range of results; especially in yields (factor of 2-5 range is common). So a working fit should still give results within the range of results of the hardware experiment.

Even though a fit may only be a 'working' fit (as opposed to the hypothetical perfect fit) when one runs a series of well planned numerical experiments one can then see a trend e.g. how properties, including yields, change with pressure or how yields scale with I_{pinch} , or with L_0 etc. And if carefully carried out, the numerical experiments can provide, much more easily, results just like hardware experiments; with the advantage that after proper reference to existing experiments, then very quickly one can extend to future experiments and predict probable results.

On scaling: Data used for scaling should be taken from yield-optimized (or at least from near optimized) situations. If one takes from the worst case situations e.g. way out in the high pressure or low pressure regions, the yield would be zero for a non-zero I_{pinch} . Such data would completely distort the scaling picture.

Not only should the pressure be changed, but there should be consideration for e.g. suitable (or even optimized) I_{pinch}/a ; as the value of I_{pinch}/a would affect the pressure at which optimized S is achieved.

On directions of work and applications: Efforts on the model code may be applied in at least two directions. The first direction is in the further development of the code; e.g. trying to improve the way the code models the reflected shock region or the pinch region.

The second direction is to apply the model to provide a solution to a particular problem. An example was when it was applied to look at expected improvements to the neutron yield of the PF1000 when L_0 is reduced.

Using the model code it was a relatively easy procedure, firing shots as L_0 was reduced in steps; optimizing the various parameters and then looking for the optimized neutron yield at the new value of L_0 . When this exercise was carried out in late 2007, for PF1000 at 35 kV, unexpectedly it was found that as L_0 was reduced from 100 nH in steps, in the region around 35 nH, I_{pinch} achieved a limiting value; in the sense that as L_0 was reduced further towards 5 nH, whilst I_{peak} continued to increase to above 4 MA, I_{pinch} dropped slightly from its maximum value of 1.05 MA to just below 1 MA. This Pinch Current Limitation Effect could have considerable impact on the future development of the plasma focus.

On numerical experiments to enhance experience and intuition: Moreover the relationship between I_{peak} and I_{pinch} is implicit in the coupling of the equations of circuit and motion within the code which is then able to handle all the subtle interplay of static and dynamic inductances and dynamic resistances and the rapid changes in distributions of various forms of energies within the system. Whilst the intuitive feel of the experienced focus exponents are stretched to the limit trying to figure out isolated or integrated features of these interplays, the simplicity of the underlying physics is captured by the code which then produces in each shot what the results should be; and over a series of shots then reveal the correct trends; provided of course the series is well planned.

So the code may also be useful to provide the numerical experimenter time-compressed experience in plasma focus behaviour; enhanced experience at much reduced time. At the same time the numerical experimenter can in a day fire a number of different machines, without restrictions by time, geography or expense. The problem then becomes one of too much data; sometimes overwhelming the experience and intuition of the numerical experimenter.

On versatility: Your numerical experiments have included examining plasma focus behaviour comparing BIG, medium size and small plasma focus, looking for common and scalable parameters. You studied neutron yields as functions of pressure, comparing computed with experimental data. In 4 modules involving some 12 hours of hands-on work you have ranged over a good sampling of plasma focus machines and plasma focus behaviour.

This was all done with one code the **RADPFV5.15de.xls** the universal plasma focus laboratory facility. We should have the confidence that if we explore the open experiments suggested in the last module of the advanced course below, that could lead us to new areas and new ideas.

End of Module 4- End of Basic Course in Plasma Focus Numerical Experiments

[Comments and interaction on the course work and other matters related to plasma focus are welcome at anytime]

Reference to this course and the Lee model code should be given as follows:

Lee S. Radiative Dense Plasma Focus Computation Package (2010): RADPF
www.plasmafocus.net <http://www.intimal.edu.my/school/fas/UFLF/>

Also see **[list of papers](#)** (pg 21-22, pg 97-202)

S Lee and S H Saw

Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and Plasma Diagnostics

Plasma Focus Numerical Experiments

Part 2 Advanced Course

Module 5: Advanced fitting

You may also wish to refer to the supplementary notes [SP2.doc](#).

Summary

This Module is divided into two parts.

For the first part we look at an advanced fitting problem; a commonly encountered situation when L_0 is given only as a nominal or very approximate value and r_0 is not even mentioned. Then there are 6 fitting parameters; and the process becomes more involved. Nevertheless we have found that, despite that, it is still possible to get a reasonable fit. In these sessions participants will be taken through that experience which will enhance our ability and confidence to fit.

In the second part is an exercise in fitting the DPF78, with bank, tube and operating parameters all provided; but with L_0 nominal and r_0 not given. The participant will fit the current curve. The properties of the DPF78 will then be placed on the comparison Excel Sheet PFcomparison.xls which you have saved from last week's work.

Part I: An advanced fitting problem using PF1000

- 5.1 Configure the code for the PF1000 using nominal L_0 , trial r_0 and trial model parameters
- 5.2 Place a measured (published) PF1000 current waveform as Sheet3
- 5.3 Place the computed current waveform on Sheet3 in the same figure
- 5.4 Vary L_0 , r_0 and the model parameters until the two waveforms achieve best match

Part II: Fitting with unknown nominal L_0 and r_0

- 5.5 Exercise 6: Fitting computed to measured current waveform for DPF78
Given measured current waveform data for the DPF78; given bank (with nominal L_0 and no r_0), tube and operating parameters; participant will fit computed to measured current waveform. Then tabulate DPF78 computed properties into the **PFcomparison.xls** file which was saved from Module 3; or use attached **PFcomparisonpf1000pf400.xls** provided for your convenience.
- 5.6 Conclusion

The material

You need **RADPFV5.15de.xls** for the following work. You should have a clean copy in your Reserve Folder. Copy and Paste a clean copy on your Desktop before the next step.

You need the file **PF1000dataNom.xls** and **DPF78dataNom.xls**. You also need the file **PFcomparison.xls**, saved from Exercise 3 of Module 3. (*or the one attached for your convenience PFcomparisonpf1000pf400.xls*)



The Abdus Salam
International Centre for Theoretical Physics



Part I: An advanced fitting problem using PF1000 (when only a nominal or wrong value of L_0 is available)

5.1 Configure the code for the PF1000 using nominal L_0 , trial r_0 and trial model parameters.

Double click on [RADPFV5.15de.xls](#) on your Desktop.

Click on **enable macros**

The worksheet opens.

Type in cell B3: PF1000; for identification purposes.

The PF1000, at 40 kV, 1.2 MJ full capacity, is one of the biggest plasma focus in the world. It is the flagship machine of the International Centre for Dense Magnetised Plasmas (ICDMP). On their website, inductance was quoted as 9 nH for short circuit.

We searched through PF1000 publications and found figures for L_0 of ‘around 20 nH’ mentioned. For this work we assume we are looking at PF1000 for the first time and all we got for L_0 is the figure $L_0=20$ nH. There is no mention of r_0 . So we use a starting value of $r_0=0.4$ mΩ; this being 0.1 of the bank impedance $(L_0/C_0)^{0.5}$ taking L_0 as 20 nH.

We use the following bank, tube parameters and operating conditions.

Bank: $L_0=20$ nH (nominal), $C_0=1332$ μF, $r_0=0.4$ mΩ(guess value).

Tube: $b=16$ cm, $a=11.55$ cm, $z_0=60$ cm

Operation: $V_0=27$ kV, $P_0=3.5$ Torr, $MW=4$, $A=1$, $At-Mol=2$

We assume that we are starting to look at PF1000 for the first time; and that we do not know the model parameters. We will use the trial model parameters recommended in the code (See cells T9-V9).

Model Parameters:

massf(f_m)=0.073, currf(f_c)=0.7, massfr(f_{mr})=0.16, currrf(f_{cr})=0.7; first try.

Configuring: Key in the following: (e.g. in cell A5 key in 20 [for 20 nH], in cell B5 key in 1332 [for 1332 μF] etc)

	A5	B5	C5	D5	E5	F5
	20	1332	16	11.55	60	0.4
Then	A9	B9	C9	D9	E9	
	27	3.5	4	1	2	
Then	A7	B7	C7	D7		
	0.073	0.7	0.16	0.7	for first try	

Fire the PF1000 with these parameters.

5.2 Place a measured (published) PF1000 current waveform as Sheet3.

We place the published PF1000 current waveform as Sheet 3.
With **RADPFV5.15de.xls** (fired as PF1000 with first try parameters) open; open **PF1000data.xls**. Copy **PF1000data.xls** into **RADPFV5.15de.xls** as Sheet3.
The measured current waveform is now displayed in the chart in Sheet3.

5.3 Place the computed current waveform on Sheet3 in the same figure

Place the computed current waveform on the same chart following the same procedure we did previously; using the strings: “=sheet1!\$a\$20:\$a\$6000” [without the quotation marks] and “=sheet1!\$b\$20:\$b\$6000” [without the quotation marks].

The pink trace is the computed current trace transferred from Sheet1.

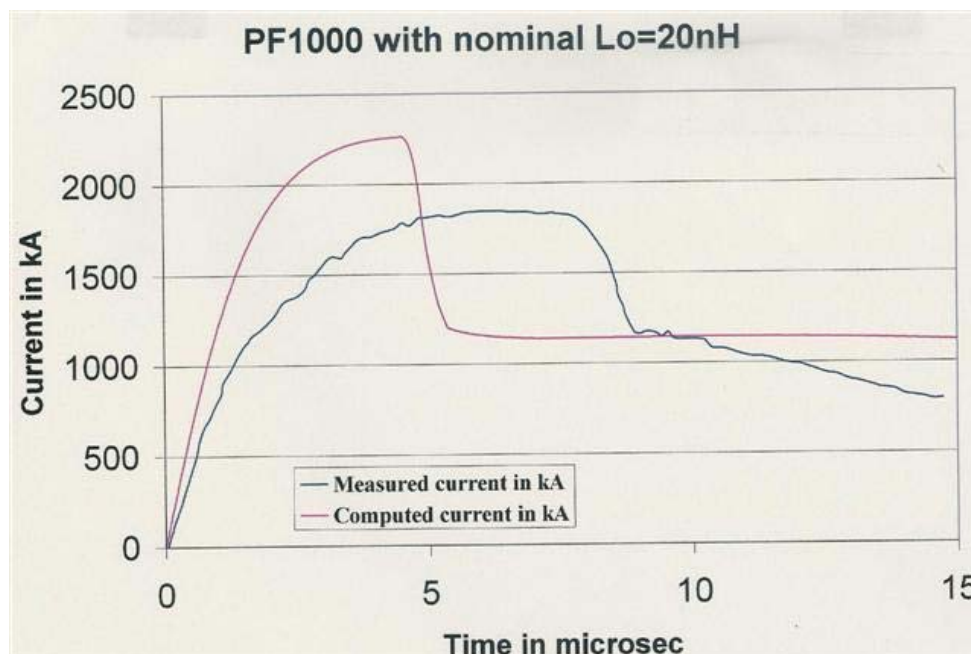


Figure 1. Comparison of traces: Note that there is very poor matching of the traces; using nominal L_o , guessed r_o and the first try model parameters.

5.4 Vary L_o , r_o and the model parameters until the two waveforms achieve the best match.

To vary model parameters:

Note: that the **computed current dip comes much too early**;
 that the **computed current rise slope much too high**;
 that the **computed current maximum is much too large**.

Suppose we do not know that L_0 is not a correct value.

Try varying axial model parameters, which as we know control the current trace up to nearly the start of the roll-over region of the current trace. To make the dip come earlier try increasing f_m ; which will slow down the axial speed (but as we know now, that will also reduce the circuit loading, leading to an even larger current; we got to try something anyway). The deviation is very large, so take a large step; say put $f_m=0.8$ (Note: *maximum allowed value of f_m is 1*). That improves the time position of the dip, but as we expected the current got even bigger. Next try increasing f_c , which will increase the dynamic loading effects on the circuit. Put f_c to its max allowed value of 1.

The time position of the dip is now good and the peak current has improved, but is still way too large. There is not much else we can do with f_m and f_c . (*you could try reducing them, but you know by now that you are not going to see any improvement*). Perhaps we could increase r_0 ; which will lower the whole current profile. Again large difference, need large change. Try $r_0=2\text{ m}\Omega$. Improvement, but not enough. Try $r_0=10\text{ m}\Omega$. Possible improvement, but looks like we have gone beyond. Next try $7\text{ m}\Omega$.

The topping profile deviation has now improved, even touching the measured current profile at one place. But the top is too droopy; and the decreased current has pushed the dip too late. At the same time the current rise rate is still too high. Try reducing f_m to 0.4.

There are now points of agreement; but the current rise slope is still too steep and the topping profile is still too droopy.

It is now clear that in all the things we have tried, the rising slope of the current profile is still too steep. How do we reduce the slope?

From capacitor discharge behaviour, we know that increasing L_0 would do it. (*So would increasing C_0 ; but in this case as in most cases we are fairly sure that the given value of C_0 is more reliable than the nominal value of L_0 .*)

So let's try $L_0=25\text{ nH}$; at last we see the slope beginning to match. Next try 30 nH ; even better. We can see now that at last we are getting onto a better track. It is therefore better to go back to more normal values of f_m and f_c (rather than the unusual values we tried in our desperation) Go back to $f_m=0.15$ and $f_c=0.7$. The matching is improving, but there is still that extra slight droop at the top. Try reducing r_0 to $6\text{ m}\Omega$.

It looks like we are getting there, but the rising slope could on average be improved by a larger L_0 , which would also lower the top. Try $L_0=33\text{ nH}$. The slope match is now pretty good on average, top still too high. Making small changes to L_0 and r_0 , one comes to a final best fit for these two bank parameters which will not be too far away from 33 nH and $6\text{ m}\Omega$. The rising slope profile and the topping profile up to the rollover region of the current trace are now fairly well fitted.

Next make adjustments to f_m and f_c until the final best fit is obtained for the axial phase up to the region of rollover from the current top to the dip.

However we note that the radial phase is yet to be fitted and currently has $f_{mr}=0.16$ and $f_{cr}=0.7$. (*We have already done this part of the fitting in **Module 3** and **Module 4** when we*

fitted the same curve for PF1000, except that then we were given the correct value of L_0 , which in that case made the fitting of the axial phase much more simple. The fitting of the radial phase as suggested below should sound familiar.)

Note that the computed current dip is too steep, and dips to too low a value. This suggests the computed radial phase has too high a speed. Try increasing the radial mass factor, say to 0.2. Observe the improvement (dip slope becomes less steep) as the computed current dip moves towards the measured. Continue making increments to $massfr$. When you have reached the $massfr$ value of 0.4; it is becoming obvious that further increase will not improve the matching; the computed dip slope has already gone from too steep to too shallow, whilst the depth of the dip is still excessive. To decrease the depth of the dip try reducing f_{cr} to say 0.68. Notice a reduction in the dip. By the time we go in this direction until f_{cr} is 0.65, it becomes obvious that the dip slope is getting too shallow; and the computed dip comes too late.

One possibility is to decrease $massfr$. Try 0.35

The fit is quite good now except the current dip could be steepened slightly and brought slightly earlier in time. Try decreasing $massfr$, say to 0.35.

The fit has improved, and is now quite good, except that the dip still goes too low.

However we can check the position of the end of radial phase which is at time=9.12 μs . Putting the cursor on the pink curve at the point $t=9.12 \mu s$, we note that the agreement of the computed curve with the measured curve up to this point is fair.

The best fit? Anyway, a good working fit!

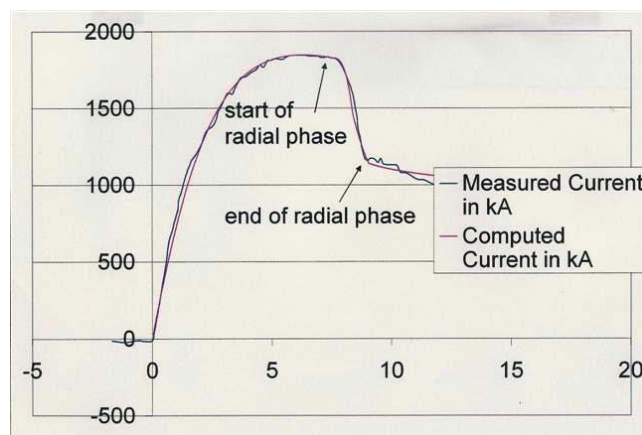


Figure 2. A good working fit

So after finding the correct values of L_0 and r_0 and fitting the model parameters, we should have gained more confidence in the ability of this method of finding a good fit. We repeat that after this fit we have confidence that the gross features of the PF1000 including axial and radial trajectories, axial and radial speeds, gross dimensions, densities and plasma temperatures, and neutron yields up to the end of the radial phase may be compared well with measured values.

Moreover the code has been tested for neutron and SXR yields against a whole range of machines and once the computed total current curve is fitted to the measured total current

curve, we have confidence that the neutron and SXR yields are also comparable with what would be actually measured.

For example, the neutron yield computed in this shot of 8.6×10^{10} is in agreement with the reported PF1000 experimental experiments; (range of $2-7 \times 10^{10}$ with best shots at 20×10^{10}).

Part II: Fitting with unknown nominal L_0 and r_0

5.5 Exercise 6: Fitting computed to measured current waveform for DPF78

We are given the following parameters for the DPF78, operating at 60 kV, 7.5 Torr D₂.

$$L_0 = 44.5 \text{ nH (nominal)} \quad C_0 = 17.2 \text{ } \mu\text{F} \quad b = 5 \text{ cm}, \quad a = 2.5 \text{ cm} \quad z_0 = 13.7 \text{ cm}$$

The DPF78 was a high voltage plasma focus operated at the IPF at Stuttgart. This current waveform (file **DPF78dataNom.xls**) was provided recently by H Schmidt.

Use our Universal Plasma Focus Laboratory code **RADPFV5.15de.xls** to configure the DPF78. Add the DPF78 data as Sheet3. Then fit the computed current waveform to the measured.

Hint 1: you need to assume a try value of r_0 in the same way we did for PF1000; i.e. try $r_0 = 0.1 * (L_0 / C_0)^{0.5}$; which will print out in cell F13 RESF=0.1 where $RESF = r_0 / (L_0 / C_0)^{0.5}$.

Hint 2: the value of RESF very seldom goes below 0.05; so don't put r_0 so small that RESF (F13) goes below 0.05.

Hint 3: The current rise slope is most controlled by value of L_0 (also by C_0 , but in this case we are given a reliable value of C_0).

Hint4: Increasing f_m has the effect of reducing axial speed and increasing I_{peak} ; reducing f_c produces similar effects.

After you are satisfied with the fit, add the DPF78 properties to the comparison tabulation that was saved from last week. **PFcomparison.xls**. Or use the one provided for your convenience: **PFcomparisonpf1000pf400.xls**.

Fill in the following questions, copy and paste and e-mail to me.

5.5.1 Questions

Q1: My best fitted values for PF1000, 27kV 3.5 Torr deuterium are:

$f_m =$	$f_c =$	$f_{mr} =$	$f_{cr} =$

Q2: Insert an image of the discharge current comparison chart in Sheet 2 here.

Copy the Chart and paste onto a fresh Excel workbook (with just the chart on one worksheet). Save this workbook and then paste the workbook here.

- Q3. Add the newly computed properties of DPF78 to the file **PFcomparison.xls** saved from last week. Or use the provided **PFcomparisonpf1000pf400.xls**. That file already contains the properties of PF1000 and PF400J. You may also calculate the ratio of PF1000/DPF78 for each of the properties; as we did last week for PF1000/PF400J. In other words we are using PF1000 as the reference; comparing PF400J as well as DPF78 with it.
- Q4. i. Study the values of r_{min} (or a_{min}), z_{max} and pinch duration? As anode radius ' a ' changes, how do r_{min} (or a_{min}), z_{max} and pinch duration change? (Hint: the relationship is linear)
- ii. How does the factor (pinch volume x pinch duration) vary with ' a '?
- iii. From above can you make a general statement about the dependence on ' a ' of focus pinch radiation yield? Justify your statement (Hint: generally the radiation yield is proportional to volume and duration of the radiating medium)
- Q5. i. Find the ratio of the axial speed of PF400J to that of PF1000.
- ii. Find the ratio the radial shock speed of PF400J to that of PF1000
- iii. You will note from (i) and (ii) that the ratio of axial speeds of the two devices is close to 1; whereas the ratio of radial shock speeds of the two devices is significantly higher than 1. Why? (Hint: Refer File2 Theory of Radiative Plasma focus from www.plasmafocus.net)

From this point we note that whilst the temperatures in the axial phase is almost the same from 1 device to another, the temperatures in the radial phase can be different by a factor as much as 4.

5.6 Conclusion

In these two sessions we experienced a common fitting situation requiring advanced fitting, when the given L_0 is either nominal or wrong and r_0 is not given. Despite having to fit these two additional parameters we found that a reasonable fit could still be achieved. The participant then proceeded to fit a similar situation with the DPF78. The properties of the DPF78 obtained in the numerical experiment are then added to the comparative tabulation obtained earlier for the PF1000 and PF400J. The file is then saved as **PFcomparisonpf1000pf400dpf78.xls**

We note that the DPF78 was a high voltage plasma focus, obviously designed to test higher voltage, higher speed operations, resulting in an unusually high value of S ; which is about a factor of 1.5 higher than the average value of S (close to 100) for most neutron-optimized plasma focus machines.

Study the comparative data in the light of the discussions last week, to strengthen and consolidate the main ideas**.

This table could be kept and added to from time to time with data from other plasma focus which you may be able to compute. Such comparative data could be useful for theses and publications.

Suggestion: *You are invited to fit your own plasma focus and add the data to PFcomparison.xls. I would appreciate a copy of all your fitted (and nominal) parameters, current trace comparison, and your **PFcomparison.xls**; to add to our database, which will be made available to all for downloading.*

****Some notes (edited) kindly provided by a participant to an earlier course:**

1. As 'a' increases, r_{min} , z_{max} , and pinch duration increases; approximately linear dependence; seen in these numerical experiments as well as in agreement with general and theoretical observations.
2. As 'a' increases, (pinch volume*pinch duration) increases; approximately to the 4th power of 'a'; (1 power from each dimension). Why is this factor important to think about?
3. S factor: additional note in comparing PF1000 to PF400J:
The ratio of radial speed/axial speed depends on a factor of $[(c^2-1)/\ln c]$.
This factor $[(c^2-1)/\ln c] \sim 0.92/0.32 \sim 2.9$ for **PF1000**; and $\sim 5.8/0.96 \sim 6$ for **PF400J**;
PF400J will have 2x radial speed as PF1000 (since axial speeds nearly the same] ; .and for supersonic plasmas: $Temp \sim speed^2$ that is the main reason why PF400J has several times higher temperature than PF1000; although same speed factor.
4. In other words same S means approximately same axial speed; and also approximately same radial speed; and also approximately same temperature for cases where 'c' is the same. In this example, 'c' is not the same and favours higher radial speeds and T in PF400J.

Y_n scales with I_{pinch} , because it is I_{pinch} that basically powers the pinching processes during which the neutrons are produced.

(You might wish to add other points.)

End of Module 5

Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics

Plasma Focus Numerical Experiments

Part 2 Advanced Course

Module 6: Soft x-ray yield of NX2 with operating pressure

Summary

In this module we look at variation of SXR with pressure; operating NX2 from short circuit (very high pressure), through optimum pressure to low pressure. In the course of these numerical experiments we take a small detour (during the NX2 experiments) to determine circuit parameters from a short circuit discharge; something very basic, but often overlooked.

The material

You should have **RADPFV5.15de.xls** on your Desktop for the next step. Please also ensure you have kept an identical original copy in a RESERVE folder. You are going to work with the desktop copy; and may be altering it. Each time you need an unaltered copy; you may copy from the reserve folder and paste it onto the desktop.

You will need the following:

[NX2pressureblank.xls](#). These files contain also tabulation blanks for your convenience. Also provided is file **HiRepHiPerformPF.doc** from which NX2 Y_{SXR} vs P_0 data for NX2 is extracted

For these NX2 SXR experiments, the steps are:

- 6.1 Configure the NX2 at 11 kV 2.6 Torr neon using fitted model parameters
- 6.2 Fire the NX2 at very high pressure, effectively a short circuit; first introduction to macro code modification
- 6.3 Detour: Exercise 7: Use this short circuit waveform **as though it were a measured current waveform**, to analyse the lightly damped L - C - R discharge; to fix bank parameters
- 6.4 Fire NX2 at 5 Torr; as an example of insufficient current drive; over-riding the model's time-match guard
- 6.5 Fire NX2 at lower pressures down to 0.5 Torr, looking for optimum SXR yield
- 6.6 Exercise 8: Place current waveforms at different pressures on the same chart, for comparison
- 6.7 Exercise 9: Tabulate results at different pressures; for comparative study; including speeds, dimensions, duration, average temperature and SXR yield
- 6.8 Discussion

The material

You need to prepare the worksheets for the experiment.

Open **RADPFV5.15de.xls**. Copy **NX2pressureblank.xls** onto **RADPFV5.15de.xls** first Sheet3, then Sheet4.

NX2pressureblank.xls has 2 worksheets, Sheet3 and Sheet4. Sheet3 has time-current data for several traces, and scrolling to the right, a table of plasma focus properties at various pressures to be filled in, and below that a normalized table; and there are also two charts; one for the current traces at various pressures and one for Y_{SXR} , I_{peak} , I_{pinch} vs pressures. Have a close look at the opened sheet to see the locations of the supplied time-current data, the blank spaces for you to fill in the other computed time-current data, the tables with the blank spaces to be filled in, and the partially filled in charts. Sheet4 has labeled spaces for the computed high pressure current data, a chart and spaces to be filled in for data to be measured from the current waveform.

Sheet3 and Sheet4 are now ready to receive the data of the numerical experiments.

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6.1 Configure the NX2 at 11 kV, 2.6 Torr neon using fitted model parameters

We use an earlier version of the NX2 with a lower inductance of 15 nH.
The parameters for that version of NX2 were successfully fitted as:

Bank: $L_0=15 \text{ nH}$, $C_0=28 \text{ }\mu\text{F}$, $r_0=2.2 \text{ m}\Omega$
Tube: $b=4.1 \text{ cm}$, $a=1.9 \text{ cm}$, $z_0=5 \text{ cm}$
Operation: $V_0=11 \text{ kV}$, $P_0= \text{Torr}$, $MW=20$, $A=10$, $At\text{-}Mol=1$
Model: $f_m=0.1$, $f_c=0.7$, $f_{mr}=0.12$, $f_{cr}=0.68$

6.2 Fire the NX2 at very high pressure, effectively a short circuit; first introduction to macro code modification

Key in 1,000,000 Torr at B9.

*[In the laboratory it is of course impossible to fire a shot at such high pressure]
In the numerical experiment at this high pressure the current sheath only moves a little down the tube, adding hardly any inductance or dynamic loading to the circuit. So it is equivalent to short circuiting the plasma focus at its input end. In the code there is a loop during the axial phase, computing step by step the variables as time is incremented. The loop is broken only when the end of the anode (non-dimensionalised $z=1$) is reached. In this case we do not reach the end of the anode. **However there is an alternative stop placed in the loop that stops the run when (non-dimensionalised time=6 ie nearly 1 full cycle time, 2π , of the short circuited discharge) is reached.** Moreover at the start of the run, the code computes a quantity ALT = ratio of characteristic capacitor time to sum of characteristic axial & radial times. Numerical tests have shown that when this quantity is less than 0.65, the total transit time is so large (compared to the available current drive time) that the radial phase will not be efficiently completed. Moreover because of the large deviation from normal focus behaviour, the numerical scheme and ‘house keeping’ details incorporated into the code may become subjected to numerical instabilities leading to error messages. To avoid these problems a time-match guard feature has been incorporated to stop the code from being run when $ALT<0.65$. When this happens one can over-ride the stop; and continue running unless the run is then terminated by Excel for e.g. ‘over-flow’ problems. In that case one has to abandon the run and reset the code.*

We want to use the NX2 in short-circuit mode to illustrate the basic but often overlooked treatment of a lightly damped L-C-R circuit for determining circuit parameters. The method we use requires determining the reversal ratio of the lightly damped discharge. For this purpose we would like to have say 3 cycles of the lightly damped discharge i.e. we should continue computing until normalized time reaches $6\pi\sim20$. Since the code has a stop placed at $t=6$, we need to make a change in this statement in the code.

We have **RADPFV5.15de.xls** opened. We will now ‘step into’ the code to edit it.

Above the worksheet, locate and click the control button ‘view’. Select ‘Macros’; Click View Macros. Dialogue box opens; select “radpf005”; click “Step into” The program code in Visual Basic appears. We have entered the code.

Scroll down to line 580. Just below this line is the Statement “If $T > 6$ Then Stop”. **Change the number ‘6’ to the number ‘20’**. Then Exit the code by clicking the ‘×’ at the extreme top right hand corner above the spreadsheet. When drop-down appears with message “This command will stop the debugger” click on the button ‘OK’; bringing us back to Sheet1.

The code is now configured to run the discharge short-circuited for 3 cycles before stopping.

Fire the high pressure shot. The Visual Basics Code appears at Statement 430 Stop; with a warning message that pressure is too high. In this case we know what we are doing, and over-ride as follows: Click on ‘Run’ (above the code sheet), and ‘continue’ [or just F5]. Another ‘Stop’ appears just below Line 485; with a warning about transit time. Click on ‘Run’ and ‘continue’; another ‘Stop’ appears below Line 488. Click on ‘Run’ and ‘Continue’.

In a little while the run has proceeded and finally the statement “If $T > 20$ Then Stop” appears; indicating we have completed more than 3 cycles of the capacitor discharge.

Now, locate the ‘×’ at the extreme right hand corner of the screen. Click on this ‘×’; pop-up appears with the message ‘This command will stop the debugger’. Click on OK, which brings you back to the Sheet1.

6.3 Detour: Exercise 7: Use this short circuit waveform as though it were a measured current waveform, to analyse the lightly damped L - C - R discharge; to fix bank parameters

Note: to fix bank parameters, you need to **measure discharge period T and reversal ratio f** ; hence determine L_0 and r_0 . Only C_0 & V_0 are assumed to be known.

Copy the current waveform data from Columns A & B and paste to Sheet3 into the columns A & B starting from A5 & B5; so that we may carry out our little ‘detour’ experiment. To save you some time the chart has been prepared in advance and the current waveform should appear; once the data is pasted correctly starting at A5 and B5.

From the current waveform: measure $3T$ (to 3 decimal places); hence obtain T . Measure the successive peak currents, recording all as positive values. Thus measure:

$$f_1 = I_2/I_1, f_2 = I_3/I_2, f_3 = I_4/I_3, f_4 = I_5/I_4 \text{ and } f_5 = I_6/I_5; \text{ and } f = (1/5)(f_1 + f_2 + f_3 + f_4 + f_5).$$

We are given C_0 and V_0 . With the measured T and f (measured from the current waveform) we calculate L_0 and r_0 and I_0 using the following approximations applicable to slightly damped L - C - R discharges:

$$L_0 = T^2 / (4\pi^2 C_0)$$

$$r_0 = -(2/\pi) \ln(f) (L_0 / C_0)^{0.5}$$

$$I_0 = \pi C_0 V_0 (1 + f) / T$$

We note from this little ‘detour’ that this method gives highly accurate results for lightly damped discharges. In practice the accuracy is limited by experimental features such as

electrical noise and electrostatic shielding of the coil which may result in a tilted zero baseline. We also note that it is important for every plasma focus to establish reliable baseline data. First, the capacitance C_0 should be reliably known or determined. Then from the value of C_0 , L_0 and r_0 may be fixed; and further I_0 deduced to calibrate the monitoring coil.

Also copy the 1,000,000 *Torr* time-current data to Sheet3 to into the columns provided for this purpose (E & F).

6.4 Fire NX2 at 5 Torr; as an example of insufficient current drive; over-riding the model's time-match guard

We now proceed to the NX2 SXR versus pressure experiment.

Key in 5 *Torr* in B9. Fire a shot.

The Visual Basics Code appears at Statement 430 Stop; with a warning message that pressure is too high. In this case we know what we are doing and over-ride as follows: Click on 'Run' (above the code sheet), and 'continue'. Another 'Stop' appears just below Line 485; with a warning about transit time. Click on 'Run' and 'continue'; another 'Stop' appears below Line 488. Click on 'Run' and 'Continue'.

In a little while the run has completed successfully. In this manner we force the code to run even though the code warns us that the pressure is too high for a good shot.

Copy the time-current data (A20-B20 to several thousand rows down) for this shot and paste into the reserved and labeled space (already done for you in columns Q & R) in Sheet2. Add the data (I_{peak} , I_{pinch} , $Peak\ v_a$, S , $Peak\ v_s$, v_p , a_{min} , z_{max} , *pinch duration* etc) (for *Temp* take average of $T_{pinchmax}$ and $T_{pinchmin}$; for the charge number z use the figure 8a in Sheet1) for this shot to the table prepared for this purpose (scroll a little to the right for this table).

6.5 Fire NX2 at lower pressures down to 0.5 Torr, looking for optimum SXR yield

In a similar way, force the code to run for 4.5 *Torr* (with an ALT=0.64; so need to force). Add data to table.

Continue with the following shots: **4 Torr** (ALT=0.68, so code runs without 'Stop' breaks) 3.5, 3.2, 3, 2.9, 2.8, 2.7, 2.6, 2.4, 2, 1.5, 1, **0.5**; adding the data for each shot to the table 1; but transferring the time-current data to sheet3 of only those shots in bold [we want to plot a few current traces to see the way the traces evolve with pressure]

6.6 Exercise 8: Place current waveforms at different pressures on the same chart, for comparison

The selected current traces are plotted onto the same chart in Sheet3. When we plot the curve for 2.6 *Torr*, note that the computed current trace falls neatly over the measured; as these have already been pre-fitted.

6.7 Exercise 9: Tabulate results at different pressures; for comparative study; including currents, speeds, dimensions, duration, average temperature and Y_{sxr} .

Discuss the results e.g. variation of I_{peak} vs P_0 , I_{pinch} vs P_0 , etc.

6.8 Discussion.

We note the way we are computing the neon SXR radiation; with power of:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} n_i^2 Z Z_n^4 (\pi r_p^2) z_f / T$$

Hence the SXR energy generated within the plasma pinch depends on the properties:

Number density n_i

Effective charge number Z

Pinch radius r_p

Pinch length z_f

Temperature T and

Pinch duration since the power is integrated over the pinch duration.

This generated energy is then reduced by the plasma self-absorption which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the SXR yield.

It was first pointed by Liu Mahe in his PhD thesis “Soft X-rays from Compact Plasma Focus” NTU/NIE 1996, that a temperature around 300 eV is optimum for SXR production. Our subsequent experience through numerical experiments suggests that around 2×10^6 K (below 200 eV) seems to be better.

Important note: **Unlike** the case of neutron scaling, **for SXR scaling there is an optimum small range of temperatures** (T window) to operate. This could be the most important point to observe for SXR scaling.

With these complicated coupled effects and the small T window I have doubts about such simplistic scaling laws as put forward from time to time: $Y_{sxr} \sim I_{pinch}^4 / r_{min}^2$???-doubtful

In this present series of experiments on the NX2 we note that a peak yield of 21J is obtained at 2.9 Torr neon at a temperature of 1.5×10^6 K (computed at the middle of the pinch duration). This compares well with experimental data in Zhang Guixin's 1999 PhD thesis, in his series of yield versus pressure experiments at 11.5 kV using the NX2 (in the configuration of our numerical experiments; our measured current waveform was taken from his series of experiments). In that series he obtained a peak yield of 20 J at 3.3 Torr with yield fall-off similar to our numerical experiments, although the curve peaks less sharply as our results.

Zhang's experimental results are plotted as black points on the chart for comparison with the computed Y_{sxr} vs pressure. Note that the computed yield at optimum pressure is comparable with the measured optimum yield; that the optimum pressure also compare well as is the fall-off of yield to either side of the optimum pressure.

End of Module 6

Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics Plasma Focus Numerical Experiments

Part 2 Advanced Course

Module 7: Advanced exercises

Summary

At the end of the course three additional exercises are given, one comparing computed and measured Y_n versus P_0 for the PF-400J. The second is an outline of an open exercise which gives a glimpse of a frontier area of plasma focus research, that of neutron yield saturation for megajoule devices. The final exercise is inspired by the newly commissioned KSU PF which provided the first clean current signals of a special class of plasma focus machines.

- 7.1 Open Exercise 1: An additional exercise comparing computed and measured Y_n versus P_0 for the PF400J
- 7.2 Open Exercise 2: An open exercise which gives a glimpse of a frontier area of plasma focus research, neutron yield saturation for megajoule devices
- 7.3 Open Exercise 3: KSU Plasma Focus

The material

You should have **RADPFV5.15de.xls** on your Desktop for the next step. Please also ensure you have kept an identical original copy in a RESERVE folder. You are going to work with the desktop copy; and may be altering it. Each time you need an unaltered copy; you may copy from the reserve folder and paste it onto the desktop.

Three additional files are provided for two additional exercises which you may complete at your leisure later. These are: [PF400Yncomparison.xls](#) and an accompanying paper ([Soto et al](#)) for the first additional exercise for you to duplicate. The other paper (Nukulin and Polukhin) goes with the second open exercise suggested as an epilogue to this course.

7.1 Open Exercise 1: An additional exercise comparing computed and measured Y_n versus P_0 for the PF400J

As an additional exercise which you can look at later, you are provided with an additional file [PF400J \$Y_n\$ Comparison.xls](#). This file records data of measured Y_n (from Leopoldo Soto's paper, also attached) and comparison with computed data using [RADPFV5.15de.xls](#). You will see that the agreement between our computed data and Soto's published data of neutron yield vs pressure may be considered to be good; features of comparison include the magnitude of the optimum yield, the optimum pressure and the fall-off on each side of the optimum pressure. You may wish to verify the comparison by running the numerical experiments yourself.

7.2 Open Exercise 2: An open exercise which gives a glimpse of a frontier area of plasma focus research, neutron yield saturation for megajoule devices

V Yu Nukulin & S N Polukhin recently (2007) published a [paper \(attached\)](#) discussing the saturation of neutron yield from megajoule PF facilities. Using an analytical method they surmised that in big plasma focus devices if storage energy is increased by essentially increasing storage capacitance C_0 then I_{peak} reaches a limiting value of around 2 MA. This is because as C_0 increases, so goes the N & P scenario, the current risetime increases and of necessity the anode length has to be increased. Thus the increased effective inductance on the circuit balances out the increase in C_0 . In other words the effective circuit impedance does not go below a limiting value. Hence I_{peak} reaches the limiting value of 2 MA. [Warning: this scenario has since been proven to be incorrect] This thesis is easily tested using our code. Say, starting with the PF1000, keeping voltage and pressure the same, we could increase C_0 starting at say 600 μF , increasing in steps of say 200 μF until 5000 μF . We could add in other criteria such as keeping I/a (current per unit anode radius) approximately constant at some value such as 160 kA/cm; ie we vary 'a' as C_0 is increased; keeping $c=b/a$ constant and r_0/Z_0 a constant where Z_0 is the surge impedance $(L_0/C_0)^{0.5}$. Then not only can we keep track of I_{peak} (which Nukulin calls I_{max}) but more importantly we can keep track of I_{pinch} as C_0 increases. We can then verify (or not) the saturation effect which they surmise (see postscript below). In a sense that already brings us to one frontier of plasma focus research, especially if we keep our minds open as we proceed. (*Note: this exercise was completed in 2008. The numerical experiments carried out showed that whilst I_{peak} does deteriorate in its scaling with increasing C_0 , the mechanism suggested by N & P is not correct. This exercise had been published in a paper (Lee S, attached scroll down to pg 155)).*

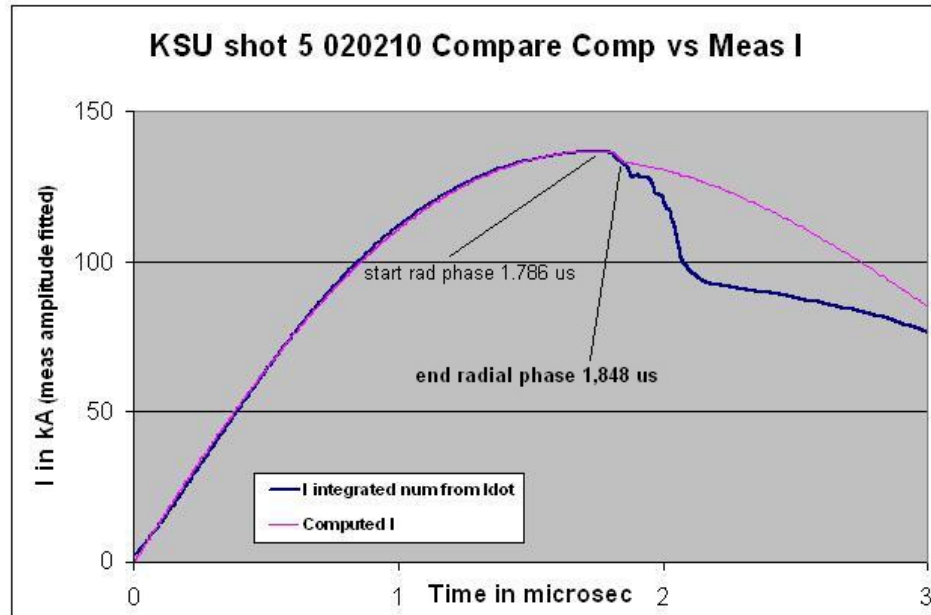
7.3 Open Exercise 3: KSU Plasma Focus

7.3.1 We select a KSU PF shot with a strong focus action (large current dip). We configure the code for the KSU PF.

We try to fit the computed current waveform to the measured waveform.
We find that it is not possible to fit.

The computed current dip is small, only fits the first part of the measured current dip. The measured dip then continues to dip to a greater depth and for a longer duration. The computed cannot be adjusted to fit however extreme the model parameters are stretched.

KSU PF cannot be fitted using the model code.

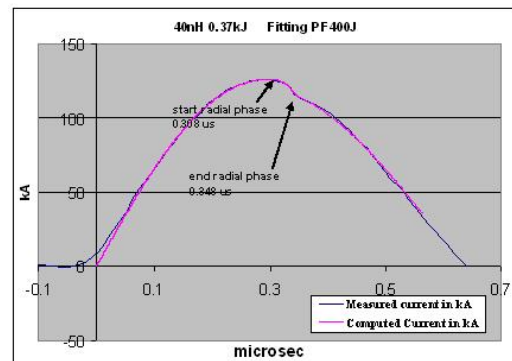
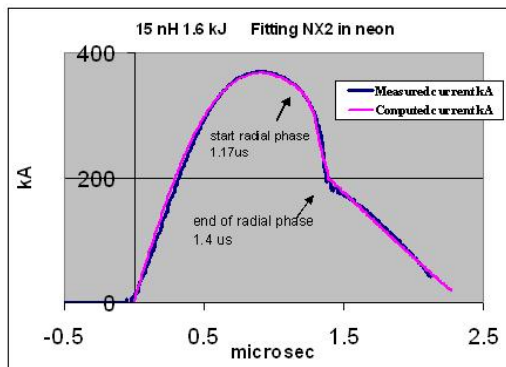
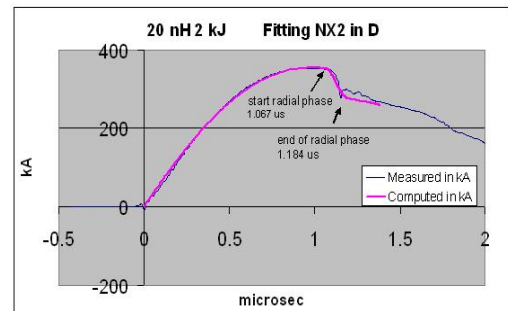
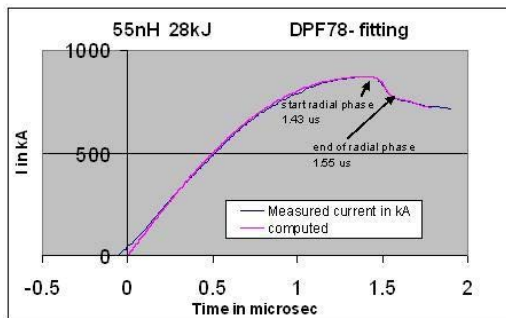
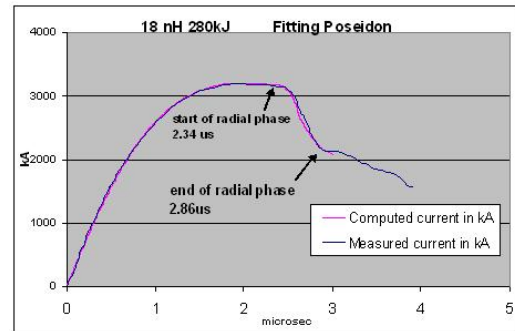
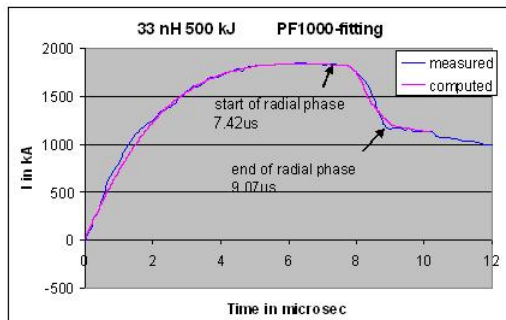


7.3.2 Review the range of plasma focus

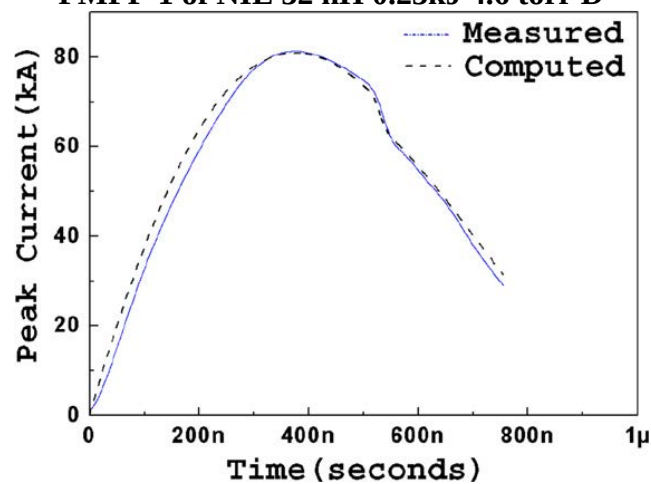
At this point it is worth while to review the comparative traces we have. Those which we are able to fit well, we call Type 1.

Type 1 are fitted well

Type 1 PF: Low Inductance 15-50 nH



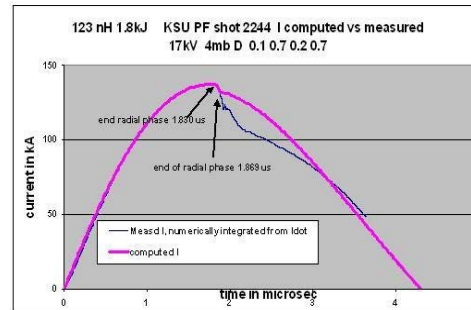
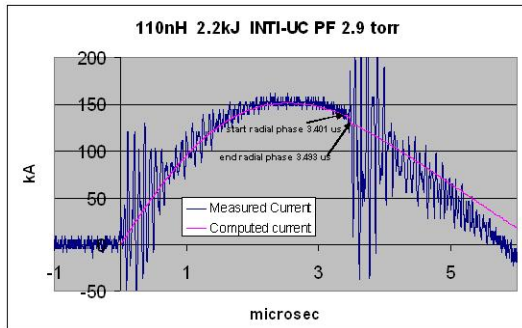
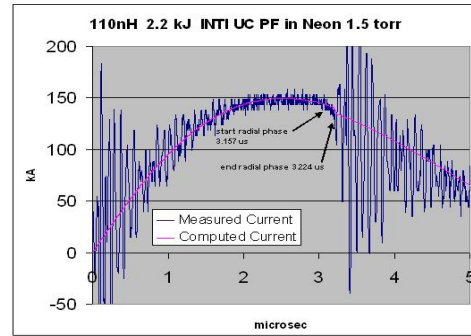
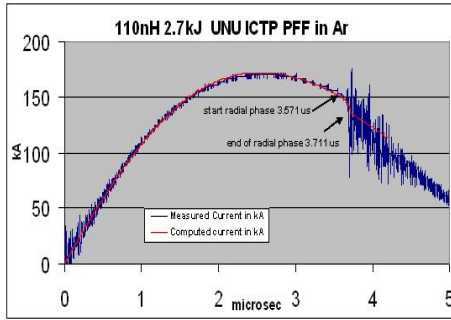
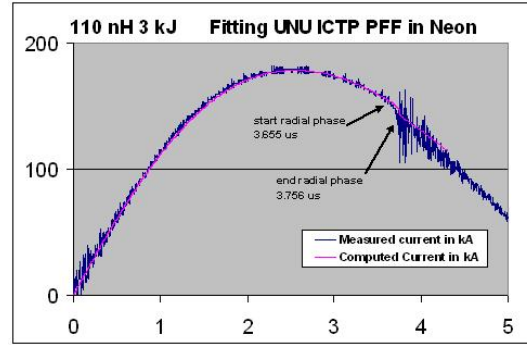
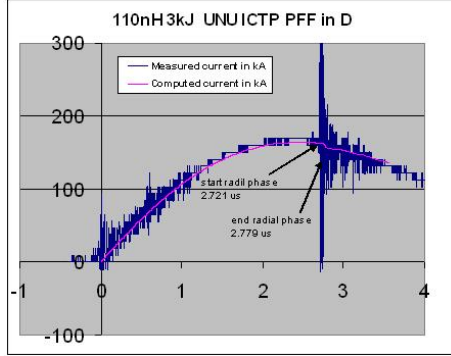
FMPF-1 of NIE 32 nH 0.23 kJ 4.6 torr D



There are also those we cannot fit well, or not at all, which we call Type 2:

Type 2 are not fitted well, or at all.

Type 2 PF: High Inductance over 100 nH.



There are also reports from our associates with various plasma focus which cannot be fitted. These include 2 plasma focus in Syria and one in Iran. In the case of the UNU ICTP PFF (110 nH), the INTI PF (110 nH) and the Syrian PF's (one is over 1000 nH the other is around 200 nH) the fitting is not so certain because the measured current traces are very noisy. But our report from Iran is that their PF (100 nH) has a measured current which cannot be fitted by the model code. We are trying to get reliable current traces for these PF's. All the reported PF's which cannot be fitted are high inductance devices.

Type 1 PF has small L_0 and can be fitted

Type 2 PF has large L_0 and cannot be fitted.

7.3.4 Explanation

The following is an attempt to explain Type 1 and Type 2 relative to the computed model:

RADPF models the electrodynamic situation using the slug model and a reflected shock for the radial phase, ending the radial phase in Phase 4. Let's call the radial phase modeled in that manner as the REGULAR radial phase.

This REGULAR radial phase, in increasing sharply the inductance of the system (constituting also a dynamic resistance) causes a dip on the current trace. Call this the regular dip RD.

At the end of the REGULAR radial phase, experimental observations point to another phase (which I have referred to for some years as 4a, ie after Phase 4, but before the final axial phase, called Phase 5) of 'instabilities' manifesting in anomalous resistance etc. These effects would also extract energy from the magnetic field and hence produce further current dips. These effects are not modeled specifically in **RADPF**. Call this the extended current dip ED.

The code models the first or regular part of the current dip RD
The code does not model the extended part of the current dip ED

However it may be argued that as long as the model parameters can be stretched sufficiently to have the computed current dip agree with the measured current dip, then in a gross sense, the modelling is energetically and mass-wise equivalent to the physical situation. Then the resulting gross characteristics from the model would give a fair representation of the actual plasma properties, even though the model has not specifically modeled ED. In other words RD is able to be stretched to also model ED, with equivalent energetic and mass implications.

Whether RD can be stretched sufficiently to cover ED depends on the relative sizes of the two effects. If RD already a big dip, then this effect may dominate and it is more likely that RD may be stretched sufficiently to cover the less prominent ED.

If RD is only a miniscule dip, and if ED is a big dip, then it is unlikely that RD can be stretched enough to encompass ED.

Large RD can be stretched to fit a small ED
Small RD cannot be stretched to fit a large ED

7.3.5 What are the criteria?

We could try some ratio of impedances or inductances. We have tried quite a Number, and found the following reasonably indicative.

Take the ratio $L_{pinch}/(L_0+L_a)$ where L_{pinch} is the inductance of the focus pinch at the end of the REGULAR radial phase, L_0 the bank static inductance and L_a the inductance of the axial part of the focus tube.

Computing the values of this quantity for PF1000, Poseidon, DPF78, NX2, UNU/ICTPPFF, INTI PF (also a UNU/ICTP PFF), PF400J and FMPF-1 and now also the KSU

PF, we have a range of devices from very big (*MJ*) to rather small (sub *kJ*) of which we have well documented fittings.

			end	end SF=	
E ₀	L ₀	C ₀	L _{pinch} /	DF/CF	
kJ	nH	uF	(L ₀ +L _a)		TYPE 1 PF
281	17.7	156	1.13	33.0	Poseidon
486	33.5	1332	1.03	12.0	PF1000
2.2	19.8	28.5	0.45	36.0	NX2
31.0	55.0	17.2	0.24	47.6	DPF78
0.4	40.0	1.0	0.113	78.1	PF400J
0.2	31.0	2.4	0.113	81.7	FMPF-1
					TYPE 2 PF
2.7	110.0	30.0	0.060	92.5	INTI PFUNUPF
1.8	123.0	12.5	0.048	126.8	KSU

Generally we see the trend that the bigger is the ratio $L_{pinch}/(L_0+L_a)$, the bigger is the current dip (seen from earlier figures of Type 1 and Type 2 fittings). When this ratio is small (primarily due to a large L_0 in the denominator), like in the case of KSU PF, the REGULAR radial phase RD is miniscule.

Next we note that from the *MJ PF* down to the sub *kJ PF*, because of the need to run these *PF*'s at about the same peak axial speed of 10 cm/μs to optimize, then the magnetic energy density per unit mass at the start of the radial phase is the same across the whole range of devices.

Thus a big RD drops the current a lot and strongly depletes the magnetic energy per unit mass at the end of the REGULAR radial phase. Hence a device with large ratio $L_{pinch}/(L_0+L_a)$ produces a big RD and ends up with relatively less energy per unit mass at the end of the REGULAR phase when compared to a device with a small value of $L_{pinch}/(L_0+L_a)$.

Thus a big RD generally tends to lead to a small ED; whereas a small RD is more conducive to lead to a larger ED.

**Type 1 (small L_0) has a big RD (& small ED) hence can be completely fitted
Type 2 (large L_0) has a small RD (& big ED)); cannot be completely fitted.**

We can strengthen this concept by asking the following question: How do we quantify the effects leading to ED?

This could be done by computing the end REGULAR radial phase drive factor $(I/a)/\rho^{0.5}$ also called the *SF*, where the current I , radius a and density (ρ) here are to be computed for the quantities prevailing at the end of the REGULAR radial phase.

We have done this for the range of machines, and note that for those machines we are tabulating, a machine with large $L_{pinch}/(L_0+L_a)$ tends to have small end REGULAR radial phase SF and vice versa. This is to be expected from the above reasoning.

Consideration of the end REGULAR radial phase strengthens above argument regarding Type 1 and Type 2

Finally, what should be done now since we cannot model KSU PF (and also the UNU ICTP PFF) using RADPF?

Well it is time to extend the model to a Phase 4a; for a start this could be done in a very approximate phenomenological manner. Alin Patran had already started on this trend with the introduction of an anomalous resistivity term in his NTU/NIE PhD thesis. In his case he only looked to compare the measured with the computed voltage spikes. We now need to model phase 4a so that the computed current trace will fit the measured current trace.

7.3.6 Conclusion

We can model Type 1 very well; and we now have a clear understanding of why the model is unable to completely fit for Type 2.

We have also started to develop a criteria for deciding the two categories of PF machines which we can classify as Type 1 and Type 2 devices.

7.3.7 Further work

We should now consider any simple method of modeling Phase 4a even if only phenomenologically e.g. by using a fitted form of anomalous resistance.

A paper has been submitted on the above T1 and T2 classification as follows:
Characterizing plasma focus devices- role of the static inductance- instability phase fitted by anomalous resistances- submitted to Plasma Physics and Controlled Fusion-
S Lee, S H Saw, A E Abdou and H Torreblanca

End of Module 7

End of Advanced Course in Plasma Focus Numerical Experiments

[Comments and interaction on the course work and other matters related to plasma focus are welcome at anytime]

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Reference to this course and the Lee model code should be given as follows:

Lee S. Radiative Dense Plasma Focus Computation Package (2010): RADPF
www.plasmafocus.net <http://www.intimal.edu.my/school/fas/UFLF/>

Also see [list of papers](#) (pg 21-22, pg 97-202)

*This Manual is prepared by S Lee and S H Saw of IPFS and INTI-IU for the Joint ICTP-IAEA Workshop on Dense Magnetized Plasmas and plasma Diagnostics
Plasma Focus Numerical Experiments*

Adapted from Manual for Collaborative Workshop for Kansas State University Plasma Focus Numerical Experiments 21 May-10 June 2010 and Manual for Fusion Energy Workshop International Conference for Nuclear and Renewable Energy Resources (NURER 2010) 4-7 July 2010, Ankara, Turkey with internet-based Part 1 from 15 April -11 May 2010

S Lee and S H Saw-
Melbourne and Kuala Lumpur 15 October 2010

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Description of Radiative Dense Plasma Focus Computation

Package RADPFV5.15dd - Lee Model code

(Supplementary Notes for Module 1)

Features

- **Numerical Experimental Facility**
- **Simulates any Mathers-type plasma focus, computes dynamics**
- **Design new plasma focus machines**
- **Thermodynamics included; 4 gases: H₂, D₂, Ne, Ar, Xe and He**
- **Model parameters to fit experimental axial, radial phase times**
- **Radiative phase computes line radiation, recombination and total yield. Computes neutron yield for deuterium operation; based on an improved beam-target model, calibrated at an experimental point. Plasma Self-absorption based on revised equations presented in File 3; appendix by N A D Khattak.**

Also includes:

Time guard feature

Choice of Tapered electrode

Quick choice of specified machines; one click loading of chosen machine;

at present 3 machines may be click-loaded: the UNU/ICTP PFF, the

NX2 and the PF1000

There are altogether 4 files in this package.

File1: PDF File "Description of Radiative Dense Plasma Focus Computation Package"

File2: PDF File "Theory of Radiative Plasma Focus Model"

File3: PDF file "Appendix by N A D Khattak".

File7: EXCEL file containing the ACTIVE SHEET AND THE PROGRAMME CODE. "Radiative Dense Plasma Focus Computation Code" RADPFV5.15

In addition, there are files for the computation of thermodynamic data needed for this code.

Hint for downloading the EXCEL FILE: Instead of left click to open the file; it is better to right click and select "save target as"; then choose a suitable location e.g. desktop. The saved EXCEL file will be only about 1M. (see last page for more hints on saving/copying)

These files may be downloaded from the following URL:

<http://www.intimal.edu.my/school/fas/UFLF/>

Introductory description

A simple 2 phase (axial and radial) model was developed by S Lee in 1983 as a component of a 3kJ plasma focus experimental package which became known as the UNU/ICTP PFF. This network of basically identical 3kJ PF machines, with different experimental and application emphases, is now operated by groups in countries including Singapore, Malaysia, Thailand, India, Pakistan,, Egypt and Zimbabwe.

The model was written as a 3 phase (non-radiative) model (in GWBASIC) for an experimental program at the 1991 Spring College in Plasma Physics at the ICTP.

The present 5-phase package (axial, radial inward shock, radial reflected shock, slow compression radiative and expanded large column phase) is re-written in Microsoft EXCEL VISUAL BASIC in order to make it available for wider usage.

The model may be adapted to any conventional Mather-type plasma focus by input of machine parameters: inductance, capacitance, electrode radii and length. And operating parameters: charging voltage and fill gas pressure. The thermodynamics (specific heat ratio and charge number as functions of temperature) are included for 6 gases namely hydrogen, deuterium, neon, argon and helium and xenon. The gases may be selected by simply inputting atomic number, molecular weight and dissociation number (2 for deuterium and hydrogen, 1 for the others).

The model has been used in many PhD and Masters Theses. It has also been used for various applications, for example, in the design of a cascading plasma focus (1991); and for estimating soft x-ray yield for the purpose of developing a SXR

source for microelectronics lithography (1997). More recently the code has been used to compute pinch current from measured total current waveform (2008). With this technique numerical experiments were run to obtain neutron scaling laws (2008). Use of the code also uncovered a Plasma Focus Pinch Current Limitation Effect (2008).

Five phases of the plasma focus are simulated by the Model code:

- 1 Axial Phase
- 2 Radial Inward Shock Phase
- 3 Radial Reflected Shock Phase
- 4 Slow Compression (Radiative) Phase
- 5 Expanded Column Axial Phase

The phases are illustrated by Fig 1 and Fig 2. More details may be obtained from:

<http://www.intimal.edu.my/school/fas/UFLF/>

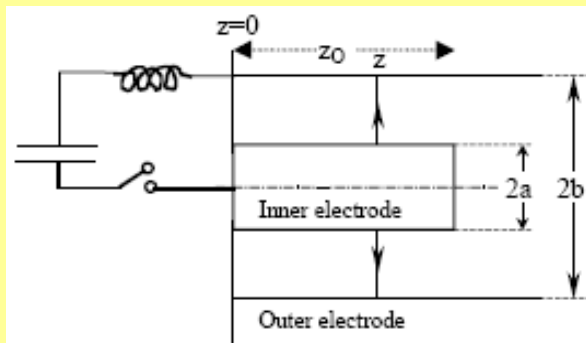


Fig 1 (a) Axial Phase

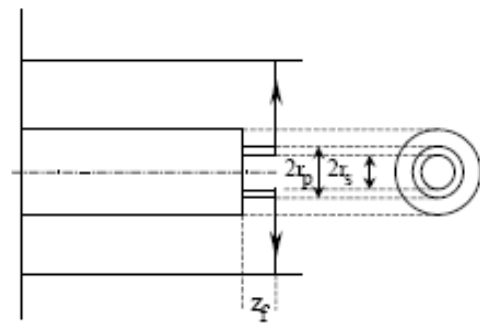


Fig 1 (b) Radial Phase

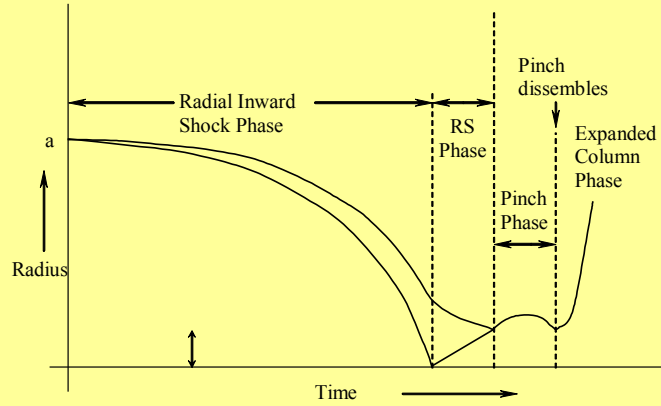


Fig 2. Schematic of radial phases

The five phases are summarised as follows (Theory and equations may be obtained from file 2 above):

1. Axial Phase: Described by a snowplow model with an equation of motion (incorporating axial phase model parameters: mass and current factors f_m and f_c) which is **coupled** to a circuit equation
2. Radial Inward Shock Phase (See Fig 1): Described by 4 **coupled** equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, also called the magnetic piston, allowing the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. The model parameters, radial phase mass and current factors f_{mr} and f_{cr} are incorporated in the radial phases. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock

front and current sheath due to the finite small disturbance speed is crucially implemented in this phase.

3. Radial Reflected Shock (RS) Phase: When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops which moves radially outwards, whilst the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the reflected shock moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the reflected shock undergoes a jump by a factor nearly 2.
4. Slow Compression (Quiescent) or Pinch Phase: When the out-going reflected shock hits the in-going piston the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three **coupled** equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects are incorporated into this phase. The duration of this slow compression phase is set as the time of transit of small disturbances across the the pinched plasma column. The computation of this phase is terminated at the end of this duration.
5. Expanded Column Phase: To simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column

inductance for further integration. In this final phase the snow plow model is used, and two coupled equations are used similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

[Note: Transition from Phase 4 to Phase 5 is observed to occur in an extremely short time. This is an important transition which merits efforts to include into the model. It would be an important next step]

Using the Code

Configuring the code

The code may be configured to any Mather-type plasma focus by inputting machine (bank and tube) parameters: inductance, capacitance, electrode radii and length; and operating parameters: charging voltage and fill gas pressure. The thermodynamics (specific heat ratio and charge number as functions of temperature) are included for 6 gases namely hydrogen, deuterium, neon, argon and helium and xenon. The gases may be selected by simply inputting atomic number, molecular weight and dissociation number (2 for deuterium and hydrogen, 1 for the others).

With the bank, tube and operating parameters specified; what remains is to specify the model parameters. As a first trial we may use: $f_m=0.08$, $f_c=0.7$, $f_{mr}=0.15$, $f_{cr}=0.7$.

Then we may run the code. The results are the following: waveforms for the total discharge current and tube voltage, axial phase trajectory and speed, radial trajectories for the shock front, current sheath and column length and the corresponding speeds, plasma temperature and radiation yields (Bremsstrahlung, line and recombination) and power; and thermodynamic quantities such as specific heat ratios and charge numbers. These are output in graphical as well as tabular forms. Also computed are plasma pinch current and neutron yield, and energy distributions, if required.

Note: **on the chronology of the development of the Lee model code**

1983: 2-phase model developed and presented by S Lee at the Spring College on Radiations in Plasmas, ICTP Trieste published in “Radiations in Plasmas” B McNamara, World Scientific pp 978-87; used in the development of the UNU/ICTP PFF and UNU, ICTP training programs and Colleges (1984, 1986 to 1991); used in PhD theses (T.Y.Tou 1986, K.H.Kwek 1988, J Ali 1990, S Mulyodrono 1993, A Serban 1995)

1991: Extension to 3-phase model (S Lee IEEE Trans Plasma Sci 1991); used for experimental program at the 1991 Spring College in Plasma Physics at the ICTP

1995: Implementation of finite small-disturbance speed correction in the radial shock phase first used in PhD thesis (Liu 1997). This is a major feature in the Lee model code.

Before this physics was implemented, radial speeds were a factor of nearly 2 too high compared with experiments. Completion of 5-phase model; used in other PhD theses (G Zhang 1999, B Shan 2000).

2000 After discussions with P Lee with a view to wider usage, the code was completely re-written in 2000 in Excel Visual Basics. Used in several recent PhD's since, notably (A Patran, D Wong, T Zhang) and in many papers. From 2003 onwards, plasma self-absorption and anode taper incorporated. Extension to Xenon.

2007 onwards: Intensive discussions with S H Saw (INTI UC) , P Lee (NTU/NIE), R S Rawat (NTU/NIE) and the AAAPT resulted in push to a new direction of applications of the code. Beam-target mechanism incorporated with realistic simulation of yield resulted in re-examination of neutron scaling laws. Plasma self-absorption and taper features completed. Technique to find I_{pinch} from measured I_{total} waveform published. A new effect of focus pinch current limitation was uncovered. All these activities resulted in the formation of Institute for Plasma Focus Studies to encourage correct usage and innovative applications of the Lee model code. Further development of the code is continuously undertaken. List of papers.

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(Supplementary Notes for Module 2 & 3)

Are the results any good?

Are there any indications that our computed results are anywhere near the actual results that may be measured on the device in actual operation?

NOT if we just guess the model parameters f_m , f_c , f_{mr} , f_{cr} . Then the results are just hypothetical; although with experience we may assign some reasonable values of the model parameters for the particular machine in its particular operating conditions. And the results may be useful for planning or designing purposes.

How do we make the results realistic?

The standard practice is to **fit the computed total current waveform to an experimentally measured total current waveform.**

From experience it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace.

The exact time profile of the total current trace is governed by the **bank parameters** namely capacitance C_o , external, or static inductance L_o and circuit resistance r_o , by the focus **tube geometry** namely electrode radii, outer 'b' and inner anode 'a', and the anode length ' z_o '; and on the **operational parameters** which are the charging voltage V_o and the fill pressure P_o and the fill gas. It also depends on the fraction of mass swept-up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The detailed profile of the discharge current during the pinch phase will also reflect the joule heating and radiative yields. At the end of the pinch phase the total current profile will also reflect the sudden transition of the current flow from a constricted pinch to a large column flow. Thus the discharge current powers all the

dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that **the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occurs in the various phases of the plasma focus.**

Our standard practice for any existing plasma focus is to obtain a measured current trace. Then we fit the computed current trace to the measured current trace. The fitting process involves adjusting the model parameters f_m , f_c , f_{mr} , f_{cr} one by one, or in combination until the computed current waveform fits the measured current waveform.

Once this fitting is done our experience is that the other computed properties including dynamics, energy distributions and radiation are all realistic.

Fitting computed current trace to experimental current trace of existing machine:

The main model parameters are the tube current flow factor CURRE (known to be 0.7 for most machines) and the mass swept-up factor (MASSF, for axial & MASSFR, for radial). First try model parameters are suggested in a table towards the right of the worksheet. These could be tried, but may be adjusted so that the time of focus, and the radial inward shock transit time, fit the experimentally observed times for each machine. The computed current trace is compared with the experimental current trace.

Features for comparison include current risetime and rising shape, peak current, current 'roll off' and dip, both shape and amplitude. Absolute values should be compared. Our experience with a number of machines shows that the fit is usually very good, occasionally almost exact..

The machine parameters and operating conditions should already have been determined and inputted into the active sheet. The model parameters are then adjusted, one by one, or in combination until best fit is obtained between the computed current trace and the experimental current trace.

First step is fitting the axial phase. This involves variation of f_m and f_c whilst observing the changes that appear on the resulting computed I_{total} trace in respect to the **risetime, rising shape and I_{peak}** ; and how these features compare with the corresponding features of the measured I_{total} trace. During this fitting an increase in f_c increases axial speed which increases dynamic resistance, thus lowering current magnitude on the rising slope. The greater rate of increase of tube inductance flattens out the rising slope. A decrease in f_m has almost the same effect. However a change in f_c has an additional subtle effect of changing the relative effect of the tube inductance. This means that increasing the speed by a certain amount by increasing f_c , then reducing it by exactly the same amount by a corresponding increase in f_m will not bring the I_{total} shape and magnitude back to the shape and value before either change is made. Thus one has to get each of f_m and f_c separately correct to get both the current shape and magnitude correct in the rising current profile.

Second step is fitting of the radial phases. We need particularly to understand the transition from the axial to the radial phase. For a plasma focus to work well, it is usually operated with a speed such that its axial run-down time is about equal to the risetime of the circuit with the device short-circuited across its back-wall. With the focus tube connected, the current risetime will be larger. At the same time the current trace is flattened out. In most cases this increased risetime will be cut short by the start of the radial phase. As this phase starts the current trace starts to roll over, at first imperceptibly, then clearly dipping and then dips sharply as the focus dynamics enters the severe pinch phase which absorbs a significant portion of the energy from the driving magnetic field. Thus, the second step in the fitting consists of adjusting f_{mr} and f_{cr} so that the computed current roll-over and the dip agree in shape, slope and extent of dip with the measured waveform.

[The rest of the notes may be left to be read in conjunction with the work of Part 3.]

Besides the model parameters, sometimes (when all else fails in the fitting process) the inductance (as published or given by the experimenters) needs to be adjusted. Very commonly the inductance L_o may be given as the short circuit bank inductance whereas it should be the ‘static’ inductance of the plasma focus; ie the inductance of the PF before the current sheet moves.

Adjustment to L_o is indicated when the computed current rise slope differs significantly from the measured slope. (adjustment to C_o will also

affect the current slope, but the value of C_o is usually more reliably given than that of L_o).

Usually also the value of stray resistance r_o needs to be guessed at as few experimenters determine this carefully if at all. We usually start with the value of r_o as 0.1 of $(L_o/C_o)^{0.5}$; and make small adjustment as necessary; noting that capacitor banks are such that the ratio of $RESF = r_o / (L_o/C_o)^{0.5}$ seldom goes below 0.05.

Sometimes, especially for PF's using very low values of C_o , it may also be necessary (when all else fails) to adjust the value of C_o (for sub-uF capacitor banks, the closely spaced connecting parallel plates and parallel connecting cables may actually significantly change the value of C_o).

In cases where there is very good fit in current profiles but the absolute values of currents don't match, it has been reasonable to suspect that the calibration constant for the current profile has been given wrongly by the experimenter. Calibration errors can be ascertained by checking the quantity of charge that has flowed out of the capacitor when the voltage across it has dropped to zero. If this quantity differs significantly from $(1/2)C_o V_o^2$; then the suspicion of calibration error is confirmed. Actually this checking is already implicit in the model.

In adjusting r_o we note that an increase of r_o lowers the current trace at all points proportionately. In adjusting L_o we note that increasing L_o lowers the slope of the rising current. When all values are properly adjusted and when f_m and f_c are correctly fitted, the measured rising profile of the computed I_{total} , usually up to the peak value I_{peak} , is found to fit the measured rising profile well in both shape and magnitude.

Two other points need to be noted^{6,7}. The measured I_{total} profile usually has a starting portion which seems to rise more slowly than the computed trace. This is due to the switching process during which, until fully switched, the spark gap presents additional resistance. It could also be compounded by the lift-off delay²². Practically this effect is compensated by shifting the whole computed trace forward in time, usually by a small amount around 50ns. A related note is that z_o may need to be reduced to account for the shape of the back-wall insulator.

A final remark in response to the general observation that the measured slope of the current dip towards the end of the radial phases is almost always steeper than can be reasonably fitted. This is indeed the case. All adjustments e.g. to L_o , C_o and r_o do not have the necessary short-time influence on this feature of the current trace. To steepen the dip slope the best we could do is to either decrease f_{mr} or increase f_{cr} ; however either of these

adjustments also tend to increase the computed depth of the dip; which often is already excessive. Moreover there are usually small but significant ‘bouncing’ features towards and beyond the bottom of the measured current dip. These features are not modeled. So the fitting has to accept the best compromise to achieve the ‘best’ fit. I tend to attribute this as a limitation of the model at this stage of its development.

Moreover this method of fitting the computed current to the measured current obviously depends on the actual plasma focus machine performing in accordance to the main features of the model. The plasma focus operated in the so-called ‘neutron optimised’ mode appears to be most suited for this model. For gases other than Deuterium, perhaps we can also identify range/modes of operations suitable for simulation with this model; e.g. a plasma focus in Neon operated to optimize SXR yield with a temperature around 100-400eV appears also to be very suited to this model code.

On the other hand, unoptimised machines, for example, may have axial phase current sheet so much fragmented that the axial phase model parameters just cannot be stretched for the model to fit the experiment. Or as another example, a plasma focus may be operated to optimize ion or electron beams; in which case conditions are manipulated for the instabilities to be so much enhanced that the radial model parameters cannot be stretched to simulate these effects. Such situations and range of operation may be outside the scope of this mode.

Despite these limitations, our experience show that the model may be used to compute plasma conditions and neutron and SXR yields with reasonable agreement over an unprecedented range of experiments, from sub-kJ PF400 (Chile) to low kJ NX2 (Singapore) and UNU/ICTP PFF (Network countries) all the way to the MJ PF1000.

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(Supplementary Notes SP3 for Module 3)

[in part extracted from [file 2Theory.pdf](#) from:
<http://www.intimal.edu.my/school/fas/UFLF/>] and from various papers

Radiation Terms

The Bremsstrahlung loss term may be written as:

$$\frac{dQ_B}{dt} = -1.6 \times 10^{-40} N_i^2 (\pi r_p^2) Z_f T^{1/2} Z^3$$

$$N_o = 6 \times 10^{26} \frac{\rho_o}{M}; \quad N_i = N_o f_{mr} \left(\frac{a}{r_p} \right)^2$$

Recombination loss term is written as:

$$\frac{dQ_{rec}}{dt} = -5.92 \times 10^{-35} N_i^2 Z^5 (\pi r_p^2) Z_f / T^{0.5}$$

The line loss term is written as:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z Z_n^4 (\pi r_p^2) Z_f / T$$

$$\text{and } \frac{dQ}{dt} = \frac{dQ_J}{dt} + \frac{dQ_B}{dt} + \frac{dQ_L}{dt} + \frac{dQ_{rec}}{dt}$$

where dQ/dt is the total power gain/loss of the plasma column.

By this coupling, if, for example, the radiation loss $\left(\frac{dQ_B}{dt} + \frac{dQ_L}{dt} \right)$ is severe, this would

lead to a large value of $\frac{dr_p}{dt}$ inwards. In the extreme case, this leads to radiation collapse, with r_p going rapidly to zero, or to such small values that the plasma becomes opaque to the outgoing radiation, thus stopping the radiation loss.

This radiation collapse occurs at a critical current of 1.6 MA (the Pease-Braginski current) for deuterium. For gases such as Neon or Argon, because of intense line radiation, the critical current is reduced to even below 100kA, depending on the plasma temperature.

Plasma Self Absorption and transition from volumetric emission to surface emission

Plasma self absorption and volumetric (emission described above) to surface emission of the pinch column have been implemented in the following manner.

The photonic excitation number (see [File 3 Appendix](#) by N A D Khattak) is written as follows:

$$M = 1.66 \times 10^{-15} r_p Z_n^{0.5} n_i / (Z T^{1.5}) \text{ with } T \text{ in eV, rest in SI units}$$

The volumetric plasma self-absorption correction factor A is obtained in the following manner:

$$A_1 = (1 + 10^{-14} n_i Z) / (T^{3.5})$$

$$A_2 = 1 / AB_1$$

$$A = A_2^{(1+M)}$$

Transition from volumetric to surface emission occurs when the absorption correction factor goes from 1 (no absorption) down to 1/e (e=2.718) when the emission becomes surface-like given by the expression:

$$\frac{dQ}{dt} = -const x Z_n^{3.5} Z^{0.5} (r_p) Z_f T^4$$

where the constant *const* is taken as 4.62×10^{-16} to conform with numerical experimental observations that this value enables the smoothest transition, in general, in terms of power values from volumetric to surface emission.

Where necessary another fine adjustment is made at the transition point adjusting the constant so that the surface emission power becomes the same value as the absorption corrected volumetric emission power at the transition point. Beyond the transition point (with A less than 1/e) radiation emission power is taken to be the surface emission power.

Neutron Yield

<http://www.intimal.edu.my/school/fas/UFLF/>

Adapted from the following papers (with modifications for erratum)

Pinch current limitation effect in plasma focus (This version includes an Erratum)

S. Lee and S. H. Saw, *Appl. Phys. Lett.* **92**, 021503 (2008), DOI:10.1063/1.2827579

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<http://link.aip.org/link/?APPLAB/92/021503/1>

Neutron Scaling Laws from Numerical Experiments (This version includes an Erratum)

S Lee and S H Saw, *J of Fusion Energy*, DOI: 10.1007/s10894-008-9132-7

published first online 20 February 2008 at <http://dx.doi.org/10.1007/s10894-008-9132-7>

"The original publication is available at www.springerlink.com."

Neutron yield is calculated with two components, thermonuclear term and beam-target term.

The thermonuclear term is taken as:

$$dY_{th} = 0.5n_i^2 (3.142) r_p^2 z_f <\sigma v> (\text{time interval})$$

where $<\sigma v>$ is the thermalised fusion cross section-velocity product corresponding to the plasma temperature, for the time interval under consideration. The yield Y_{th} is obtained by summing up over all intervals during the focus pinch.

The beam-target term is derived using the following phenomenological beam-target neutron generating mechanism¹⁷, incorporated in the present RADPFV5.13. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modeling each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with proportionality constant. The yield is then calibrated against a known experimental point.

The beam-target yield is written in the form: $Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$ where n_b is the number of beam ions per unit plasma volume, n_i is the ion density, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n- branch¹⁸, v_b the beam ion speed and τ is the beam-target interaction time assumed proportional to the confinement time of the plasma column.

Total beam energy is estimated¹⁷ as proportional to $L_p I_{pinch}^2$, a measure of the pinch inductance energy, L_p being the focus pinch inductance. Thus the number of beam ions is $N_b \sim L_p I_{pinch}^2 / v_b^2$ and n_b is N_b divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that⁴ $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$ where U is the disruption-caused diode voltage¹⁷. Here 'b' is the cathode radius. We also assume reasonably that U is proportional to V_{max} , the maximum voltage induced by the current sheet collapsing radially towards the axis.

$$\text{Hence we derive: } Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 ((\ln b/r_p)) \sigma / V_{max}^{1/2} \quad (1)$$

where I_{pinch} is the current flowing through the pinch at start of the slow compression phase; r_p and z_p are the pinch dimensions at end of that phase. Here C_n is a constant which in practice we will calibrate with an experimental point.

The D-D cross-section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute σ . The code computes V_{max} of the order of 20-50 kV. However it is known¹⁷, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50-150keV¹⁷, and for smaller lower-voltage machines the relevant energy¹⁹ could be lower at 30-60keV. Thus to align with experimental observations the D-D cross section σ is reasonably obtained by using beam energy equal to 3 times V_{max} .

A plot of experimentally measured neutron yield Y_n vs I_{pinch} was made combining all available experimental data^{2,4,12,13,17,19-22}. This gave a fit of $Y_n = 9 \times 10^{10} I_{pinch}^{3.8}$ for I_{pinch} in the range 0.1-1MA. From this plot a calibration point was chosen at 0.5MA, $Y_n = 7 \times 10^9$ neutrons. The model code²³ RADPFV5.13 was thus calibrated to compute Y_{b-t} which in our model is the same as Y_n .

Notes on **The total current and I_{peak} , the plasma current and I_{pinch}**

Extracted From: [Computing Plasma Focus Pinch Current from Total Current Measurement](#)

S. Lee, S. H. Saw, P. C. K. Lee, R. S. Rawat and H. Schmidt, *Appl Phys Letters* 92, 111501 (2008) DOI:10.1063/1.2899632

The total current I_{total} waveform in a plasma focus discharge is easily measured using a Rogowski coil. The peak value I_{peak} of this trace is commonly taken as a measure of the drive efficacy and is often used to scale the yield performance of the plasma focus. This is despite the fact that yields should more consistently be scaled to focus pinch current I_{pinch} , since it is I_{pinch} which directly powers the emission processes. The reason many researchers use I_{peak} instead of I_{pinch} for scaling is simply that while I_{peak} is easily measured, I_{pinch} , which is the value of the plasma sheath current I_p at time of pinch, is very difficult to measure even in large devices where it is possible to place magnetic probes near the pinch. This measurement is also inaccurate and perturbs the pinch. In a small device, there is no space for such a measurement.

The relationship between I_{pinch} and I_{peak} is not simple and has only been recently elaborated. It primarily depends on the value of the static inductance L_0 compared to the dynamic inductances of the plasma focus. As L_0 is reduced, the ratio $I_{\text{pinch}} / I_{\text{peak}}$ drops. Thus, yield laws scaled to I_{peak} will not consistently apply when comparing two devices with all parameters equal but differing significantly in L_0 . Better consistency is achieved when yield laws are scaled to I_{pinch} . In this paper, we propose a numerical method to consistently

Distinguishing the I_{total} waveform from the I_p waveform

A measured trace of I_{total} is commonly obtained with a Rogowski coil wrapped around the plasma focus flange through which is fed I_{total} discharged from the capacitor bank between the coaxial electrodes across the back wall. A part of I_{total} , being the plasma sheath current I_p , lifts off the back-wall insulator and drives a shock wave axially down the coaxial space. We denote f_c as the current fraction I_p / I_{total} for the axial phase and f_{cr} for the radial phases. In modeling it is found that a reasonable value for initial trial for f_c is 0.7 with a similar first trial value for f_{cr} . However in a DPF78 experiment f_c was found to vary from 0 at the start of the axial phase rising rapidly above 0.6 for the rest of the axial phase. In the radial phase f_{cr} was found to stay above 0.6 before dropping to 0.48 at the start of the pinch and then towards 0.4 as the pinch phase progressed. These Stuttgart results confirm a complex relationship between the waveforms of I_{total} and I_p .

The performance of a plasma focus is closely linked to the current I_{pinch} actually participating in the focus pinch phase rather than the total current flowing in the circuit. It is a common practice to take I_{peak} or some representative fraction of it as I_{pinch} . Another practice is to take the value of I_{total} at the time of the pinch as I_{pinch} . Whilst in their special cases this practice could be justifiable, the distinction of I_p from I_{total} should generally be clearly made. We emphasize that it should be the value of I_p at the time of pinch which is the relevant value for the purpose of yield scaling. The practice of associating yield

scaling with the total current waveform (i.e. taking I_{peak} or I_{total} at estimated pinch time) would be justifiable if there were a linear relationship between the waveforms of I_{total} and I_p . However as shown by the Stuttgart experiments the actual relationship is a very complex one which we may ascribe to the interplay of the various electro-dynamical processes including the relative values of static inductance L_o , tube inductance and the dynamic resistances which depend on the tube geometry and plasma sheath speeds. This relationship may change from one machine to the next. Whilst these electro-dynamical processes and other relevant ones such as radiation are amenable to modeling there are other machine effects such as back wall restriking (for example due to high induced voltages during the pinch phase) which can almost unpredictably affect the relationship between I_{total} and I_p during the crucial radial phases. Hence it is not only simplistic to discuss scaling in terms of the I_{total} waveform (i.e. taking I_{peak} or the value of I_{total} at the estimated time of pinch) but also inconsistent. One of the most important features of a plasma focus is its neutron production. The well-known neutron yield scaling, with respect to current, based on various compilations of experimental data, is $Y_n \sim I_{\text{pinch}}^x$ where x is varied in the range 3–5. In a recent paper, numerical experiments using a code was used to derive a scaling with $x = 4.7$. Difficulties in the interpretation of experimental data ranging across big and small plasma focus devices include the assignment of the representative neutron yield Y_n for any specific machine and the assignment of the value of I_{pinch} . In a few larger machines attempts were made to measure I_{pinch} using magnetic probes placed near the pinch region, with uncertainties of 20%. Moreover the probes would have affected the pinching processes. In most other cases related to yield scaling data compilation or interpretation I_{pinch} is simply assigned a value based on the measurement of peak total current I_{peak} or the value of total current at the observed current dip.

The difficulties in distinguishing I_{pinch} from I_{total} are obviated in numerical experiments using the Lee Model [In a typical simulation, the I_{total} trace is computed and fitted to a measured I_{total} trace from the particular focus. Three model parameters for fitting are used: axial mass swept-up factor f_m , current factor f_c and radial mass factor f_{mr} . A fourth model parameter, radial current factor, f_{cr} may also be used. When correctly fitted the computed I_{total} trace agrees with the measured I trace in peak amplitude, rising slope profile and topping profile which characterize the axial phase electro-dynamics. The radial phase characteristics are reflected in the roll-over of the current trace from the flattened top region, and the subsequent current drop or dip. Any machine effects, such as restrikes, current sheath leakage and consequential incomplete mass swept up, not included in the simulation physics is taken care of by the final choice of the model parameters, which are fine-tuned in the feature-by-feature comparison of the computed I_{total} trace with the measured I_{total} trace. Then there is confidence that the computed gross dynamics, temperature, density, radiation, plasma sheath currents, pinch current and neutron yield may also be realistically compared with experimental values.

A note on scaling:

Scaling of yields to say I_{pinch} should be carried out using yields which are at optimum, or at least near optimum. If one indiscriminately uses any data one may end up with

completely trivial or misleading results. For example if a point is used at too high or low pressure (away from the optimum pressure) then there may be zero yield ascribed to values of I_{pinch} .

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- ²³S Lee **Radiative Dense Plasma Focus Computation Package (2008) RADPF**:
<http://www.intimal.edu.my/school/fas/UFLF/>

Reference to this course and the Lee model code should be given according to the following format:

Lee S. Radiative Dense Plasma Focus Computation Package (2008): RADPF
www.plasmafocus.net www.intimal.edu.my/school/fas/UFLF/

References

Theoretical Basis: Plasma Focus Model (Radiative)-S Lee Model

<http://www.intimal.edu.my/school/fas/UFLF/>

(This revision, 17 March 2008, conforms to RADPFV5.13.8, including beam-target neutron yield and plasma self-absorption with smooth transition from volumetric to surface emission)

This model has been developed for Mather-type (1) plasma focus machines. It was developed for the 3kJ machine known as the UNU/ICTP PFF (2,3) (United Nations University/International Centre for Theoretical Physics Plasma Focus facility, which now forms an international network. In principal there is no limit to energy storage and electrode configuration, though house-keeping may need to be carried out in extreme cases, in order to keep within efficient ranges e.g. of graph plotting.

For details of the computing package, go back to the introductory section.

<http://www.intimal.edu.my/school/fas/UFLF/>

The model has been used for various applications, for example, in the design of a cascading plasma focus (Ref 4); and for estimating soft x-ray yield (Ref 5) for the purpose of developing a SXR source for microelectronics lithography (Ref 6); and recently in uncovering a pinch current limitation effect (Ref 7, 2008) , throwing new light on neutron scaling laws (Ref 8, 2008) and as an experimental technique (Ref 9, 2008) to compute focus pinch current from a measured discharge current waveform.

The 5-phase model is described in some detail in the following sections:

- 1 Axial Phase**
- 2 Radial Inward Shock Phase**
- 3 Radial Reflected Shock Phase**
- 4 Slow Compression (Radiative) Phase**
- 5 Expanded Column Axial Phase**

1

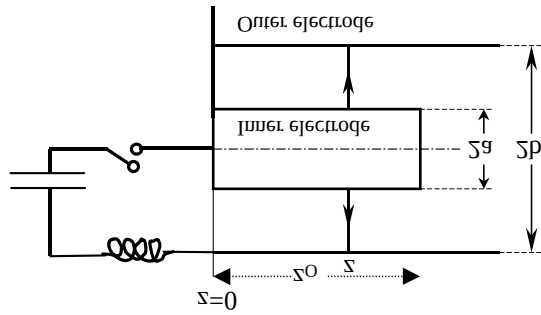


Fig 1 (a)

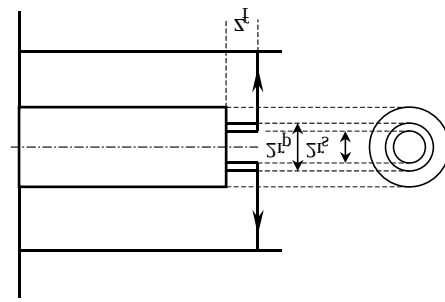


Fig 1 (b)

Rate of change of momentum at current sheath, position z , is

$$\frac{d(mv)}{dt} = \frac{d}{dt} \left([\rho_o \pi (b^2 - a^2) z] f_m \frac{dz}{dt} \right) = \rho_o \pi (c^2 - 1) a^2 f_m \frac{d}{dt} \left(z \frac{dz}{dt} \right)$$

Magnetic force on current sheath is

$$F = \int_a^b \left[\left(\frac{\mu I f_c}{2\pi r} \right)^2 / (2\mu) \right] 2\pi r dr = \frac{\mu f_c^2}{4\pi} \ln(c) I^2$$

f_m = fraction of mass swept down the tube in the axial direction

 f_c = fraction of current flowing in piston

Equation of motion:

$$\begin{aligned} \rho_o \pi (c^2 - 1) a^2 f_m \frac{d}{dt} \left(z \frac{dz}{dt} \right) &= \frac{\mu f c^2}{4\pi} (\ln c) I^2 \\ \therefore \frac{d^2 z}{dt^2} &= \left[\frac{f_c^2}{f_m} \frac{\mu (\ln c)}{4\pi^2 \rho_o (c^2 - 1)} \left(\frac{I}{a} \right)^2 - \left(\frac{dz}{dt} \right)^2 \right] / z \end{aligned} \quad \text{-- (I)}$$

Circuit (current) Equation

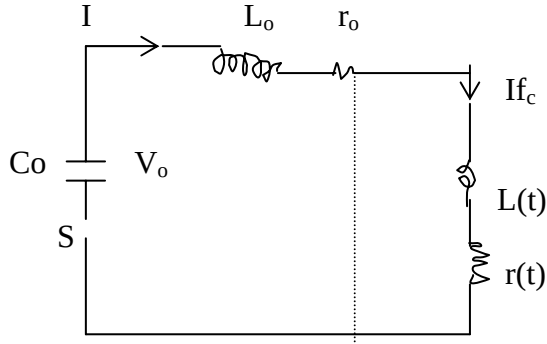


Fig 2 Circuit schematic

Ignore $r(t)$, plasma resistance. This is the approximation which is generally used for electromagnetic drive.

$$\begin{aligned} \frac{d}{dt}[(L_o + Lf_c)I] + r_o I &= V_o - \int \frac{Idt}{C_o} \\ (L_o + Lf_c) \frac{dI}{dt} + If_c \frac{dL}{dt} + r_o I &= V_o - \int \frac{Idt}{C_o} \quad \text{-- (II)} \\ \frac{dI}{dt} &= \left[V_o - \frac{\int Idt}{C_o} - r_o I - If_c \frac{\mu}{2\pi} (\ln c) \frac{dz}{dt} \right] / \left[L_o + \frac{f_c \mu}{2\pi} (\ln c) z \right] \end{aligned}$$

Equations (I) and II) are the generating equations of the model. They contain the physics built into the model.

They are coupled equations.

The equation of motion is affected by the electric current I .

The circuit equation is affected by the current sheath motion $\frac{dz}{dt}$ and position z .

Normalise the equations to obtain scaling parameters

Replace variables t, z, I by non-dimensionalised quantities as follows:

$$\tau = t/t_o, \quad \zeta = z/z_o, \quad \iota = I/I_o$$

where the normalising quantities t_o, I_o and Z_o are carefully chosen to be relevant, characteristic, convenient quantities, reflecting the physics of the problem.

Choices:

z_o is the length of the anode,

t_o is $\sqrt{L_o C_o}$ (noting that $2\pi\sqrt{L_o C_o}$ is the cycle time of L_o - C_o discharge circuit)

I_o is V_o/Z_o where $Z_o = \sqrt{L_o/C_o}$ is the surge impedance (noting that I_o is the peak current of the L_o - C_o discharge circuit with capacitor C_o charged initially to V_o .)
Normalising, we have:

Equation of motion:

$$\frac{z_o}{t_o^2} \frac{d^2 \zeta}{d\tau^2} = \left[\frac{f_c^2}{f_m} \frac{\mu \ln c}{4\pi^2 \rho_o (c^2 - 1)} \left(\frac{I_o}{a} \right)^2 t^2 - \frac{z_o^2}{t_o^2} \left(\frac{d\zeta}{d\tau} \right)^2 \right] / \zeta z_o$$

$$\frac{d^2 \zeta}{d\tau^2} = \left[\frac{f_c^2}{f_m} \frac{\mu \ln c}{4\pi^2 \rho_o (c^2 - 1)} \left(\frac{I_o}{a} \right)^2 \frac{t_o^2}{z_o^2} t^2 - \left(\frac{d\zeta}{d\tau} \right)^2 \right] / \zeta$$

which we write as

$$\frac{d^2 \zeta}{d\tau^2} = \frac{\left[\alpha^2 t^2 - \left(\frac{d\zeta}{d\tau} \right)^2 \right]}{\zeta} \quad \text{-- (I.1)}$$

Obtain first scaling parameter:

We note, by inspection,

$$\alpha^2 = t_o^2 / \left\{ \left[z_o^2 / (I_o / a)^2 \right] (f_m / f_c^2) \left[4\pi^2 \rho_o (c^2 - 1) / \mu \ln c \right] \right\}, \text{ which we thus define in this manner.}$$

By inspection of equation (I.1), we note α is dimensionless.

Hence since t_o has the dimension of time we may define a time value t_a where

$$t_a = \left[\frac{4\pi^2 (c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{\sqrt{f_m}}{f_c} \frac{z_o}{(I_o / a) / \sqrt{\rho}}$$

identifying this quantity as the characteristic axial transit time of the CS down the anode axial phase.

We may then think of α as:

$$\alpha = (t_o/t_a) - \text{scaling parameter.}$$

ratio of characteristic electrical discharge time to characteristic axial transit time.

We may further identify a characteristic axial transit speed $V_a = z_o/t_a$

$$v_a = \left[\frac{\mu \ln c}{4\pi^2 (c^2 - 1)} \right]^{1/2} \frac{f_c}{\sqrt{f_m}} \frac{(I_o / a)}{\sqrt{\rho}}$$

The quantity $\left(I_o/a\right)/\sqrt{\rho}$ is the S (speed or drive) factor of electromagnetically driven devices, focus, pinches etc.

Normalising the circuit (current) Equation, we have:

$$\frac{I_o}{t_o} \frac{d\iota}{d\tau} = \left[v_o - \frac{I_o t_o}{c_o} \int \iota d\tau - r_o I_o \iota - f_c \frac{\mu}{2\pi} (\ln c) I_o \frac{z_o}{t_o} \iota \frac{d\zeta}{d\tau} \right] / \left[L_o + \frac{f_c \mu}{2\pi} (\ln c) z_o \zeta \right]$$

and substituting in $I_o = V_o / \sqrt{L_o / C_o}$, $t_o = \sqrt{L_o C_o}$, we have

$$\frac{d\iota}{d\tau} = \left[1 - \int \iota d\tau - f_c \left[\left(\frac{\mu}{2\pi} (\ln c) z_o \right) / L_o - (r_o / Z_o) \right] \iota \frac{d\zeta}{d\tau} \right] \left[1 + f_c \left[\frac{\mu}{2\pi} (\ln c) z_o \right] \zeta / L_o \right]$$

$$\text{write: } \frac{d\iota}{d\tau} = \left(1 - \int \iota d\tau - \beta \iota \frac{d\zeta}{d\tau} - \delta \iota \right) / (1 + \beta \zeta) \quad \text{-- (II.1)}$$

Second scaling parameter

We note $L_a = \frac{f_c \mu}{2\pi} (\ln c) z_o$ is the inductance of the axial phase when CS reaches the end $z = z_o$.

Hence $\beta = \frac{L_a}{L_o}$ is the ratio of load to source inductance and since the device is electromagnetic, the electrodynamics is determined strongly by this scaling parameter.

The third scaling parameter $\delta = r_o / Z_o$ is the ratio of circuit stray resistance to surge impedance. This acts as a damping effect on the current.

(I.1) and (II.1) are the Generating Equations that may be integrated step-by-step.

Calculate voltage across input terminals of focus tube:

$$V = \frac{d}{dt}(L I f_c) = f_c I \frac{dL}{dt} + f_c L \frac{dI}{dt} \quad \text{where} \quad L = \frac{\mu}{2\pi} (l_n c) \quad \text{-- (II.10)}$$

$$\text{Normalized form } v = \frac{V}{V_o} = \beta \iota \frac{d\zeta}{d\tau} + \beta \zeta \frac{d_c}{d\tau} \quad \text{-- (II.11)}$$

Integration

Define initial conditions:

$$\tau = 0, \quad \frac{d\zeta}{d\tau} = 0, \quad \zeta = 0, \quad \iota = 0, \quad \int \iota d\tau = 0, \quad \frac{d\iota}{d\tau} = 1, \quad \frac{d^2 \zeta}{d\tau^2} = \alpha \sqrt{2/3}$$

Set time increment: D = 0.001

Increment time: $\tau = \tau + D$

Next step values are computed using the following linear approximations:

$$\frac{d\zeta}{d\tau} = \frac{d\zeta}{d\tau} + \frac{d^2\zeta}{d\tau^2} D$$

$$\zeta = \zeta + \frac{d\zeta}{d\tau} D$$

$$\iota = \iota + \frac{d\iota}{d\tau} D$$

$$\int \iota d\tau = \int \iota d\tau + \iota D$$

Use new values of $\frac{d\zeta}{d\tau}$, ζ , ι and $\int \iota d\tau$ to calculate new generating values of $\frac{d\iota}{d\tau}$ and $\frac{d^2\zeta}{d\tau^2}$ using generating equs (I.1) and (II.1).

Increment time again and repeat calculations of next step values and new generating values.

Continue procedure until $\zeta = 1$.

Then go on to radial phase inward shock computations.

2 Radial Inward Shock Phase (Slug model)

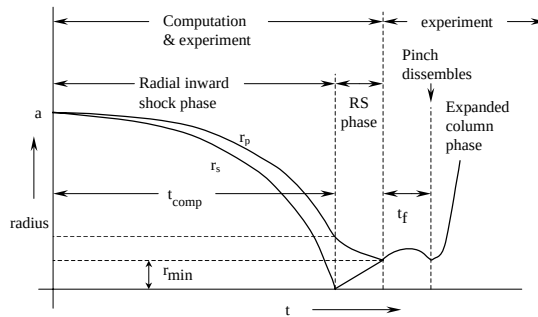


Fig 3. Schematic of radial phases

The snowplow model is used for axial phase just to obtain axial trajectory and speed (from which temperature may be deduced) and to obtain reasonable current profile. As the CS is assumed to be infinitesimally thin, no information of density is contained in the physics of the equation of motion, although an estimate of density may be obtained by invoking additional mechanisms e.g. using shock wave theory.

In the radial phase however, a snowplow model (with an infinitesimally thin CS) would eventually (in the integration) lead to all current flowing at $r = 0$, with infinite inductance and density.

We thus replace the snow plow model by a slug model . In this model, the magnetic pressure drives a shock wave ahead of it, creating a space for the magnetic piston (CS) to move into.

The speed of the inward radial shock front (see Fig 1b) is determined by the magnetic pressure (which depends on the drive current value and CS position r_p).

The speed of the magnetic piston (CS) is determined by the first law of thermodynamics applied to the effective increase in volume between SF and CS, created by the incremental motion of the SF.

The compression is treated as an elongating pinch.

Four generating equations are needed to describe the motion of (a) SF, (b) CS (c) pinch elongation and the electric current (d); to integrate for the four variables r_s , r_p , z_f & I .

Motion of Shock Front:

r_p piston	r_s SF
P_m vacuum	P ρ T
	ρ_o, P_o, T_o

From Shock Wave theory, shock pressure $P = \frac{2}{\gamma+1} \rho_o v_s^2$ where shock speed v_s into ambient gas ρ_o causes the pressure of the shocked gas (just behind the shock front) to rise to value P .

If we assume that this pressure is uniform from the SF to the CS (infinite acoustic [small disturbance] speed approximation) then across the piston, we may apply $P = P_m$ where

$$P_m = (\mu I f_c / 2\pi r_p)^2 / 2\mu$$

$$\text{Thus: } v_s^2 = \frac{\mu (I f_c)^2}{8\pi^2 r_p} \times \frac{\gamma+1}{2\rho_o f_{mr}}$$

where I is the circuit current and $I f_c$ is the current flowing in the cylindrical CS, taken as the same f_c as in the axial phase, and $\rho_o f_{mr}$ is the effective mass density swept into the radial slug; where f_{mr} is a different (generally larger) factor than f_m of the axial phase.

$$\text{Thus } \frac{dr_s}{dt} = - \left[\frac{\mu(\gamma+1)}{\rho_o} \right]^{1/2} \frac{f_c}{\sqrt{f_{mr}}} \frac{I}{4\pi r_p} \quad \text{-- (III)}$$

Elongation speed of CS (open-ended at both ends)

The radial compression is open at one end. Hence an axial shock is propagated in the z -direction, towards the downstream anode axis. We take z_f as the position of the axial CS (rather than the SF). The pressure driving the axial shock is the same as the pressure driving the inward radial shock. Thus the axial shock speed is the same as the radial shock speed. The CS speed is slower, from shock wave theory, by an approximate factor of $2/(\gamma+1)$. Thus the axial elongation speed of the CS is:

$$\frac{dz_f}{dt} = - \left(\frac{2}{\gamma + 1} \right) \frac{dr_s}{dt} \quad \text{-- (IV)}$$

In this modelling we treat the elongation in a very approximate fashion, as its effect on the compressing column is relatively secondary. The main mechanism controlling the state of the plasma column is the radial compression. The radial CS (piston) speed is hence treated with more care as follows:

Radial piston motion

We inquire:

For an incremental motion, dr_s , of the shock front, at a driving current I , what is the relationship between plasma slug pressure P and plasma slug volume V ?

We assume an adiabatic relationship (7) (infinite small disturbance speed – for which we will apply a correction subsequently) to a fixed mass of gas in the slug during the incremental motion dr_s . We have

$PV^\gamma = \text{constant}$ or

$$\frac{\gamma dV}{V} + \frac{dP}{P} = 0$$

where slug pressure $P \sim v_s^2$

$$\text{so } \frac{dP}{P} = \frac{2dv_s}{v_s}$$

but $v_s \sim \frac{I}{r_p}$ (see section on Motion of Shock Front, above)

$$\text{so } \frac{dP}{P} = 2 \left(\frac{dI}{I} - \frac{dr_p}{r_p} \right)$$

Now slug volume $V = \pi (r_p^2 - r_s^2) z_f$

and at first sight $dV = 2\pi (r_p dr_p - r_s dr_s) z_f + \pi (r_p^2 - r_s^2) dz_f$ – not correct!

But here we note that although the motion of the piston dr_p does not change the mass of gas in the slug, the motion of the shock front, dr_s , does sweep in an amount of ambient gas.

This amount swept in is equal to the ambient gas swept through by the shock front in its motion dr_s . This swept-up gas is compressed by a ratio $(\gamma+1)/(\gamma-1)$ and will occupy part of the increase in volume dV .

The actual increase in volume available to the original mass of gas in volume V does not correspond to increment dr_s but to an effective (reduced) increment drs ($2/(\gamma+1)$). (Note γ is specific heat ratio of

the plasma e.g. $\gamma = 5/3$ for atomic gas, $\gamma = 7/5$ for molecular gas; for strongly ionising argon γ has value closer to 1 e.g. 1.15.)

Thus, the more correct interpretation is:

$$dV = 2\pi \left(r_p dr_p - \frac{2}{\gamma+1} r_s dr_s \right) z_f + \pi (r_p^2 - r_s^2) dz_f$$

$$\text{Thus we have: } \frac{\gamma dV}{V} = \frac{2\gamma \left(r_p dr_p - \frac{2}{\gamma+1} r_s dr_s \right) z_f + \gamma (r_p^2 - r_s^2) dz_f}{z_f (r_p^2 - r_s^2)}$$

and adding together dP/P and $\gamma dV/V$ we have

$$\frac{2\gamma \left(r_p dr_p - \frac{2}{\gamma+1} r_s dr_s \right) z_f + \gamma (r_p^2 - r_s^2) dz_f}{z_f (r_p^2 - r_s^2)} + 2 \frac{dI}{I} - \frac{2 dr_p}{r_p} = 0$$

Rearranging and putting dr_p as the subject we have

$$\frac{dr_p}{dt} = \frac{\frac{2}{\gamma+1} \frac{r_s}{r_p} \frac{dr_s}{dt} - \frac{r_p}{\gamma I} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dI}{dt} - \frac{1}{(\gamma+1) z_f} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma} \frac{r_s^2}{r_p^2}} \quad \text{-- (V)}$$

where we are reminded r_p = radial piston position
 r_s = radial shock front position
 z_f = axial piston position

Circuit Equation during radial phase

The inductance of the focus tube now consists of the full inductance of the axial phase and the inductance of the radially imploding & elongating plasma pinch.

$$\text{Thus } L = \frac{\mu}{2\pi} (\ln c) z_o + \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) z_f \text{ where both } z_f \text{ and } r_p \text{ vary with time.}$$

Thus the circuit (current) equation is

$$\left\{ L_o + f_c \frac{\mu}{2\pi} (\ln c) z_o + f_c \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) z_f \right\} \frac{dI}{dt} + f_c I \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) \frac{dz_f}{dt} - f_c I \frac{\mu}{2\pi} \frac{z_f}{r_p} \frac{dr_p}{dt} + r_o I = V_o - \frac{\int Idt}{C_o}$$

$$\text{Giving } \frac{dI}{dt} = \frac{V_o - \frac{\int Idt}{C_o} - r_o I - f_c \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) I \frac{dz_f}{dt} + f_c \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dt}}{L_o + f_c \frac{\mu}{2\pi} (\ln c) z_o + f_c \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) z_f} \quad \text{-- (VI)}$$

Generating equations (III), (IV), (V), (VI) form a close set of equations which may be integrated for r_s , r_p , z_f and I .

Normalization

For this phase the following normalization is adopted.

$\tau = t/t_o$, $\iota = I/I_o$ as in axial phase but with $\kappa_s = r_s/a$, $\kappa_p = r_p/a$, $\zeta_f = z_f/a$ ie. distances are normalized to anode radius, instead of anode length.

After normalization we have:

$$\text{Radial shock speed} \quad \frac{d\kappa_s}{d\tau} = -\alpha\alpha_1 \iota / \kappa_p \quad \text{-- (III.1)}$$

Axial column elongation speed (both ends of column defined by axial piston)

$$\frac{d\zeta_f}{d\tau} = -\frac{2}{\gamma+1} \frac{d\kappa_s}{d\tau} \quad \text{-- (IV.1)}$$

$$\text{Radial piston speed: } \frac{d\kappa_p}{d\tau} = \frac{\frac{2}{\gamma+1} \frac{\kappa_s}{\kappa_p} \frac{d\kappa_s}{d\tau} - \frac{\kappa_p}{\gamma\iota} \left(1 - \frac{\kappa_s^2}{\kappa_p^2} \right) \frac{d\iota}{d\tau} - \frac{1}{\gamma+1} \frac{\kappa_p}{\zeta_f} \left(1 - \frac{\kappa_s^2}{\kappa_p^2} \right) \frac{d\zeta_f}{d\tau}}{(\gamma-1)/\gamma + (1/\gamma)(\kappa_s^2/\kappa_p^2)} \quad \text{-- (V.1)}$$

$$\text{current: } \frac{d\iota}{d\tau} = \frac{1 - \int \iota d\tau + \beta_1 [\ln(\kappa_p/c)] \iota \frac{d\zeta_f}{d\tau} + \beta_1 \frac{\zeta_f \iota}{k_p} \frac{d\kappa_p}{d\tau} - \delta \iota}{\{1 + \beta - (\beta_1) [\ln(\kappa_p/c)] \zeta_f\}} \quad \text{(VI.1)}$$

where the **scaling parameters** are $\beta_1 = \beta/(F \ln c)$, $F = z_o/a$ and

$$\alpha_1 = \left[(\gamma+1)(c^2-1)/(4 \ln c) \right]^{1/2} F [f_m/f_{mr}]^{1/2}$$

Note that whereas we interpret $\alpha = t_o/t_a$, we may interpret $\alpha_1 = t_a/t_r$ where t_r is a characteristic radial transit time.

The **scaling parameter** $\alpha\alpha_1$ may then be interpreted as $\alpha\alpha_1 = \frac{t_o}{t_a} = t_o/t_r$

We note that α_1 the ratio of characteristic axial transit to characteristic radial compression inward shock transit time is essentially a geometrical ratio

$$F \left[(c^2 - 1) / 4 \ln c \right]^{1/2} \simeq 20 \quad (\text{if } F \simeq 16 \text{ and } c \simeq 3)$$

(ie. axial transit time is characteristically 20 times longer than radial shock transit) modified by the thermodynamic term $(\gamma + 1)^{1/2}$ and the mass swept up ratio $(f_m / f_{mr})^{1/2}$. Including all 3 factors, the ratio of axial to radial characteristic times is typically 40.

We also note from the scaling parameter $\alpha\alpha_1$ that

$$t_r = \frac{4\pi}{[\mu(\gamma + 1)]^{1/2}} \frac{\sqrt{f_{mr}}}{f_c} \frac{a}{(I_o / a / \sqrt{\rho})}$$

and characteristic speed of inward shock to reach focus axis is:

$$v_r = a / t_r = \frac{[\mu(\gamma + 1)]^{1/2}}{4\pi} \frac{f_c}{\sqrt{f_{mr}}} \frac{(I_o / a)}{\sqrt{\rho}}$$

The ratio of characteristic radial and axial speeds is also essentially a geometrical one, modified by

thermodynamics. It is $v_r / v_a = \left[\frac{(c^2 - 1)(\gamma + 1)}{4 \ln c} \right]^{1/2}$ with a value typically 2.5.

Note that the radial characteristic speed has the same dependence as the axial transit speed on drive factor $S = (I_o / a) / \sqrt{\rho}$.

Calculate voltage V across PF input terminals

As in the axial phase, the voltage is taken to have only inductive component.

$$V = \frac{d}{dt} (LI)$$

$$\text{where } L = \frac{\mu}{2\pi} (\ln c) z_o + \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) z_f$$

$$V = \frac{\mu}{2\pi} \left[(\ln c) z_o + \left(\ln \frac{b}{r_p} \right) z_f \right] f_c \frac{dI}{dt} + \frac{\mu}{2\pi} \left[\left(\ln \frac{b}{r_p} \right) \frac{dz_f}{dt} - \frac{z_f}{r_p} \frac{dr_p}{dt} \right] f_c I \quad - (VI.10)$$

We may also write in normalised form $v = V/V_o$
(normalised to initial capacitor voltage V_o)

$$v = \left[\beta - \beta_1 \left(\ln \frac{\kappa_p}{c} \right) \zeta_f \right] \frac{d\iota}{d\tau} - \beta_1 \iota \left[\left(\frac{\zeta_f}{\kappa_p} \right) \frac{d\kappa_p}{d\tau} + \left(\ln \frac{\kappa_p}{c} \right) \frac{d\zeta_f}{d\tau} \right] \quad - (VI.12)$$

The generating equations (III.1), (IV.1), (V.1), (VI.1) may now be integrated using the following initial conditions:

τ = the time that axial phase ended

$$\kappa_s = 1$$

$$\kappa_p = 1$$

$\zeta_f = 0$ (taken as a small number such as 0.00001 to avoid numerical difficulties for equation V.1)

ι = value of current at the end of the axial phase.

$\int \iota d\tau$ = value of ‘flowed charge’ at end of axial phase.

The integration (step-by-step) may proceed with the following algorithm: (taking smaller time increment of $D = 0.001/100$)

Using initial values (above) of κ_s , κ_p , ζ_f and ι

$\frac{d\kappa_s}{d\tau}$, $\frac{d\zeta_f}{d\tau}$, $\frac{d\kappa_p}{d\tau}$ and $\frac{d\iota}{d\tau}$ are sequentially calculated from generating equation (III.1), (IV.1), (V.1), (VI.1).

$$\kappa_s = \kappa_s + \frac{d\kappa_s}{d\tau} D$$

$$\zeta_f = \zeta_f + \frac{d\zeta_f}{d\tau} D$$

Then sequentially using linear approximation: $\kappa_p = \kappa_p + \frac{d\kappa_p}{d\tau} D$

$$\iota = \iota + \frac{d\iota}{d\tau} D$$

$$\int \iota d\tau = \int \iota d\tau + \iota D$$

Time is then incremented by D , and the next step value of $\frac{d\kappa_s}{d\tau}$, $\frac{d\zeta_f}{d\tau}$, $\frac{d\kappa_p}{d\tau}$, $\frac{d\iota}{d\tau}$ are computed from (III.1), (IV.1), (V.1) and (VI.1), followed by linear approximation for κ_s , ζ_f , κ_p , ι and $\int \iota d\tau$.

The sequence is repeated step-by-step until $\kappa_s = 0$.

Correction for finite acoustic (small disturbance) speed.

In the slug model above we assume that the pressure exerted by the magnetic piston (current I , position r_p) is instantaneously felt by the shock front (position r_s). Likewise the shock speed $\frac{dr_s}{dt}$ is instantaneously felt by the piston (CS). This assumption of infinite small disturbance speed (SDS) is implicit in equations (III) and (V) (or in normalised form (III.1) and (V.1)).

Since the SDS is finite, there is actually a time lapse Δt communicating between the SF and CS. This communication delay has to be incorporated into the model. Otherwise for the PF, the computation will yield too high values of CS and SF speed.

Consider the instant t , SF is at r_s , CS at r_p , value of current is I . SF actually feels the effect of the current not of value I but of a value I_{delay} which flowed at time $(t-\Delta t)$, with the CS at $r_{p\text{delay}}$. Similarly the piston ‘thinks’ the SF speed is not $\frac{dr_s}{dt}$ but $\left(\frac{dr_s}{dt}\right)_{\text{delay}}$ at time $(t-\Delta t)$.

To implement this finite SDS correction we adopt the following procedure:

Calculate the SDS, taken as the acoustic speed.

$$SDS = \left(\frac{\gamma P}{\rho} \right)^{1/2} \text{ or } \left(\frac{\gamma R_o}{M} D_c T \right)^{1/2}$$

$$\text{or } \left(\frac{\gamma D_c k T}{M m_i} \right)^{1/2}$$

where γ = specific heat ratio, M = Molecule Weight,

R_o = universal Gas constant = 8×10^3 (SI units)

m_i = mass of proton, k = Boltzmanns constant.

D_c = departure coefficient = $DN (1+z)$

where Z , here, is the effective charge of the plasma

$$Z = \sum_r^J r \alpha_r, \text{ summed over all ionization levels } r = 1 \dots J.$$

DN = dissociation number, e.g. for Deuterium $DN = 2$, whereas for argon $DN = 1$.

The temperature T may be computed for the shocked plasma as

$$T = \frac{M}{RoD} \frac{2(\gamma-1)}{(\gamma+1)^2} \left(\frac{dr_s}{dt} \right)^2$$

Calculate the communication delay time as

$$\Delta T = (r_p - r_s) / SDS$$

In our programme using the Microsoft EXCEL VISUAL BASIC, data of the step-by-step integration is stored row-by-row, each step corresponding to one row. Thus the ΔT may be converted to Δ (row number) by using Δ (row number) = $\Delta T / (\text{timestep increment})$ this Δ (row number being, of course, rounded off to an integer.

The correction then involves ‘looking back’ to the relevant row number to extract the corrected

$$\text{values of } I_{\text{delay}}, r_{\text{pdelay}}, \left(\frac{dr_s}{dt} \right)_{\text{delay}}.$$

Thus in the actual numerical integration, in equation (III.1), ι and κ_p are replaced by ι_{delay} and κ_{pdelay}

and in equation (V.1) $\frac{dk_s}{d\tau}$ is replaced by $\left(\frac{dk_s}{d\tau} \right)_{\text{delay}}$

Radial Reflected Shock Phase

When the inward radial shock hits the axis, $\kappa_s = 0$. Thus in the computation, when $\kappa_s \leq 1$ we exit from radial inward shock phase. We start computing the RS phase.

The RS is given a constant speed of 0.3 of on-axis inward radial shock speed.

In this phase computation is carried out in real (SI) units.

Reflected Shock Speed:

$$\frac{dr_r}{dt} = -0.3 \left(\frac{dr_s}{dt} \right)_{on-axis}$$

Piston speed:

$$\frac{dr_p}{dt} = \frac{-\frac{r_p}{\gamma I} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dI}{dt} - \frac{1}{\gamma + 1} \frac{r_p}{z_f} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma - 1}{\gamma} + \frac{1}{\gamma} \frac{r_s^2}{r_p^2}}$$

Use the same equation as V except put $\left(\frac{dr_s}{dt} \right) = 0$ and $r_s=0$

Elongation speed:

Use same equation as Eq IV.

$$\frac{dz_f}{dt} = - \left(\frac{2}{\gamma + 1} \right) \left(\frac{dr_s}{dt} \right)_{on-axis}$$

Circuit Equation:

Use the same equation as Eq VI.

$$\frac{dI}{dt} = \frac{V_o - \frac{\int Idt}{C_o} - r_o I - f_c \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) I \frac{dz_f}{dt} + f_c \frac{\mu}{2\pi} \frac{z_f}{r_p} I \frac{dr_p}{dt}}{L_o + f_c \frac{\mu}{2\pi} (\ln c) z_o + f_c \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) z_f}$$

Continue integrating seamlessly.

Tube Voltage

Use the same equation as Eq (VI.10).

$$V = \frac{\mu}{2\pi} \left[(\ln c) z_o + \left(\ln \frac{b}{r_p} \right) z_f \right] f_c \frac{dI}{dt} + \frac{\mu}{2\pi} \left[\left(\ln \frac{b}{r_p} \right) \frac{dz_f}{dt} - \frac{z_f}{r_p} \frac{dr_p}{dt} \right] f_c I$$

In this phase as the RS (position r_r) moves outwards, the piston (position r_p) continues moving inwards.

Eventually r_r increases until its value reaches the decreasing value of r_p .

We make the assumption that the RS is sufficiently attenuated when it reaches the piston, so that its overpressure is negligible.

In that case, the piston may not be pushed outwards, but will continue to move inwards, although its inward speed may be gradually reduced.

4 Slow Compression Phase

In this phase the piston speed is:

$$\frac{dr_p}{dt} = \frac{-\frac{r_p}{\gamma I} \frac{dI}{dt} - \frac{1}{\gamma+1} \frac{r_p}{z_f} \frac{dz_f}{dt} + \frac{4\pi(\gamma-1)}{\mu\gamma z_f} \frac{r_p}{f_c^2 I^2} \frac{dQ}{dt}}{\frac{\gamma-1}{\gamma}} \quad \text{-(XX)}$$

Here we have included energy loss/gain terms into the equation of motion.

The plasma gains energy from Joule heating;
and loses energy through Bremsstrahlung & line radiation.
Energy gain term will tend to push the piston outwards.
Energy loss term will have the opposing effect.

Using Spitzer form for resistivity, for the plasma column:

To estimate the temperature, T, we use:

$$\frac{dQ_J}{dt} = RI^2 f_c^2 \quad \text{where} \quad R = \frac{1290Zz_f}{\pi r_p^2 T^{3/2}} \quad \text{where}$$

$$T = \frac{\mu}{8\pi^2 k} I^2 f_c^2 / (DN_o a^2 f_{mr})$$

Radiation Terms

The Bremsstrahlung loss term may be written as:

$$\frac{dQ_B}{dt} = -1.6 \times 10^{-40} N_i^2 (\pi r_p^2) Z_f T^{1/2} Z^3$$

$$N_o = 6 \times 10^{26} \frac{\rho_o}{M}; \quad N_i = N_o f_{mr} \left(\frac{a}{r_p} \right)^2$$

Recombination loss term is written as:

$$\frac{dQ_{rec}}{dt} = -5.92 \times 10^{-35} N_i^2 Z^5 (\pi r_p^2) Z_f / T^{0.5}$$

The line loss term is written as:

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} N_i^2 Z Z_n^4 (\pi r_p^2) Z_f / T$$

$$\text{and } \frac{dQ}{dt} = \frac{dQ_J}{dt} + \frac{dQ_B}{dt} + \frac{dQ_L}{dt} + \frac{dQ_{rec}}{dt}$$

where dQ/dt is the total power gain/loss of the plasma column

By this coupling, if, for example, the radiation loss $\left(\frac{dQ_B}{dt} + \frac{dQ_L}{dt} \right)$ is severe, this would lead to a

large value of $\frac{dr_p}{dt}$ inwards. In the extreme case, this leads to radiation collapse, with r_p going rapidly to zero, or to such small values that the plasma becomes opaque to the outgoing radiation, thus stopping the radiation loss.

This radiation collapse occurs at a critical current of 1.6 MA (the Pease-Braginski current) for deuterium. For gases such as Neon or Argon, because of intense line radiation, the critical current is reduced to even below 100kA, depending on the plasma temperature.

Plasma Self Absorption and transition from volumetric emission to surface emission

Plasma self absorption and volumetric (emission described above) to surface emission of the pinch column have been implemented in the following manner.

The photonic excitation number (see File 3 Appendix by N A D Khattak) is written as follows:

$$M = 1.66 \times 10^{-15} r_p Z_n^{0.5} n_i / (Z T^{1.5}) \text{ with } T \text{ in eV, rest in SI units}$$

The volumetric plasma self-absorption correction factor A is obtained in the following manner:

$$A_1 = (1 + 10^{-14} n_i Z) / (T^{3.5})$$

$$A_2 = 1 / A B_1$$

$$A = A_2^{(1+M)}$$

Transition from volumetric to surface emission occurs when the absorption correction factor goes from 1 (no absorption) down to $1/e$ ($e=2.718$) when the emission becomes surface-like given by the expression:

$$\frac{dQ}{dt} = -const x Z_n^{3.5} Z^{0.5} (r_p) Z_f T^4$$

where the constant *const* is taken as 4.62×10^{-16} to conform with numerical experimental observations that this value enables the smoothest transition, in general, in terms of power values from volumetric to surface emission.

Where necessary another fine adjustment is made at the transition point adjusting the constant so that the surface emission power becomes the same value as the absorption corrected volumetric emission power at the transition point. Beyond the transition point (with A less than 1/e) radiation emission power is taken to be the surface emission power.

Neutron Yield

<http://www.intimal.edu.my/school/fas/UFLF/>

Adapted from the following paper (with modifications for erratum)

Pinch current limitation effect in plasma focus

S. Lee and S. H. Saw, Appl. Phys. Lett. 92, 021503 (2008), DOI:10.1063/1.2827579

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Neutron yield is calculated with two components, thermonuclear term and beam-target term.

The thermonuclear term is taken as:

$$dY_{th} = 0.5n_i^2 (3.142)r_p^2 z_f <\sigma v> (\text{time interval})$$

Where $<\sigma v>$ is the thermalised fusion cross section-velocity product corresponding to the plasma temperature, for the time interval under consideration. The yield Y_{th} is obtained by summing up over all intervals during the focus pinch.

The beam-target term is derived using the following phenomenological beam-target neutron generating mechanism¹⁷, incorporated in the present RADPFV5.13. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modeling each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with proportionality constant. The yield is then calibrated against a known experimental point.

The beam-target yield is written in the form: $Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$

where n_b is the number of beam ions per unit plasma volume, n_i is the ion density, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D-D fusion reaction, n -branch¹⁸, v_b the beam ion speed and τ is the beam-target interaction time assumed proportional to the confinement time of the plasma column.

Total beam energy is estimated¹⁷ as proportional to $L_p I_{pinch}^2$, a measure of the pinch inductance energy, L_p being the focus pinch inductance. Thus the number of beam ions is $N_b \sim L_p I_{pinch}^2 / v_b^2$ and n_b is N_b divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that⁴ $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$ where U is the disruption-caused diode voltage¹⁷. Here 'b' is the cathode radius. We also assume reasonably that U is proportional to V_{max} , the maximum voltage induced by the current sheet collapsing radially towards the axis.

$$\text{Hence we derive: } Y_{b-t} = C_n n_i I_{pinch}^2 z_p^2 ((\ln b/r_p)) \sigma / V_{max}^{1/2} \quad (1)$$

where I_{pinch} is the current flowing through the pinch at start of the slow compression phase; r_p and z_p are the pinch dimensions at end of that phase. Here C_n is a constant which in practice we will calibrate with an experimental point.

The D-D cross-section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute σ . The code computes V_{max} of the order of 20-50 kV. However it is known¹⁷, from experiments that the ion energy responsible for the beam-target neutrons is in the range 50-150keV¹⁷, and for smaller lower-voltage machines the relevant energy¹⁹

could be lower at 30-60keV. Thus to align with experimental observations the D-D cross section σ is reasonably obtained by using beam energy equal to 3 times $V_{\max}$.

A plot of experimentally measured neutron yield Y_n vs I_{pinch} was made combining all available experimental data^{2,4,12,13,17,19-22}. This gave a fit of $Y_n = 9 \times 10^{10} I_{\text{pinch}}^{3.8}$ for I_{pinch} in the range 0.1-1MA. From this plot a calibration point was chosen at 0.5MA, $Y_n = 7 \times 10^9$ neutrons. The model code²³ RADPFV5.13 was thus calibrated to compute Y_{b-t} which in our model is the same as Y_n .

Column elongation

Whereas in the radial RS phase we have adopted a ‘frozen’ elongation speed model, we now allow the elongation to be driven fully by the plasma pressure.

$$\frac{dz_f}{dt} = \left[\frac{\mu(\gamma + 1)}{\rho_o} \right]^{\frac{1}{2}} \frac{If_c}{4\pi r_p} \quad - \text{(XXI)}$$

Circuit current equation

$$\frac{dI}{dt} = \frac{V_o - \frac{\int Idt}{C_o} - \frac{\mu}{2\pi} \left(\ln \frac{b}{r_p} \right) \frac{dz_f}{dt} If_c + \frac{\mu}{2\pi} \frac{z_f}{r_p} \frac{dr_p}{dt} If_c - I(Rf_c + r_o)}{L_o + \frac{\mu}{2\pi} f_c \left((\ln c) z_o + \left(\ln \frac{b}{r_p} \right) z_f \right)} \quad - \text{(XXII)}$$

Equations (XX), (XXI) and (XXII) are integrated as coupled equations for r_p , z_f and I . At each step the value of $\frac{dQ}{dt}$ is also evaluated as above.

The total energy radiated by Bremsstrahlung (Q_B) and line radiation (Q_L) may also be evaluated.

Voltage across focus terminals

$$V = \frac{\mu f_c}{2\pi} I \left[\left(\ln \frac{b}{r_p} \right) \frac{dz_f}{dt} - \frac{z_f}{r_p} \frac{dr_p}{dt} \right] + \frac{\mu f_c}{2\pi} \left[\left(\ln \frac{b}{r_p} \right) z_f + (\ln C) z_o \right] \frac{dI}{dt} + RI$$

Instability resistance/impedance not included in slow compression phase

From experiments, it is well known that after a brief period (few ns), the quiescent column is rapidly broken up by instabilities. One effect is a huge spike of voltage, partially observed at focus tube terminals. This voltage spike is responsible for driving ion beams (forward direction) and REB (negative direction, up the anode) with energies typically 200keV.

We could model this by including a suitable time varying resistance/impedance into the $\frac{dI}{dt}$ equation; and adjusting this function to suit the observed voltage/beam energy characteristics. There is a complication of this ‘anomalous’ resistance in our modelling. If we include this resistance also

into the joule heating term in the piston motion Eq (XX), the sudden increase in $\frac{dQ_J}{dt}$ will blow the piston outwards, leading to a huge negative voltage ‘spike’; not experienced experimentally. The model may be more realistic if at the moment of introducing the ‘anomalous’ resistance, the piston motion is ‘frozen’, or even allowed to continue inwards, as the magnetic field in such ‘small Magnetic Reynolds Number’ situation will diffuse inwards – no piston blow-out!

The final result of this instability mechanism is the breaking up of the focus pinch into a large expanded current column.

5 Expanded Column Axial Phase

We model the expanded column axial phase (3,4) in the following manner.

In the expanded column phase we assume that the current flows uniformly from anode to cathode in a uniform column having the same radius as the anode and a length of z .

The normalised equations (same normalisation as in axial phase):

$$\text{Circuit current: } \frac{d\iota}{d\tau} = \frac{1 - \int \iota d\tau - \beta \iota \frac{d\zeta}{d\tau} e - \delta \iota}{1 + \beta + \beta(\zeta - 1)e}$$

$$\text{where } e = \left(\ln c + \frac{1}{z} \right) / \ln c$$

$$\text{Motion: } \frac{d^2 \zeta}{d\tau^2} = \frac{\alpha^2 \iota^2 e_1 - h^2 \left(\frac{d\zeta}{d\tau} \right)^2}{1 + h^2 (\zeta - 1)}$$

$$\text{where } h = [c^2 / (c^2 - 1)]^{1/2}$$

$$e_1 = \left(\ln c + \frac{1}{4} \right) / (\ln c)$$

The initial conditions for ι and $\int \iota d\tau$ are the last values of ι and $\int \iota d\tau$ from the last phase. The initial value of ζ is $\zeta = 1 + \zeta_f$ where ζ_f is the last length of the focus column, but normalised to z_0 , rather than a .

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HIGH REPETITION HIGH PERFORMANCE PLASMA FOCUS AS A POWERFUL RADIATION SOURCE

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ABSTRACT

Basic operational characteristics of the plasma focus are considered from design perspectives to develop powerful radiation sources. Using these ideas we have developed two Compact Plasma Focus (CPF) devices operating in neon with high performance and high repetition rate capacity for use as an intense Soft X-ray (SXR) source for microelectronics lithography. The NX1 is a four-module system with a peak current of 320 kA when the capacitor bank ($7.8\mu\text{F} \times 4$) is charged to 14kV. It produces 100J of SXR per shot (4% wall-plug efficiency) giving at 3 Hz, 300W of average SXR power into 4π . The NX2 is also a four-module system. Each module uses a rail-gap switching 12 capacitors each with a capacity of $0.6\mu\text{F}$. The NX2 operates with peak currents of 400kA at 11.5kV into water-cooled electrodes at repetition rates up to 16 Hz to produce 300W SXR in burst durations of several minutes. SXR lithographs are taken from both machines to demonstrate that sufficient SXR flux is generated for an exposure with only 300 shots. In addition flash electron lithographs are also obtained requiring only 10 shots per exposure. Such high performance compact machines may be improved to yield over 1 kW of SXR, enabling sufficient exposure throughput to be of interest to wafer industry. In deuterium the neutron yield could be over 10^{10} neutrons per sec over prolonged bursts of minutes.

INTRODUCTION

For Mather's type plasma focus operation it is observed experimentally /1,2/ that the quantity designated as the drive parameter $S = (I/a)/\rho^{1/2}$ (where I is the driving current and ρ the operational gas density), a measure of speed, both axial and radial, has an optimum value for each gas of operation. Thus for deuterium the average axial speed for optimum neutron yield appears to be just below 6 cm/ μsec corresponding to a peak axial speed of 9-10 cm/ μsec and a peak radial speed of some 25 cm/ μsec as the plasma focus radial shock goes on axis. That the optimum speed should be so low for optimum neutron yield is surprising since one would expect from D-D fusion cross-section consideration that the fusion yield should be enhanced by an increase in speed which should boost the focus ions above the 1 keV observed for focus operation at the above mentioned optimum speeds. The speed limitation may be caused by a force-field flow-field decoupling effect /1/. An effort to achieve yield enhancement by breaking through the speed limit has been made /3/.

Operating in noble gases for the generation of soft x-ray (SXR) an optimum speed may be more readily understood. For example in neon the compressed plasma in the focus should have a temperature of some 400 eV if the radiation is required to be predominantly in the 0.8-1.4 nm for the purpose of microelectronics lithography. We have used a model computing plasma dynamics in the axial, radial, and radial reflected shock phases, incorporating a quasi-equilibrium radiative phase /4,5,6/ to examine for example the optimum axial speed required to set the stage for optimum radiation in the 0.8-1.4 nm range. This model is used to correlate with experimental results which indicate an optimum average axial speed of 4.5 cm/ μsec .

It is important to note that the optimum speed for each of deuterium and neon operation remains nearly constant for the range of machines surveyed. This is particularly remarkable for deuterium operation where a value of S nearly constant at $90 \text{ kA/torr}^{1/2}$, corresponding to a peak axial speed of just less than $10 \text{ cm}/\mu\text{sec}$, is tabulated /1/ over a wide range of machines from training machines of 3 kJ /2/ to machines of 300 kJ . This means that for each gas the plasma temperatures in each of the dynamic phases, and by inference also in the compressed radiative phase, are identical for all machines, big and small, when optimized.

We next note that the quantity S is dependent on $D=(I/a)$ linearly whilst it depends only on the half power of ρ . Note also that over a two decade range of stored energy the optimized operational pressure has a range of only 2 /1/. Thus in a relative sense the density ρ and hence the quantity D may in the first approximation be considered also as constant when comparing different machines, all optimized. This clearly agrees with the design tendency to increase the anode radius proportionally with the available drive current. But there is also a fundamental significance.

For each gas, since we are dealing with the same compressed temperature and essentially the same density, radiation yield will depend on the product of compressed plasma volume and lifetime. Again since we are dealing with the same dynamical speeds and compressed temperatures any reasonable modelling /4,7,14/ will show that each dimension of the pinched plasma is proportional to the anode radius, as is the lifetime of the compressed plasma. Thus radiation yield is proportional to a^4 . And since D is essentially constant for each gas, radiation yield, at least for neutrons and SXR, is proportional to I^4 . Such a scaling is energetically possible since whenever energy is taken from the circuit by the plasma such an energy extraction will reflect in a lowering of the current. Such a self-regulating mechanism will self-consistently limit the extraction of energy from the circuit.

Thus for a given stored energy, yield performance is related to current. Circuit inductance needs to be minimized. Our modelling also indicates the importance of minimizing the ratio of generator impedance to total impedance for efficient transfer of energy to the plasma pinch. Practically this is again accomplished by minimizing all the inductances from the capacitor bank through the switches right up to the collector flanges of the plasma focus head. Thus improving circuit performance should improve yield performance.

For applications, whilst the peak rates of yield may have significance for some time-resolved experiments, for other applications such as SXR lithography for microelectronics application there is a need for high average yield rates sustained over at least a duration of minutes, even for demonstration purposes. Thus ability to operate at high repetition rates in a prolonged burst is necessary.

The length of the anode is also of crucial importance /2,7/. Computation and experience agree that a strong focus with optimum energy coupled into the focus pinch so as to emit intense radiation, is achieved when the radial compression starts (end of axial phase) at a time t_{ax} where t_{ax} is equal to t_r , t_r being a hypothetical risetime of the capacitor bank with a value between the short-circuited risetime and the risetime of the circuit loaded hypothetically with the full axial load. As a rule of thumb the short-circuited risetime may be used for a first estimate of the optimum anode length.

Thus for the deuterium focus with an optimum average axial speed of say $5.5 \text{ cm}/\mu\text{sec}$ the anode length should be 5.5 cm per μsec short-circuited bank risetime. For the neon focus taking the optimum average axial speed to be $4.5 \text{ cm}/\mu\text{sec}$ would give us an indicative optimum anode length of 4.5 cm for every μsec of shortcircuited bank risetime.

What about the value of $D=(I/a)$? From a survey of experiments it is found that the current per unit anode radius has a design range of $150\text{-}220 \text{ kA/cm}$, for optimum neutron yield. We have used this range also as an indicative design range for our SXR facilities.

Based on the above considerations we have developed two high repetition rate compact plasma focus facilities, the NX1 and the NX2 to be powerful SXR sources for microelectronics lithography /8,9,10/.

We note that the requirements for a point SXR lithography source may be expressed as follows: point source dimension less than 1 mm (focussed plasma viewed end-on) with emission in the wavelength range 0.8-1.4nm, and average SXR power of 1 kW at source over 4π delivered over a prolonged burst. This last requirement indicates what is needed from industrial wafer throughput considerations. For a resist with

$100\text{mJ}/\text{cm}^2$ sensitivity exposed at a distance which cannot be less than 30cm /9/ from the point source, 1kW will deliver the required $100\text{ mJ}/\text{cm}^2$ in 2 sec assuming beamline transmission ratio of 0.5. A 2 sec exposure time per field may be sufficient for industrial wafer throughput purposes. For demonstration purposes even a 100W source is useful.

Other practical design features include compact footprint with ample space for a stepper to be integrated eventually into the facility.

2. APPARATUS

The plasma focus soft x-ray sources used in these experiments are low energy $\sim 2\text{kJ}$ plasma focus operated in neon. A general view of the NX2 is shown in Fig 1. The design enables measurement of SXR yield at the same time as lithographic exposure is made. The footprint of

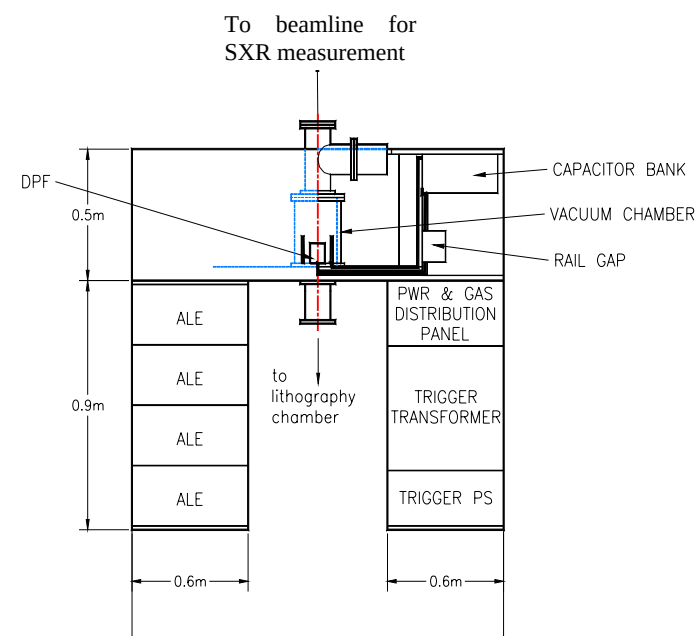


Figure 1 General overview of the NX2 apparatus

the machine is 1.6 m x 1.6 m. There is a clear space for the integrated development of a stepper. The system is completely shielded against electromagnetic radiation.

2.1 Electrical system

Both the NX1 and NX2 plasma focuses are driven by $30\mu\text{F}$ capacitor bank charged by ALE Systems model 802 high voltage capacitor chargers. The capacitor banks are connected to the focus via four switches (pseudo spark switches for the NX1 and rail gap switches for the NX2). Table 1 summarizes the characteristics of the electrical systems. A schematic of the electrical system is shown in Fig 2.

	Charging Voltage (kV)	Energy (kJ)	Repetition rate (Hz)	Current (kA)	Short circuit rise time or quarter period (μs)
NX1	12	2.2	3*	280	1.5
NX2	11.5	1.9	16	400	1.0

Table 1 Summary of electrical characteristics

* limited by available charging power.

2.2 Focus chamber

Three chambers have been used in the NX1 (see Fig 3) with oxygen-free copper anode lengths 3.5, 4.5 and 5.5 cm respectively. Three anode lengths were tried with the NX2. The electrode dimensions are summarized in table 2. The NX2 stainless steel electrodes are cooled by water circulated through the electrodes using two Bay Voltex RRS-1650-AC chillers with a total cooling capacity of 9.6kW

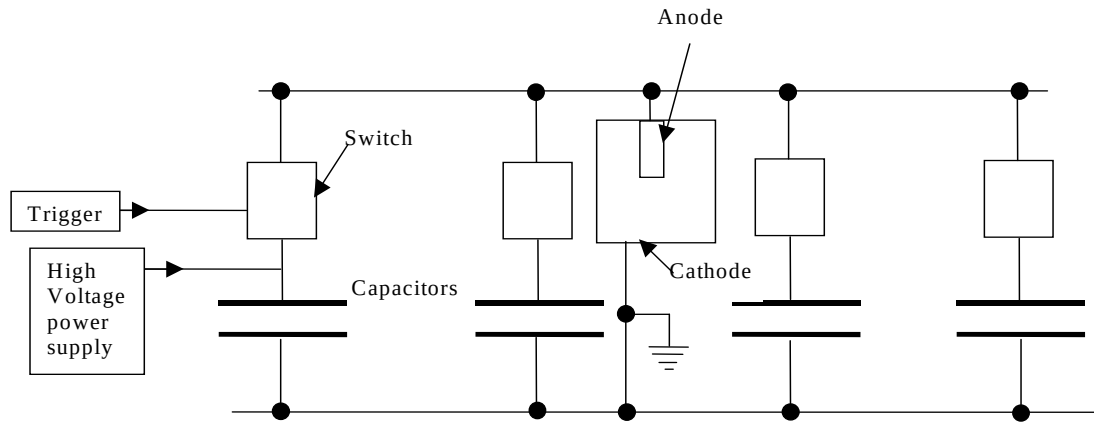


Figure 2. Schematic of electrical system

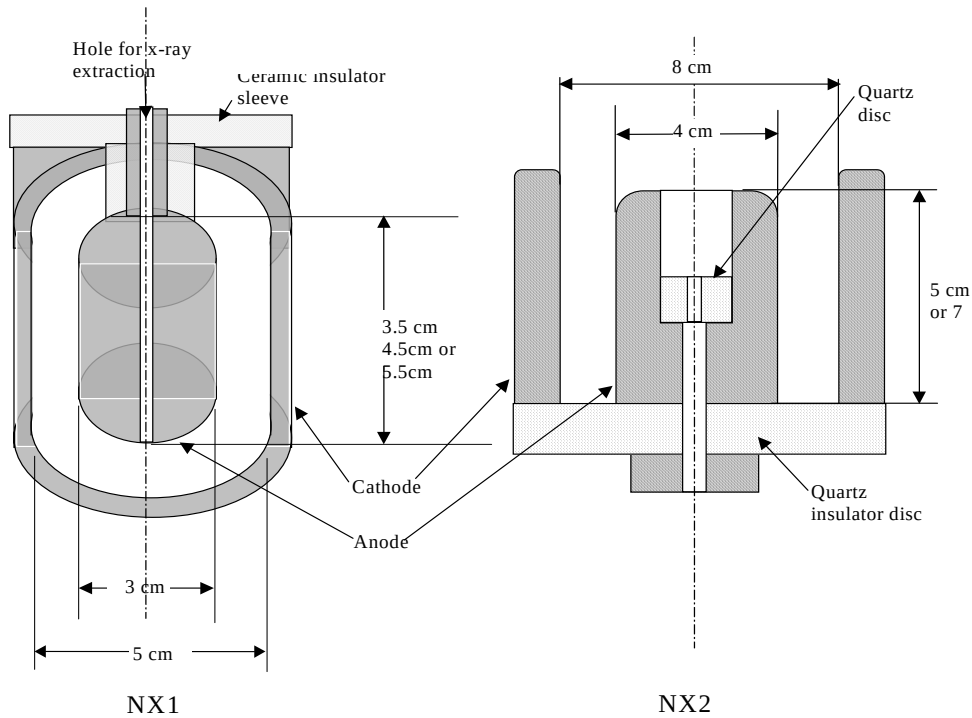


Figure 3. Schematic of focus electrodes and chamber

	Anode diameter(cm)	Cathode diameter (cm)	Anode length (cm)	Electrode material	Insulator material
NX1	3	5	3.5, 4.5, 5.5	Copper	Ceramic
NX2	4	8	5, 7	Stainless steel	Quartz

Table 2 Summary of focus electrodes

3. EXPERIMENT

The diagnostics were 1mm^2 area $10\mu\text{m}$ thick PIN diodes and 3mm^2 photoconducting diamond (PCD) filtered by aluminium, mylar and beryllium foils. The setup used for the experiments is shown in Fig 4. Both the PCDs and the PIN diodes were used on both machines. In the case of the NX1, the x-ray was

detected through the extraction hole through the anode. This means that there is also an electron beam travelling along the same path. The electrons are excluded by the application of a magnetic field to deflect the electrons and also by the 10 μm beryllium which scatters the electrons. The energy of the electrons have been determined in a previous experiment /11/.

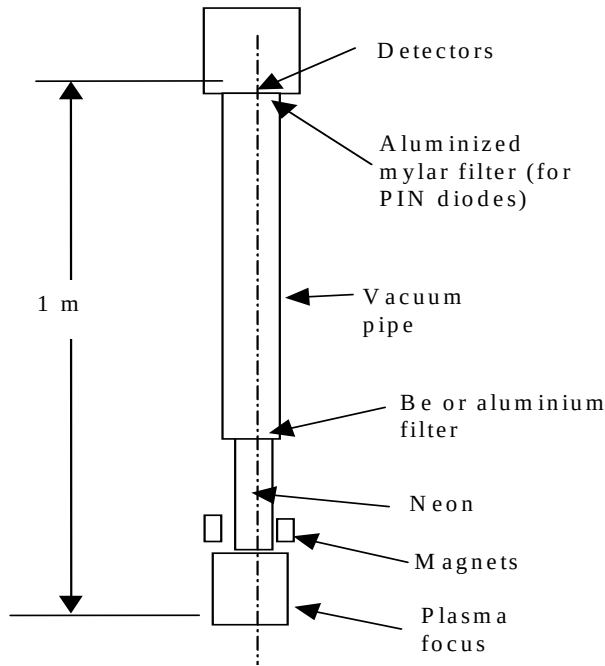


Figure 4 Experimental set up for soft x-ray measurement

The initial pressure of neon was varied and the optimum pressure was found for the various electrode lengths and for charging voltages of 10kV and 14kV. Most of the datapoints were repeated 5-10 times for the NX1 and 20-200 times for the NX2. A fast acquisition system consisting of a tektronix TDS380 oscilloscope connected to a computer was used so that the x-ray for every shot up to a repetition rate of about 10 HZ could be obtained.

4. RESULTS

To obtain the SXR yield from the PIN diode pulse, the area under the oscilloscope trace is obtained and the total amount of SXR is calculated using a sensitivity factor into which has been folded the sensitivity versus wavelength characteristics of the PIN diode, the spectrum of the neon focus emission which had been separately obtained earlier using a crystal spectrograph /6/ and the absorption of the beamline gas path and filters. The PIN diode measurements are cross-calibrated against a calibrated PCD detector. The PCD has a flat sensitivity over the range of SXR spectrum considered.

Hence interpretation of the yield is more reliable. The measurements using the two detectors agree to within 20% on the NX2. All results are adjusted to the PCD calibration.

The results from the x-ray yield measurements are shown in Fig 5. Fig 5a shows that with the NX1, we obtained up to 5% conversion into soft x-ray from the capacitor bank energy and corresponding wall plug efficiency of 4% with the 4.5cm anode at 12kV. The x-ray yield varies within 50% of the maximum when the pressure is within 20% of the optimum. The typical variation of the x-ray yield when other factors like neon pressure, charging voltage are kept constant is about $\pm 35\%$ of the average x-ray yield.

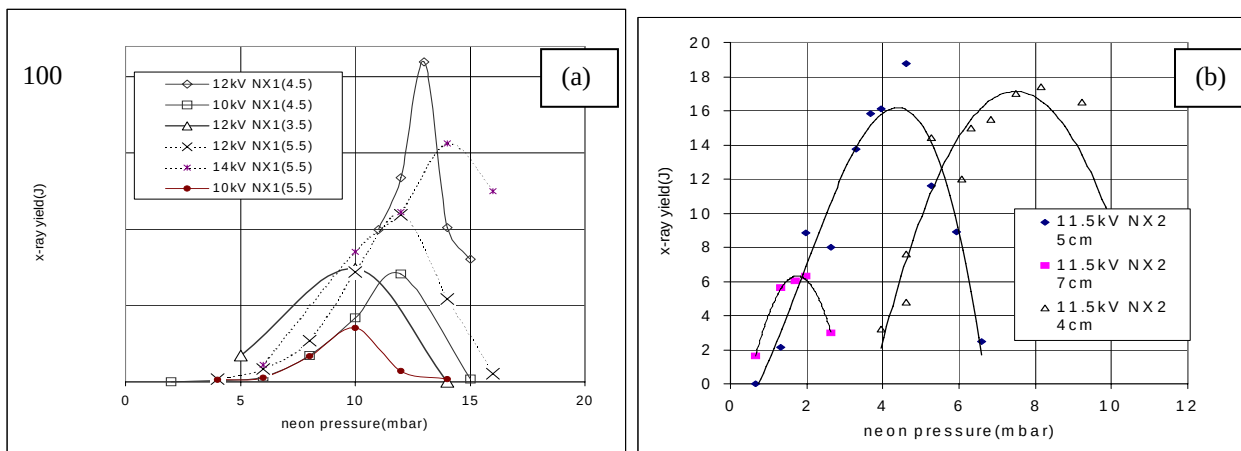


Figure 5. X-ray yield for different neon pressures for a) NX1 and b) NX2.

Figure 6 shows some representative oscilloscope current traces obtained using the NX1 with 4.5cm anode operated with a charging voltage of 10kV and the NX2 with the 5cm anode operated at 11.5kV with 4mbar neon. Some current dips as low as 60% of the peak current when there are multiple focus within a short time of each other. More typical dips drop the current to 80% of the maximum which is what the 1st dip associated with the 1st focus event in figure 6a. It can be seen that the energy transfer into the plasma is more efficient for the NX2 as the dip shown in figure 6b dips to about 65% of the peak current with only one focus event.

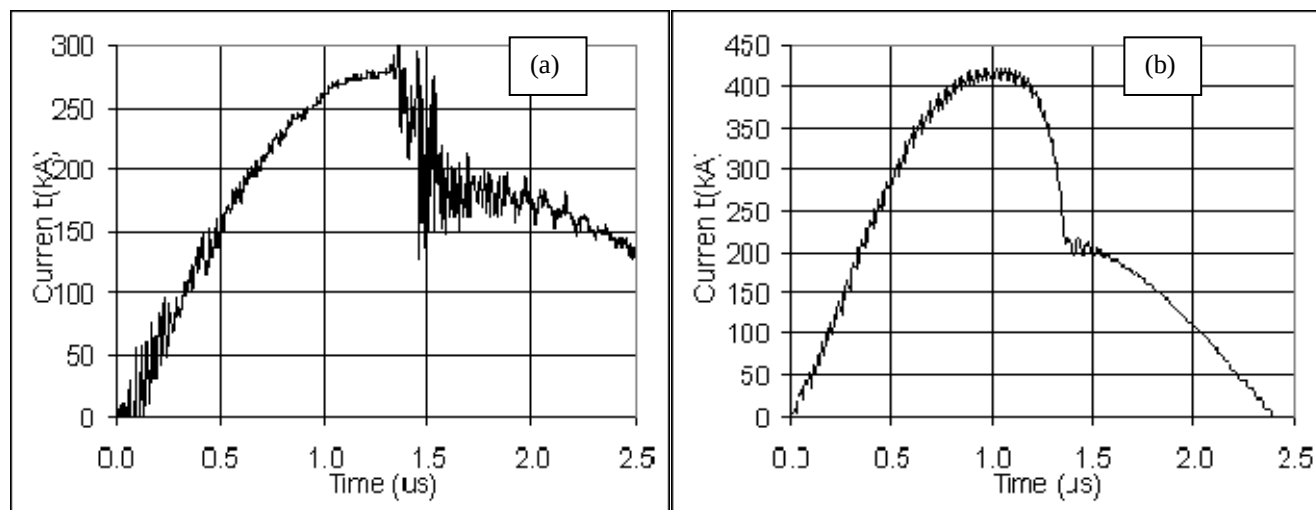


Figure 6 Some representative oscilloscope traces obtained from (a) the NX1 and (b) the NX2

Table 3 shows the parameters for maximum x-ray yield for some of the configurations we tried. It can be seen that the best SXR yield is at an average velocity of $4.5 \mu\text{scm}^{-1}$. With the shorter electrode, it is not possible to run the focus at a higher velocity as the focus would occur at a time too long before the natural current peak such that not enough of the capacitor bank energy has been converted to the magnetic field energy driving the plasma. However if the anode is made too long and the velocity pushed too high, the final focus temperature will become too high for efficient production of neon K shell lines.

Mac- hine	Bank Voltage/ Energy (kV/kJ)	Length/ Equival- ent* (cm/cm)	Opti- mum Pressure (mbar)	Meas- ured Current (kA)	t_{ax}^{**} (μs)	Average Axial Speed ($\text{cm}\mu\text{s}^{-1}$)	D (kAcm^{-1})	S (kA cm^{-1} $\text{torr}^{-1/2}$)	SXR yield (J)
NX1	10/1.5	5.5/6	10	230	1.50	4.0	153	56	20
NX1	12/2.2	5.5/6	12	280	1.45	4.1	187	62	55
NX1	14/2.9	5.5/6	14	320	1.40	4.3	213	66	80
NX1	10/1.5	4.5/5	12	230	1.25	4.0	153	56	35
NX1	12/2.2	4.5/5	13	280	1.10	4.6	186	60	105
NX1	12/2.2	3.5/4	10	280	1.10	3.6	187	68	35
NX2	11.5/1.9	7	2	340	1.30	5.4	170	139	7
NX2	11.5/1.9	5	4	400	1.15	4.4	200	115	18
NX2	11.5/1.9	4	7	410	1.05	3.8	205	90	15

Table 3 Comparative performance of NX1 and NX2

*The equivalent length takes into account that the NX1 is curved so that the run down length is slightly longer.

** t_{ax} =time at end of axial phase, or start of compression, taken as 0.25/0.3 μs before SXR pulse for the NX1/NX2.

Figure 7 shows SXR lithographic exposures to confirm the SXR flux of NX1 and NX2 /12/. The resists used have a sensitivity rated at $100\text{mJ}/\text{cm}^2$ and are placed 40cm from the focus. Magnets are placed to deflect the electron beams associated with the plasma focus /13/ to ensure that the exposure are by SXR. The mask is a $1\text{ }\mu\text{m}$ thick gold mesh with grid separation of $5\text{ }\mu\text{m}$. The NX1 beamline has a 3 times poorer transmission ratio than the NX2 beamline. These lithographs confirm that the NX1 produces more than 3 times the SXR yield per shot when compared to the NX2.

Figure 8 shows a flash electron lithograph exposed on PMMA with 10 shots of NX1. For electron lithographs the deflecting magnets were removed. The exposure was used to estimate the electron beam current as $50\text{ }\mu\text{A}$ over the 5 Hz burst . The electron energy was estimated as 30keV /12/.

5. CONCLUSION

We note that the NX1 and the NX2 have quite different yield performance. For each machine the observed speed at optimum yield for each anode length generally does correlate with the drive parameter S. However the value of S is significantly higher (up to 2 times) for equivalent speeds for the NX2 compared with the NX1. On the basis of machine scaling for neutrons /1/ we would have expected constant value of S for optimum operation. This difference may be the cause of the large difference in yield. Despite higher circuit performance the yield performance of the NX2 is significantly lower than the NX1. This may be due to the significantly lower value of S for the NX1 which could be related to a higher operational density (up to 3 times) of NX1 for equivalent speeds and D, when compared to the NX2. The higher optimum operational density at equivalent temperatures obviously favours a higher SXR yield for NX1. This yield superiority of NX1 could perhaps be ascribed to differences in electrode materials (oxygen-free copper for NX1 compared to stainless steel for the NX2), chamber configurations (carefully shaped channel and closed outer electrode for the NX1), perhaps even to the differences in backwall insulation materials and configuration.

In any case it appears that the NX1 chamber has the more promising features with maximum SXR yield over 100J and wall plug efficiency of 4%, compared to 18J and 1% for the NX2. By incorporating cooling in the NX1 chamber and increasing the charging capacity so that the NX1 chamber may be fired at 10Hz, 1kW of SXR power may be achieved which will expose a field at 30cm in less than 2sec on a $100\text{mJ}/\text{cm}^2$ resist, assuming a beamline transmission of 0.5. This should be sufficient for industrial throughput demands applied to microelectronics lithography aimed at $0.15\text{ }\mu\text{m}$ design rules.

Moreover, by using deuterium we expect a neutron yield of better than 10^9 per shot and 10^{10} neutrons per shot when operating the cooled NX1 in a prolonged burst at 10Hz. Such a powerful compact neutron source will have interesting applications.

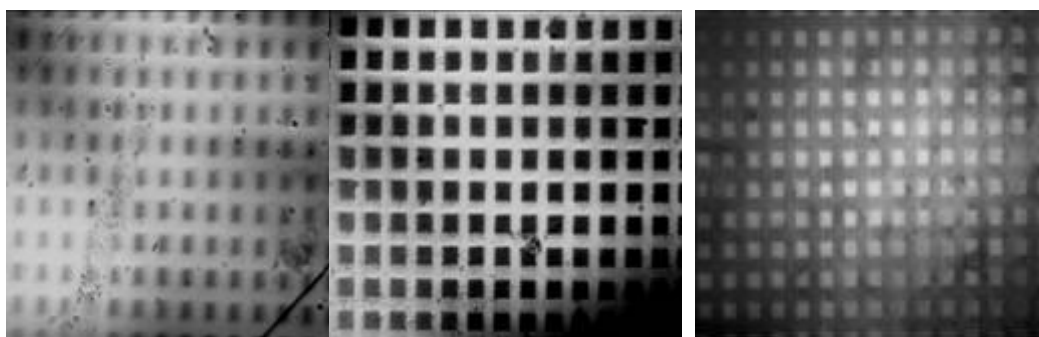


Figure 7 Test exposure (a) NX1 (200 shots & 400 shots) and (b) NX2 (300 shots)

Resist has a rated sensitivity of $100\text{mJ}/\text{cm}^2$

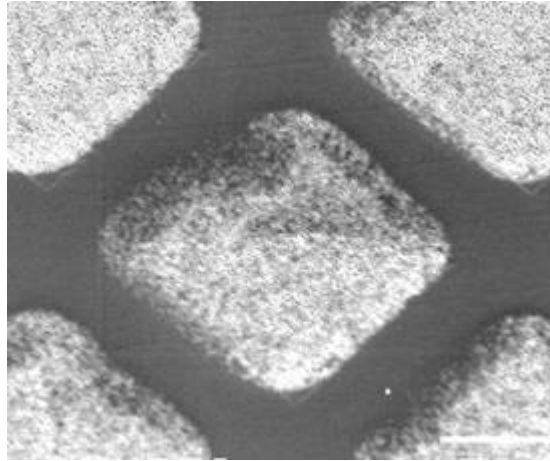


Figure 8 Flash electron lithograph on PMMA, 10 shots (NX1)

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Plasma dynamics in the PF-1000 device under full-scale energy storage: II. Fast electron and ion characteristics versus neutron emission parameters and gun optimization perspectives

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Abstract

Electron and ion beam dynamics of the PF-1000 facility were investigated for the first time at its upper energy limit (≈ 1 MJ) in relation to neutron emission, the pinch's plasma ('target') characteristics and some other parameters with the help of a number of diagnostics with ns temporal resolution. Special attention was paid to the temporal and the spatial cross correlations of different phenomena. Results of these experiments are in favour of a neutron emission model based on ion beam–plasma interaction with three important features: (1) *the plasma target is hot* and confined during a few '*inertial confinement times*'; (2) the *ions* of the main part of the beam are *magnetized* and *entrapped* around the pinch plasma target for a period *longer than the characteristic time of the plasma inductive storage system* and (3) *ion–ion collisions* (both *fusion collisions*, due to head-on impacts and *Coulomb collisions*) are *responsible for neutron emission*. Analysis has shown that one of the ways for achieving a future improvement in the neutron yield of the PF-1000 facility may be by changing the geometry of the device. It may ensure an increase in both the discharge current and the initial working gas pressure, eventually resulting in the neutron yield boost.

1. Introduction

Dense plasma focus (DPF) [1, 2] is a gas-discharge installation of Z-pinch class. It has two coaxial electrodes with the internal one spanned by an insulator. After applying a voltage

between the electrodes and breakdown of a filling gas along the insulator's surface such a discharge undergoes two general phases [1–4].

(1) A relatively long magneto-hydrodynamic (MHD) stage (several microseconds), during which a plasma-current sheath (PCS) is accelerated to the chamber axis and imploded

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on it thus forming a ‘pinch’; after this implosion (so-called ‘first compression’) the plasma column is confined for a time interval Δt equal to several (n) inertial periods of time $\Delta t = n\tau = nr/v_i = nr/(3kT/m_D)^{1/2}$ (where r is the pinch radius, v_i is the thermal ion velocity, k is the Boltzmann constant, T is the plasma temperature and m_D is the deuteron ion mass) and eventually it is disturbed by MHD instabilities (mainly by the Rayleigh–Taylor one) having an increment of about 10^8 s^{-1} .

(2) A short kinetic (K) stage (with an assortment of characteristic times of microinstabilities ranging in the interval 10^{-13} – 10^{-10} s) when the pinch, already MHD-perturbed, is destroyed by micro- and macroinstabilities during the above period Δt .

As was measured by many researchers right from the start of the DPF phenomenon investigations (50 years ago, see references in [1, 2]) a consequence of the perturbation of a plasma column (pinch) is the generation of powerful beams of fast electrons (e) and ions (i) having particle energy in the range extending to hundreds of keV and a few MeV (for electrons and deuterons, respectively). After generation of these e- and i-beams, intense emissions of hard x-rays (HXR) and fusion neutrons (N) are produced. The latter type of radiation takes place in the operation of a DPF with deuterium (DD) as in the case of PF-1000 or with a deuterium–tritium (DT) mixture used for the filling of its chamber as a working gas under the initial pressure of a few Torr.

Energy E_c , stored in capacitors for a subsequent release to the discharge, occupies a range from just a few Joules [3, 4] to about 1 megaJoule (MJ) [2]. The main interest in the installation of its highest energy level operation is connected with a favourable scaling law for the neutron production yield— $Y_n \sim E_c^2$ or $Y_n \cong 10^{10} I_p^4$ (sometimes $Y_n \sim I^5$ [1, 2], in particular for the same device [4]). The scaling type is valid for deuterium as a working gas, where I is a discharge current and I_p is the part of the total current which flows through the dense plasma pinch, measured in MA. The operation of a DPF with the deuterium–tritium mixture produces an increase in the neutron yield by a factor of about 100 [5]. This means that on the level of a 10 MA current a DPF might produce the same neutron yield as modern pulsed fission reactors, differing from them however by a much shorter neutron pulse duration—a few hundreds of ns—and by an almost monochromatic spectrum centred near 14 MeV. This would open opportunities for many applications in science (e.g. in neutron spectrometry due to the very high ‘quality’ of the source, $q \geq 10^{37}$ [neutrons s^{-3}]) and technology (e.g. in radiation material sciences—see paper I [6]).

As was shown in many publications (see first of all [7] and also the references in [1, 2]) the major mechanism responsible for neutron emission in DPF is the interaction of deuterons of ‘medium’ energy (50–150 keV) with a ‘target’ (pinch), which is a hot (≤ 1 keV) and relatively dense ($\leq 10^{19} \text{ cm}^{-3}$) plasma (the so-called ‘gyrating particle model’—(GRM)). Here we use the term ‘medium energy’ for the ion energy of a magnitude well above the thermal one for ions of the pinched plasma (~ 1 keV) but much lower than the upper limit of the accelerated deuterons registered ($\sim$ a few MeV). The Larmor radius of these medium-energy deuterons in the magnetic field of the pinch is much smaller compared with the size of the pinch

itself thus providing an opportunity to entrap them for a period longer than a simple direct fly-out time.

Paper I presented our recent results taken during the investigation of the PF-1000 facility on an energy level close to the maximal one (~ 0.85 MJ). In that paper we paid special attention to different scenarios of the dynamics of the pinch’s plasma (target) depending on various modes of the device operation. Here we shall concentrate our attention on the generation of charged particle beams and their interaction with the above-mentioned plasma and with the anode. The concept of the fast particle generation mechanism in DPF was developed in [3, 4]. It is based on electron magneto-hydrodynamics (EMH) theory [8] and uses the model of a ‘virtual plasma diode’, which appears due to anomalous resistivity, springing up in the pinch and constituting the current abrupt phenomenon. Within this diode, first fast electrons are accelerated towards the anode and then they are magnetized and substituted by fast ions. The magnitudes of both beam currents are of the order of the total discharge current. The ion beam is aimed in the direction opposite to the electron stream—towards the cathode. Processes of interaction of both e- and i-beams with targets also have a non-trivial character, and they were the subject matter of the above-cited works. In this paper we examine the e- and i-beams dynamics on the upper level of the DPF energy in relation to the target evolution (paper I) and the neutron emission in view of the above-mentioned models. So the issue is whether the same physics is true at an energy level of the device an order of magnitude higher than those previously exploited.

2. The apparatus

The PF-1000 device [9] is a DPF of the Mather type [1, 2] operating with deuterium as a working gas at an energy level up to 1 MJ. It was designed about 30 years ago and manufactured on the basis of the technology used in those days. However, this is the only kind of facility being used at the present time with which one can investigate mechanisms of neutron generation within a DPF on an MJ energy level.

Yet its neutron yield at the present time is quite far from those deduced from the above-mentioned law of neutron yield dependence on energy stored in the bank (it should be on the level of 10^{13} neutrons per shot whereas the yield is actually equal to 6×10^{11} for the best shot and 2×10^{11} for the typical ‘good’ ones as seen in this paper). The PF-1000 construction (described in [9] and in paper I [6]) consists of the following main units (positioned on three different floors of the IPPLM building).

- Condenser bank, coaxial cables and a collector of diameter 3 m (figure 1(a) and the left-hand side of figure 1(b)), charger and pulsed electrical circuit with high-pressure spark-gaps.
- Vacuum chamber (figure 1(b) with coaxial electrodes of Mather-type geometry [1, 2] (see also figure 2 of paper I [6]) and vacuum/gas handling systems.

The cylindrical copper anode ($\varnothing = 230$ mm, $l = 600$ mm) is closed by a lid, having the same or a slightly larger diameter as the tube, i.e. a circular hat-shaped ‘cap’ at its end. If the anode has this cap of a larger diameter, it would be an obstacle

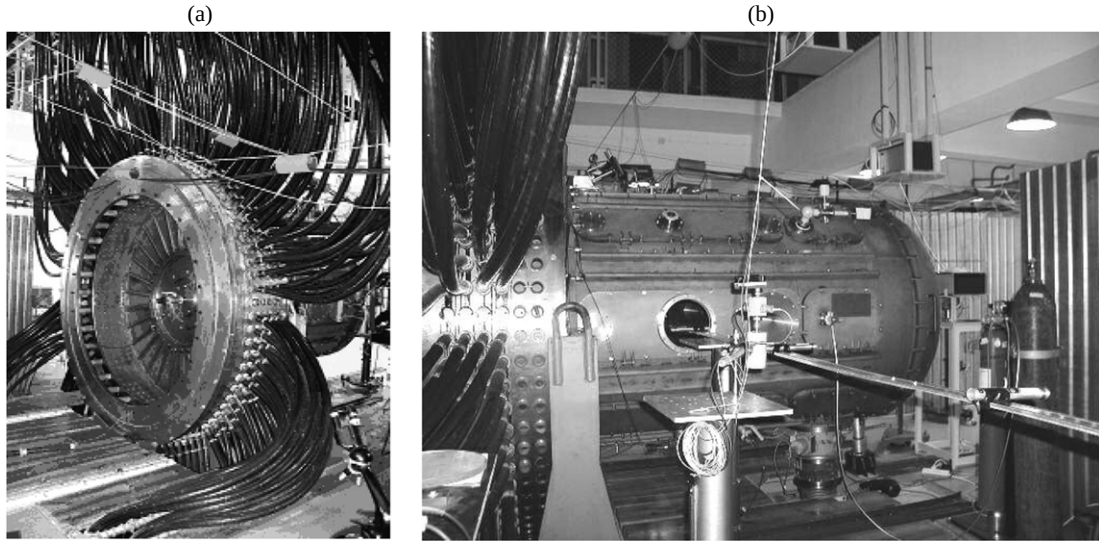


Figure 1. PF-1000: current collector with cables (a) and vacuum chamber with the collector (b).

for a PCS, which will bifurcate into two parts (see paper I [6]). Two modifications of the cathode electrode geometry of squirrel cage type differing essentially by their inter-electrode gaps, as well as by the lengths and the shape of the rods, were used in this set of experiments. The cathode stainless steel bars were much longer than the anode in the first case whereas both electrodes were equal in length in the second configuration. Their modes of operation, resulting mainly in differing MHD *plasma dynamics*, are also discussed in paper I.

An alumina insulator envelops the anode at its lower part. Its main part extends 113 mm along the anode into the vacuum chamber. The condenser bank of total capacitance $1320 \mu\text{F}$ (264 capacitors of $5 \mu\text{F}$ capacitance and 40 nH inductance each) is charged in these experiments to voltage U_0 ranging between 27 and 36 kV. It corresponds to discharge energies E_c within the limits from 480 kJ to 850 kJ. Usually its energy and voltage are equal to 810 kJ and 35 kV, respectively. The energy increase in this set of experiments with PF-1000 compared with the previous ones (see, e.g., [10]) was made by the *bank capacitance increase* (not by the voltage change as in other previous experiments made in Frascati, Stuttgart and Düsseldorf). And as usual the electrode sizes were increased in comparison with previous experiments to match the external and internal inductances of the gun and to equalize the current quarter of the period with the plasma collapse time at this energy magnitude. Typical oscilloscope traces of current and voltage taken at the charging voltage of battery equal to 27 kV are presented in figures 2 (a) and (b), respectively. They look similar in all the cases of electrode geometries.

3. Experimental arrangement

To study the MHD evolution of plasma a *streak* camera with a slit parallel to the anode surface and a three/four *frame optical* camera with an exposure time of about 1 ns were employed (see their description in paper I [6]). Two types of three/four *frame soft x-ray* (SXR) cameras—one based on an open microchannel plate (MCP) device in conjunction with a pinhole camera and

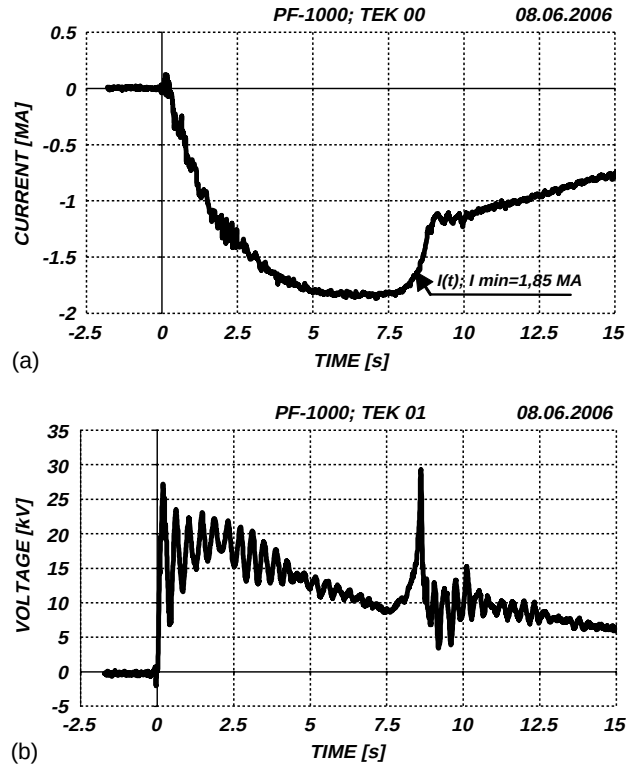


Figure 2. Current (a) and voltage (b) waveforms of the PF-1000 taken at $U_0 = 27$ kV.

another just a time-integrated pinhole camera with an x-ray film registration—have been applied in order to obtain plasma images in the SXR range (see details in paper I [6]).

Fast electron beams generated in the DPF were investigated registering HXR radiation produced by them on the anode. This was done with the help of photomultiplier (PM) tubes plus scintillators (PM+S). These beams were also registered directly by means of Cerenkov-type detectors (figure 3) positioned down-stream in relation to the principal *e-beam direction of propagation* (i.e. at the backside of the anode).

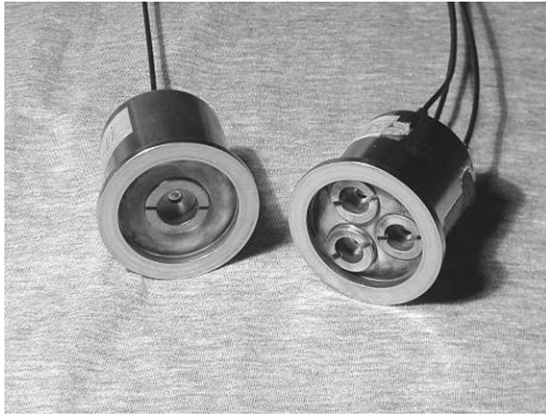


Figure 3. Cerenkov detectors.

These detectors were composed of rutile crystals covered with Cu foils of different thicknesses, and they were coupled with fast PMs through optical cables. The energy of fast electrons which can be registered by this method depends on the crystals used and the filters placed in front of them: without foils, >35 keV; with foils, either above 80 keV or above 120 keV. Signals from them were observed with a fast oscilloscope in correlation with signals from other diagnostic tools.

In order to define the emission characteristics of fast ions (deuterons) in relation to the neutron production process (i.e. ion fluxes, ion angular distributions, ion source location etc) direct ion measurements of fast deuteron beams were performed within the PF-1000 device. An angular distribution of fast deuterons has been measured with nuclear track detectors (of the PM-355 type), placed at a distance of 550 mm from the inner electrode. The semi-ring, where a number of ion track detectors is located, can be seen in figure 4. Each of these detector samples was covered with different Al-foil filters. Ions have been registered at various angles to the electrode axis. To investigate the spatial distribution of fast ions and their trajectories beyond the pinch, miniature ion pinhole cameras have been used. They were positioned at a number of angles to the anode's axis. These cameras were also equipped with the same nuclear track detectors. In order to estimate the energy of the observed deuterons, the detector samples were shielded with Al-foils of different thicknesses. Some temporal and spatial characteristics of the ion beam and its dynamics were obtained by means of an optical multi-frame camera.

Time-resolved SXR signals were measured by means of PIN diodes covered with different filters and by PM tubes also shielded by different foils. Signals from these two types of SXR measuring techniques were compared with other oscilloscope traces (voltage waveforms, dI/dt signals, Cerenkov-detector signals and neutron/HXR pulses) in order to determine their cross correlation. The SXR signal detected by the PIN diodes was also used for synchronization purposes and for the determination of the temporal relation between the maximum of SXR radiation of the plasma and the frame images recorded by means of optical and x-ray diagnostics. Special electrical and optical synchronization arrangements allowed synchronization of the optical diagnostics and other DPF time-resolved diagnostics with a precision of 5 ns.

We investigated the neutron production process by measuring its time evolution, absolute neutron yield and its

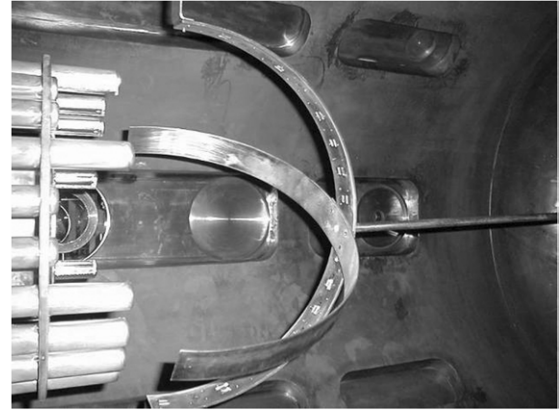


Figure 4. Scheme of track detectors mounting.

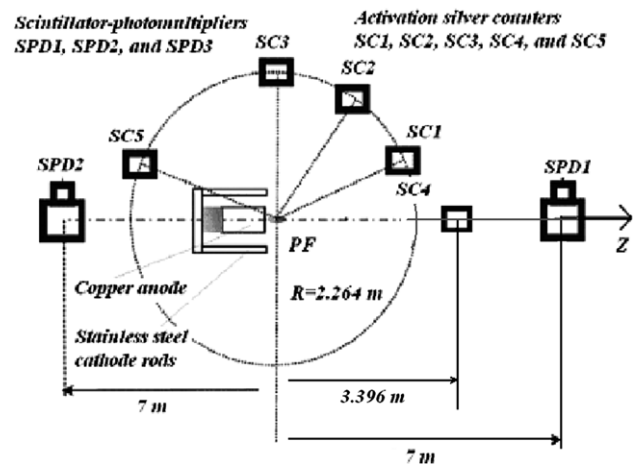


Figure 5. Electrode set-up also showing the positions of the neutron activation counters (SCs) and the fast probe (PM+ scintillator) detectors (SPD) in relation to the Z-axis of the facility.

anisotropy on the basis of both time-integrated methods and time-resolved registration of neutron pulses at different angles to the electrode axis as well as by their comparison with time-resolved and time-integrated measurements of SXR and HXR radiation and the fast electron and ion beams. The total neutron yield (Y_{tot}), i.e. the number of neutrons produced during a single discharge ('shot') and emitted in various directions, was measured taking into consideration the data received by means of five silver-activation counters (SCs) placed at equal distances (3.396 m) in the head-on direction for ion propagation, i.e. in the direction of the Z-axis (SC4), as well as at different angles to it (30° , 60° , 90° and 150° for SC1, SC2, SC3 and SC5, respectively) around the PF-1000 experimental chamber (figure 5).

Independently these measurements were verified by the use of indium-activated counters and so-called 'bubble detectors'. Calibration of *all* detector types was made simultaneously by placing the AmBe isotope neutron source inside the DPF chamber at the position of the plasma pinch. Three scintillator-PM detectors (SPD), located at different angles (all of them at a 7 m distance from the electrode outlet), were used to perform time-resolved measurements of the HXR radiation and N emission with their pulses being separated on the oscilloscope traces due to the corresponding time-of-flight (TOF).

4. Experimental results

Our main series of observations at the PF-1000 facility devoted to the beam generation investigation were carried out in the operational regime with the deuterium pressure $\cong 2\text{--}4$ Torr and with a discharge energy of 810 kJ. In these series the chamber had long cathode rods. The anode's cylinder was smooth and even with its flush-mounted head (i.e. *without* a protrusion).

The obtained results, in relation to the discharge current, can be summarized as follows.

(1) The amplitude of the *total* discharge current measured by the Rogowski coil, positioned in close vicinity to the cathode rods inside the DPF chamber, in typical discharges was 2.5–2.6 MA, being sometimes close to 3 MA for the best DPF shots. However, it is much less than the expected one for such an energy of the bank according to the following *experimental* scaling law valid for DPF devices within the above-mentioned energy range (increasing a bank, one has to pay attention that an inductance magnitude is decreased not simply linearly with the number of capacitors but as a root-square dependence on them because of the cables' length and the adding of pre-collectors [3, 4]):

$$I_{\text{tot}} \cong U_0 \times (N)^{1/2} \times (C/L)^{1/2} \cong 3.3 \times 10^4 \times 16.3 \times (C[F]/L[H])^{1/2} \cong 6.0 \text{ MA}, \quad (1)$$

where $U_0 = 33$ kV is the initial charging voltage of the bank, $N = 264$ is the number of capacitors constituting the bank and $C = 5 \mu\text{F}$ and $L = 40$ nH are the capacitance and the inductance of a single capacitor of the bank. Unfortunately attempts to increase this magnitude by a maximization game with the initial pressure and the charging voltage of the device were not crowned with success.

(2) No attempts to measure the *pinch* current I_p were made during these experiments, i.e. there are no experimental data on the part of the total current flow actually through the dense plasma column (see paper I [6]). However, substitution of the *total measured current value* into the neutron scaling law gives the following figure expected of the PF-1000 operation:

$$Y_n = 10^{10} \times I_{\text{tot}}^4 = 10^{10} \times (3[\text{MA}])^4 = 8.1 \times 10^{11}, \quad (2)$$

that is, only 35% more than the best experimental magnitude and 4 times more than the mean one. The latter result signifies that the estimated *average pinch current* was about 1.4 times less compared with the total one. This is a reasonable value: $I_p \cong 2$ MA, which is consistent with the data known from the literature [1, 2] and deduced from the other results in paper I [6].

(3) Plasma dynamics was investigated in paper I [6]. Here we present four frame-by-frame pictures taken in the visible range in one shot (figure 6). They demonstrate plasma pinch formation ('first compression'—(1), (2)), filament creation (3), i.e. a process of free-streaming of runaway electrons along the Z-axis in its opposite direction, and the start of MHD perturbation of the boundary of a pinch (4). The exposure time is 1 ns and time intervals between the frames is $\sim 10\text{--}20$ ns.

(4) A diagram of the angular distribution of neutron emission is presented in figure 7. It was measured by the above-mentioned 5 SCs for shot No 3121 produced at an initial pressure of

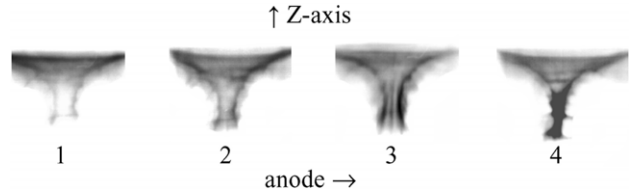


Figure 6. The time sequence of plasma compression stages.

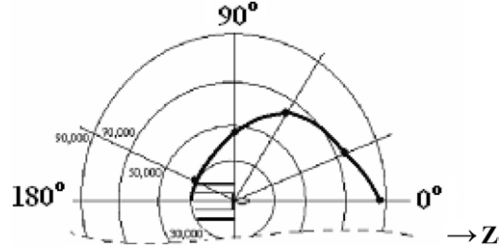


Figure 7. Angular distribution of the PF-1000 neutron yield.

465 Pa and a charging voltage of 35 kV (thus, the total energy in the bank was about 0.810 MJ). One can see that under these conditions the anisotropy of the emission measured in the laboratory coordinate frame has a so-called 'normal' character (i.e. it is characterized by a preferential direction of neutron irradiation at 0° to the Z-axis) and its magnitudes are equal to about 1.8 for the ratio Y_{0°/Y_{90° and to $\cong 0.65$ for the ratio $Y_{180^\circ}/Y_{90^\circ}$.

(5) Under these conditions the total neutron emission yield calculated by integrating the data over all 5 silver counters was of the order of $5 \times 10^{10}\text{--}2 \times 10^{11}$ neutron/shot with the maximum neutron yield of 6×10^{11} neutron/shot measured by silver and supported by indium activation counters. Bubble detectors (positioned only at the angle of 90° in these experiments) gave us, in these discharges, a 30% smaller value.

(6) Two neutron pulses were observed in most cases. The second pulse was higher in amplitude by four–ten times compared with the first one (figure 8). The duration of each pulse (FWHM) as well as the interval between them at their registration in the 'head-on' direction were about 150 ns (except for the case of a 'cusp geometry'—see paper I). Compared with smaller devices the first pulse is relatively much larger (usually it was 1–5% of the second one in small DPF whereas here it is 10–25%) and they both have greater longevity (thus in the range of the DPF bank energy from 0.1 to 800.0 kJ the neutron pulse duration increases from 4–5 ns to 150 ns, i.e. it is roughly proportional to the current value).

However, one can see the difference in the neutron pulse shapes (and their duration) for two dissimilar directions of investigation. Namely, at 90° -observation ('side-on') both pulses are longer and look smoother compared with the 'head-on' measurements (0°).

It should be mentioned here that the data in figures 7 and 8 have a rather crude character. Indeed the environment, walls, columns, ceiling and floor, made up of concrete, elements of the DPF construction and capacitors of the main bank in particular, are positioned at distances of 1.5 through 10 m from the chamber or from the counters. It means that the TOF of *primary scattered streams* of 2.45 MeV neutrons along the

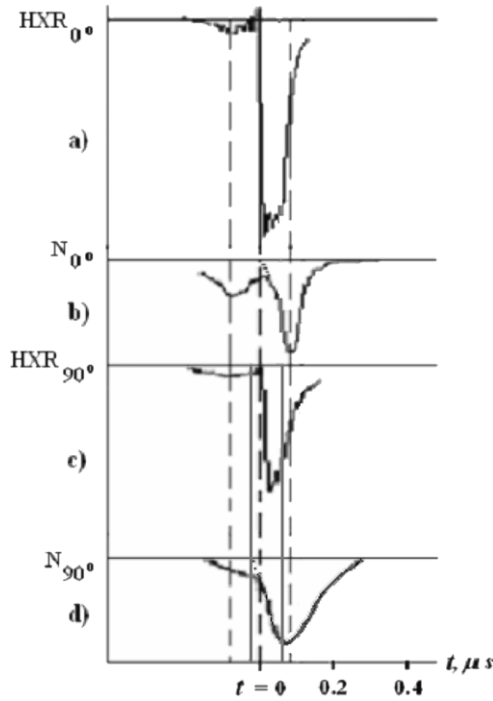


Figure 8. HXR pulses (a) and (c) versus neutron pulses (b) and (d) taken at 0° and 90° to the Z-axis after moving them forward according to their real (HXR) and assumed (N) TOF.

paths from these elements to the counters (1.5–30 m) is about 75–1000 ns which makes the absolute measurements position-dependent and problematic. Moreover our analysis has shown inequality of the two directions. Namely, the PM tube situated in the Z-direction has almost nothing in the hall for single-reflected neutrons to it (a hatch beneath the chamber and a door and a window behind the detector are wide) whereas the side-on PM tube has a wall and columns just near it. These ‘obstacles’ scatter neutrons and thus give additional primary streams onto the scintillators. Taking into consideration the particular TOF of scattered neutrons it should be admitted that the real shapes of the neutron pulses are distorted especially after their maxima (i.e. in their ‘tails’). However, it is evident that such an environment cannot influence the position of the N-pulse *summit* in both cases. This is why the rise-time of pulses in both directions is similar (see figure 8), whereas the pulse decay time is much longer at the side-on observation.

At the same time our activation counters summarize the total yield over a long period (1 min) (figure 7). So they are irradiated by *repeatedly scattered* neutron streams (low-intensity but long-lasting radiation). This is why the data on the neutron yield anisotropy seem to be exaggerated in the normal direction to the Z-axis.

In any case these results gave us important and qualitatively correct as well as quantitative information. In the future they need to be verified by a computer simulation of the neutron field evolution in this particular environment, which would give a possibility of better interpretation of the numerical results obtained in such an experiment.

(7) As we mentioned in paper I it was found that for each value (p_0) of an initial filling pressure there is an optimal charging voltage (U_0), which ensures the maximal neutron

yield (Y_n). Thus, the initial pressure increase produced *in parallel* to the charging voltage rise during the operation of the facility (provided that for *each initial pressure* we used the above *optimal* initial charging voltage) resulted in a further increase of the neutron yield. *No saturation in the neutron yield was found by this strategy till 4 Torr and 35 kV.*

(8) HXR and neutron signals presented in figure 8 are essentially the same as those given in figure 9 of paper I. But in this case we have moved forward both HXR pulses of each oscilloscope trace ((a), 0° and (c), 90°) by their TOF to a 7 m distance (23.3 ns). At the same time for both neutron emission pulses we have adopted the same procedure for *their* TOF ((b), 0° and (d), 90°) as if these neutron pulses consist *exactly* of 2.45 MeV neutrons, namely, by 323 ns. It is clearly seen that the *first pulses* of both HXR and N emissions almost *coincide in both cases* (0° and 90°) after their correction by TOF. This means that in the PF-1000 device runaway electrons are accelerated during or slightly earlier in comparison with a so-called first compression phase as in other devices [3,4], and neutron emission produced during this period of time (i.e. in the first pulse) has an energy spectrum centred at 2.45 MeV.

In contrast, second N pulses in the two directions start and have their peaks at later moments compared with the second HXR pulses. Namely, they have their maximal value during a decay time (droop) of the HXR pulses. Moreover the second pulse maximum *in the head-on case* comes earlier than those in the *side-on case* and ‘runs over’ the HXR pulse. It means (and we might expect it from our anisotropy measurement and the literature) that the spectrum of neutrons irradiated at 0° has higher energy than the neutrons propagating at 90° . Comparing these data with figure 7 one can really see that with the angle variation higher neutron yield corresponds to higher energy of neutrons. It supports results usually being received in smaller devices. We shall discuss it in more detail later in correlation with other diagnostics data.

Two special features of these oscilloscope traces in correlation with frame pictures and with fast particle and neutron generation mechanisms are the following.

- (a) The start moment of the second pulse of the HXRs ($t = 0$) being very sharp (in fact it is out of the temporal resolution of our diagnostics) precisely coincides with the appearance of the disruption at the plasma column as was described above. It is seen in figure 6 (4) and is shown in figure 9 by a ‘↔’-mark. The sequence of pictures shows the column break with the plasma streaming along the circumference and having a meniscus shape (see discussion of this phenomenon later in this paper). Pictures are taken for the anode with an obstacle and short cathode rods—see paper I [6].
- (b) The second neutron pulse starting at the same time or a bit later compared with the second HXR pulse reaches its peak when the HXR pulse is over already.

(9) The group of traces (figure 10) taken in a single shot with the whole set of diagnostics presents examples of typical waveforms displaying from top to bottom (a) HXRs from PMT (8–30 keV), (b) SXR from PMT (3–8 keV), (c) SXR from the PIN diode (0.8–4 keV), (d) fast electrons from Cerenkov detectors, (e) the Rogowski dI/dt signal and finally neutrons

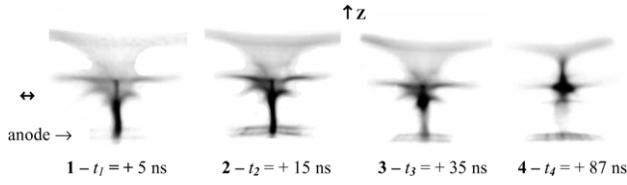


Figure 9. Experimental images of the current cutoff phenomenon (marked by a sign: ' $\leftrightarrow$ ') taken by the optical image camera with 1 ns time resolution.

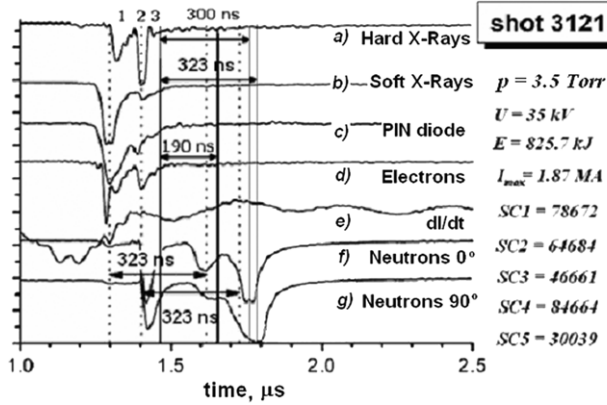


Figure 10. Typical set of registered signals illustrating correlation of various signals versus temporal evolution of neutron emission with its anisotropy data.

plus very HXRs (>80 keV) taken at 90° (f) and 0° (g) by PM tubes + scintillators.

All traces are moved according to the TOF of the corresponding type of radiation, as well as the delay times of related detecting systems (the dead time of PM tubes, cables, oscilloscopes, etc) except neutron pulses which move with their HXR pulses *synchronously* (i.e. by the TOF of HXR).

From these traces, in correlation with figures 6 and 9, one can observe the following features.

- SXR pulses from PMT practically coincide with the same from PIN diodes ((b), (c)) and with maximal plasma compression.
- HXRs (a) in the range 8–30 keV (presumably produced by runaways) correctly reflect fast electron signals (d) taken by Cerenkov detectors (pulses 1, 2 and 3) with one exception: the first and most intense pulse from Cerenkov detectors does not correlate with the above HXR trace; probably it is related to HXR of higher energy than the range 8–30 keV, because it is correlated with the beginning of the very HXRs seen on the two bottom traces.
- The 1st maximum of the neutron signal $Y(t)$ ((f), (g)) appears 323 ns after the SXR pulse maximum (PIN, PMT); this time interval is exactly equal to the TOF of 2.45 MeV neutrons from the pinch to the PMT; these maxima of both first neutron pulses delayed by 323 ns practically coincide with their first HXR maxima, and it is so for both (0° and 90°) directions again as for the shot reflected in figure 8; synchronization with figure 6 shows that the 1st neutron pulse coincides with maximal plasma compression and heating of at least its electron component.
- The second maxima of the neutron signals registered in both directions correlates with the second, relatively small,

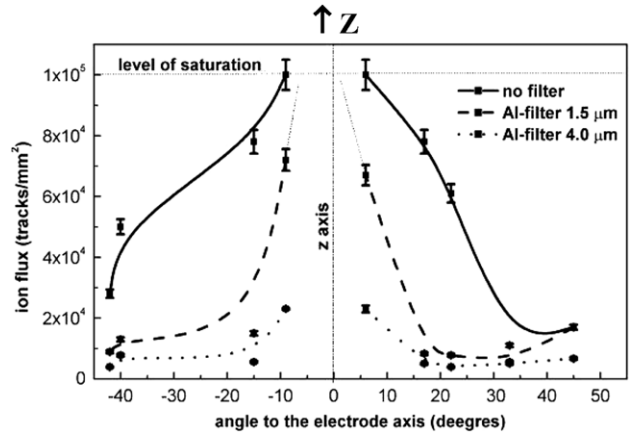


Figure 11. Angular distributions of fast deuterons deduced from track detector samples subjected to direct ion irradiation.

SXR pulse as well as with the HXR signals and the electron beam pulse. Again, as in the case of figure 8, the *side-on* N pulse is late for the start of its HXR pulse by 323 ns. However, the *head-on* N pulse is delayed only by 300 ns in relation to the moment when the *side-on* pulse appears inside the chamber; this means that the head-on neutrons have higher energy. Thus, if one moves this head-on pulse to its real place on the trace (the start should coincide with the start of the N pulse seen at 0°) it should be delayed by 23 ns in comparison with the side-on N pulse. After this procedure one can see that *both* second N pulses registered in two directions (0° and 90°) start with the beginning of HXR pulses, as in figure 8. Again, as in figure 8, both second pulses reach their maxima at the decay of their HXR pulses.

(10) The samples of nuclear track detectors were irradiated by fast deuterons emitted from a single PF-1000 shot, the same as in figure 10. After the irradiation these samples were etched under standard conditions and scanned with an optical microscope. The optical analysis shows the ion crater densities ranging from 10^3 to 10^5 craters mm^{-2} with this number increasing to the Z-axis up to a saturation level. To understand the results obtained we have to take into account the energy losses of D^+ ions in Al foils (D^+ ions of energy >250 keV can penetrate through about a $1.5 \mu\text{m}$ Al-foil and 500 keV ions through a $4 \mu\text{m}$ Al-foil) and the detection characteristics of the used detector [11, 12]. Thus, one could estimate that the uncovered detector samples recorded D^+ ions of energy above 80 keV, samples covered with a $1.5 \mu\text{m}$ Al foil registered D^+ ions more energetic than 330 keV and samples masked with a $4 \mu\text{m}$ Al foil revealed tracks of ions of energy >580 keV. It was, however, observed that this *fast* ion emission is not reproducible from shot to shot (at least less reproducible than the neutron yield), but its absolute yield as usual decreases with an increase in the filling pressure. Angular distributions of primary deuterons, having different energies and obtained in the above-mentioned shot, are presented in figure 11. In fact, the area of our track detector near the Z-axis was damaged in spite of its distant position from the pinch.

(11) Our miniature ion pinhole cameras show the track distributions as presented in figure 12. It is clearly seen

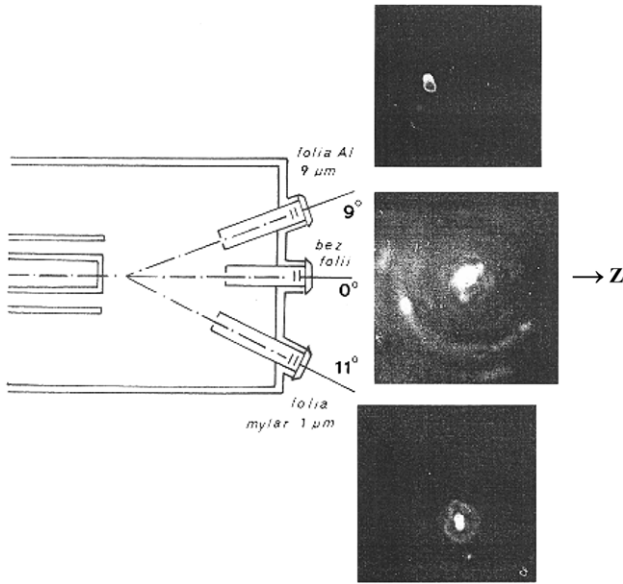


Figure 12. Spatial distribution of tracks received by three ion pinhole cameras placed at different angles to the Z-axis of the chamber.

that the image (an ‘autograph’ of the fast ion beam) has a tubular structure with an additional very bright maximum on the Z-axis.

(12) In the frame pictures (see figure 13) taken in the visible range with a 1 ns time resolution we found some structure within a zone above the pinch (on the right-hand side of frames (b) and (c)) which has good correlation with figure 12. This *tubular-conical* formation with a central stem (also of conical shape) always appears *after* the pinching period. (The pinching process is always accompanied by a characteristic *hemispherical* shock wave—see figure 13(a)).

These structures become visible right after the moment of current abruption. They changed from shot to shot by cone angle (e.g. the half-angle in (b) is 20° whereas in (c) it is about 35°) and by their transverse ‘layers’ (bright discs). For example, in both figures the thicknesses of these discs are less than 3 mm. But in (b) one may see just two of them separated at a distance of 10 mm whereas in (c) there are three discs with distances between them of 5 and 8 mm from left to right, respectively.

5. Discussion

In paper I we presented results on the main parameters of pinch plasma measurements as well as on their evolution during the period of generation of hard radiation. If the GPM [7] is valid this pinch is presumably a target, which would be irradiated by a fast ion beam generated within a DPF after current abruption phenomenon. The Bennett equation being possibly correct during the period of plasma confinement time (the so-called ‘first compression’) can help in the estimation of plasma density during the first neutron pulse (see paper I [6]): $0.8 \times 10^{19} \text{ cm}^{-3}$. The pinch radius also determines the *maximum* value of the azimuth magnetic field at the pinch’s border:

$$B_{\max} = 0.2I/r^2 = 2 \text{ MG}, \quad (3)$$

where $I = 2 \text{ MA}$ and $r = 0.45 \text{ cm}$. This means that the Larmor radii of fast (100 keV) electrons and deuterons, widely presented in the discharge, are accordingly [13]

$$r_{\text{Be}} \geq 3.37(W_{\perp})^{1/2}/B_{\varphi} \quad \text{and} \quad r_{\text{Bd}} \geq 204(W_{\perp})^{1/2}/B_{\varphi}, \quad (4)$$

where transverse energy $W_{\perp}$ is in eV, B_{φ} is in Gauss and r is in cm. It gives estimations for the *minimal* values of $r_{\text{Be}} \geq 5 \times 10^{-4} \text{ cm}$ and $r_{\text{Bd}} \geq 3 \times 10^{-2} \text{ cm}$. They both appear to be much less than the pinch diameter. As for the compressed B_z component being of the order of 10^5 G (see paper I [6]), these values are, respectively, 100 μm and 0.6 cm.

As was found in paper I the pinch’s column during the first neutron pulse is straight and has a height of 10 cm with a radius of $\sim 0.45 \text{ cm}$. Later on this plasma column is widened and disturbed by instabilities. All pinch parameters start to fall with the characteristic time of the order of the above plasma *confinement time* ($\sim 150 \text{ ns}$). As was shown in paper I the coefficient of the pinch longevity is 10–15 times larger in comparison with the ideal MHD confinement time. And as a consequence its *effective* expansion velocity v_{exp} is also 10–15 times lower compared with the implosion one v_{im} , i.e. $v_{\text{exp}} \leq 0.5 \times 10^7 \text{ cm s}^{-1}$. It is supported by the frame-by-frame pictures showing that to the moment of the maximum of the second neutron pulse the radius of the pinch is 1 cm.

Strong perturbations of the plasma sheath surface can be found after the confinement period in all its frame images. The pinch usually breaks into two (or sometimes several) cylinder regions along the column. It looks like a fast penetration (emission) of plasma into the surrounding magnetic field at the periphery of the pinch in the form of a meniscus, i.e. a disc centred at a certain point on the pinch axis and slightly concaved up to the cathode. For a pinch column it gives the impression of being like a gap creation, which ‘breaks’ the column in the transverse direction to its length. Examples are shown both in figures 9 and 13 of this paper and in some figures of paper I, but in particular in figures 7(d) and 14 of paper I [6]. It is seen that the plasma does *not* push away (disappear) from this gap, but becomes of a lower density compared with the adjacent parts of the pinch above and below this ‘gap’. A typical size of this zone along the Z-axis (the gap’s width) *seen in the figures* is about 1 mm, which is much larger than the electron Larmor radius, but is comparable to the ion one for the B_{φ} field, which is lower than its above-mentioned maximum during the process of its diffusion into the pinch plasma (see equation (4)).

We shall start our discussions concerning fusion events taking place in PF-1000 with estimations of the possibility of a *thermonuclear* mechanism for neutron generation during first and second pulses. We shall assume that the first pulse is produced by a compression of a fast moving ($3.5 \times 10^7 \text{ cm s}^{-1}$) plasma-current sheath (PCS) with the transformation of the energy of its direct movement (quasi-cylinder shock wave) into heat with additional subsequent adiabatic compression (the temperature increase factor is about 1.4 [14]). As for the second neutron pulse we shall propose at first that it is generated by the second subsequent plasma compression.

Taking into consideration plasma parameters during the first period of the plasma confinement (see paper I [6])—its ion density $n_i \cong 0.8 \times 10^{19} \text{ cm}^{-3}$, ion temperature $T_i \cong 1.3 \text{ keV}$, pinch dimensions $R_p \cong 0.45 \text{ cm}$ and $h_p \cong 10 \text{ cm}$ and pinch longevity $\tau \cong 1.5 \times 10^{-7} \text{ s}$, we have [13]

$$Y = (n_i^2/4)(\sigma v)_{\text{DD}} \pi R_p^2 h_p \tau \cong 1.5 \times 10^{10} \text{ n pulse}^{-1}. \quad (5)$$

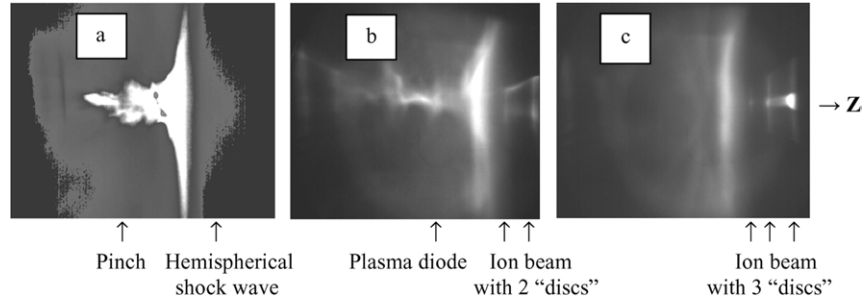


Figure 13. 1 ns pictures taken in the visible range for three different shots and demonstrating a shock wave produced by a cumulative stream during the plasma pinching (a) and an ion beam structure as it appears after the current abruption phenomenon (b) and (c).

This amount is ten times less than the total neutron yield of the device ($(1-2) \times 10^{11}$ n pulse $^{-1}$), and it coincides with the results of measurements made by PM tubes (figures 8 and 10).

As for the more hypothetical ‘second compression’ (see, e.g., figure 9 the 4th frame taken at the moment $t = +87$ ns, the picture looks as it should for this event, being however 100 ns earlier than one can expect according to the second neutron pulse maximum), we have to nevertheless substitute figures really seen for the moment of the 2nd neutron pulse maximum ($t \cong +180$ ns): for the pinch radius $R_p \cong 1$ cm, ion density $n_i \cong 2 \times 10^{18}$ cm $^{-3}$ (calculated due to pinch radius change) with other parameters having the same values. In this case in order to obtain from formula (5) an experimentally measured real neutron yield 2×10^{11} n pulse $^{-1}$ we have to assume that the ion temperature during this interval of time should be increased to the level of $T_{i\text{eff}} \geq 4$ keV.

The most dubious data in both cases are the ion temperatures, which were not measured here directly (and have not been reliably measured in all previous experiments). Let us check the possibility of the existence of these temperatures in a real situation. We can estimate the mean-free path of ions in our plasmas by using the following equation [15]:

$$l_{ii} = 3 \times 10^{18} T_i^2 / n_i, \quad (6)$$

where l_{ii} is in cm, T_i in keV and n_i in cm $^{-3}$. The results are shown in table 1.

It is clearly seen that if for the first compression the mean-free path of ions is smaller compared with the pinch’s dimensions, the plasma of the ‘second one’ should be collisionless.

Estimations of the characteristic *collision rates* can be made using equations [15, 16]

$$v_e = 2.9 \times 10^{-6} n \lambda T_e^{-3/2}, \quad (7)$$

$$v_i = 3.4 \times 10^{-8} n \lambda T_i^{-3/2}. \quad (8)$$

These estimations have supported the above conclusion: for the first compression stage equilibrium for electrons and for ions is established for less than 0.2 ns and 20 ns, respectively, which is short compared with the duration of this phase (150 ns). The time to achieve *thermal equilibrium* between ions and electrons (here we have presumably ion heating by SW and adiabatic compression) would be $(m_d/m_e)^{1/2} \cong 60$ times longer. It would mean that even here electrons might not have enough time to reach the level of ion temperature. In contrast, the above time intervals for the hypothetical ‘second compression’ (20 ns and 2 μ s for electrons and ions separately

and even larger for their mutual equilibrium) are substantially longer than the duration of the second neutron pulse.

One can now see that the first pulse can be explained *in principle* on the basis of shock wave/adiabatic plasma compression and heating. Conversely, for an explanation of the generation mechanism of the second neutron pulse we have to come up with other ideas.

According to our analysis of the data obtained here we have evidence of the virtual plasma diode creation at the PF-1000 facility. That is we have here the same phenomena which were investigated in the FLORA device [3, 4, 14] and proved in those experiments by almost the same set of diagnostics as here but additionally supported there by 1 ns multi-frame laser interferometry.

Indeed till the moment of the current maximum the main part of electric energy accumulated previously in the bank is concentrated as magnetic energy near the pinch column, i.e. in the ‘plasma inductive storage’. Then we have an abruption in the current and the formation of a plasma diode on the pinch. The effects in the diode lead to an evolution following the scenario (compare the experimental sequence of frames in figure 9 presented in the lower part of the double-pinch structure—see paper I [6]—with the schematic drawing of figure 14(a) made for this lower pinch and with figure 14(b), where the region of the EMH effects [8] is shown on an enlarged scale).

- (1) *Rayleigh–Taylor instability* development on the surface of the pinch (with an increment in the unstable state development of the order of 10^8 s $^{-1}$).
- (2) Formation in a certain pinch region along the perimeter of the pinch column (*above* plasma neck but *below* plasma widening—i.e. in a region shown by the symbol ‘ $\leftrightarrow$ ’) of the *right-hand-triple* of vectors $\{\mathbf{H}_\varphi, \text{grad } n, \mathbf{V}_h\}$, where the vector $\mathbf{H}_\varphi$ is the azimuth magnetic field of current, $\text{grad } n$ is the plasma density gradient and $\mathbf{V}_h$ is the vector of the magnetic field penetration direction.
- (3) Fast increase in the *anomalous resistivity* R_{an} in this ring-like region of a skin-layer because of a number of microinstabilities (with increments of the order of 10^{13-11} s $^{-1}$) provoked by the EMH field effects, which resulted in *plasma disc-like release* outwards from the pinch (practically with gas-dynamic velocity of the penetration into the ‘confining’ magnetic field $\mathbf{H}_\varphi$) as well as in the fast *meniscus-like convective penetration* of the magnetic field into plasma (anomalous resistivity R_{an} can be deduced from the electro-technical measurements of current and voltage across the pinch column and also evaluated from EMH theory [8]).

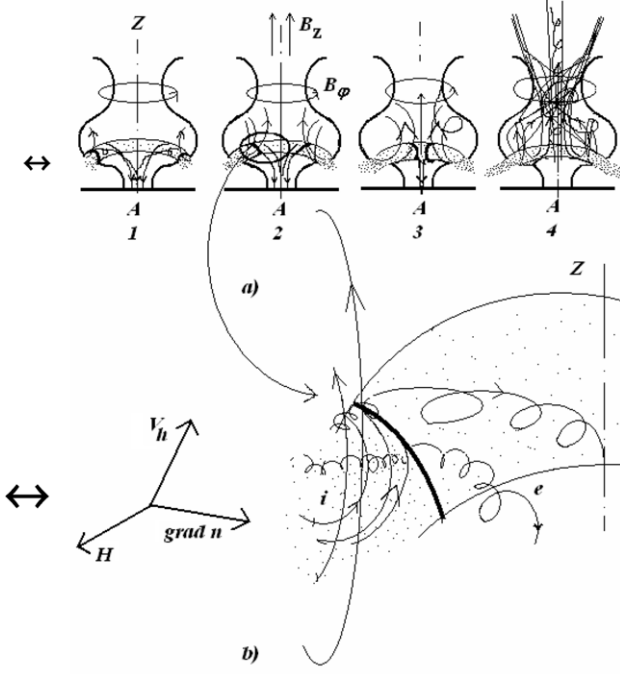


Figure 14. Model pictures of the plasma diode evolution (a) and (b) based on [3, 8, 14].

- (4) *Current cutoff* within the skin-layer due to R_{an} followed by induction of a high vortex *electric field* $\mathbf{E}_{ind}$ according to Maxwell's equation: $\partial \mathbf{H} / \partial t \sim \text{rot } \mathbf{E}_{ind}$ inside the gap of the magnetic field penetration; thus, this process forms a virtual 'plasma diode' with the 'anode' width of the order of the pinch diameter and the 'anode-cathode' separation determined by a subsequent process of e-beam formation based on a parapotential model [16, 17], which permits the carrying, through the gap, of the e-beam I_b of about the same current magnitude as the previous collisional current I_c :

$$I_b \cong 8500 \beta \gamma r / d [\text{A}] \approx I_c, \quad (9)$$

where β and γ are relativistic factors, r/d is the so-called *aspect ratio*, i.e. the ratio of the radius of the diode $r \approx r_p$ to the distance d between the virtual anode and the cathode, and I_b and I_c are in amperes.

- (5) *Acceleration of electrons* by the above field ($\mathbf{E}_{ind} \sim (\Delta \mathbf{H} / \Delta t) \times d$), generated during the cutoff process, where $\Delta \mathbf{H} \sim 0.2 I_p / r_p$, Δt is the current's cutoff time ruled by increments of microinstabilities and measured by the rise-time of the second HXR pulse and d is the diode separation, defined from equation (5) and estimated from figure 9 or from figures 7(d) and 14(a) of paper I.
- (6) *Self-focussing of the e-beam* inside the pinch and the *propagating* of it right up to the anode (see, in figure 9, images related to the moments $t_1 = +5$ ns and $t_2 = +15$ ns and the schematic drawing in figure 14(a) in the sequence 1, 2, 3).
- (7) *Interaction of fast electrons* with the anode surface, resulted in production of *HXRs* and *vapours* of anode material glowing in the visible and the SXR range (figure 9, frames 2 and 3).

Table 1. Estimates of the mean-free paths of ion-ion collisions in *thermal* plasma for the first and second neutron pulses for different diameters and heights of the pinch.

Neutron pulse	Time of pulse maximum (ns)	$T_{i \text{ eff}}$ (keV)	n_i (cm^{-3})	R_p (cm)	h_p (cm)	l_{ii} (cm)
First	0	1.3	8×10^{18}	0.45	10	0.6
Second	+180	4	2×10^{18}	1	10	24

- (8) *Magnetization of the fast electrons* after penetration of the magnetic field up to the centre of the virtual diode; this comes from the fact that the Larmor radius of electrons is short compared with the diode gap d —see the disappearance of the *filament* luminescence between the diode and the anode in figure 9 ($t_3 = +35$ ns and $t_4 = +87$ ns) and the corresponding explanation in figures 14(a) and (b).
- (9) *Substitution of the e-beam by the i-beam*, taking place *simultaneously* with the above process; it becomes possible because the Larmor radius of the accelerated ions is larger than d ; the currents of the fast electron and ion beams carry approximately the total pinch current I_p during the time interval τ , for the period of which the plasma inductive storage releases its magnetic energy into anomalous resistivity of the virtual diode:

$$\tau = L_{int} / R_{an}, \quad (10)$$

where L_{int} is the internal inductance of the discharge chamber (probably with some part of the external one related to the part of the electrical circuit adjoining the DPF chamber).

Later on, these parts of the fast ions, which were accelerated to *medium* energy (i.e. to the energy at which their Larmor radius in the magnetic field inside the pinch is of the order, or less than, the pinch radius), should be magnetized. It takes place mainly above the virtual diode gap, i.e. inside the upper part of the pinch [3, 4]. They gyrate during the time interval determined by their confinement time, which could be, in principle, longer than that given by (10) and even longer than the plasma diode existence time. During this *confinement time of fast ions* they interact with the pinch plasma ('hot target') [3–5].

To 'fill' the whole pinch volume by these medium-energy ions, to confine them there and to give them the possibility of going away eventually from the pinch we have to suppose the existence of a longitudinal component of magnetic field (along the Z-axis) $\mathbf{H}_z$. Its origin, magnitude ($|\mathbf{H}_z| \sim 0.1 |\mathbf{H}_\phi|$) and stabilizing effect on the pinch were discussed in paper I [6] (see also the corresponding references there).

Now let us see how this overall physical picture is reflected in our full-scale experiment at the PF-1000 facility fulfilled with a number of different diagnostics and what are the magnitudes of the parameters appearing in the above-mentioned model. In these discussions we shall widely use the results of paper I giving us the temporal behaviour of dense plasma ('pinch', 'target') parameters.

Let us examine the possibility of applying to our case the above-mentioned model based on the virtual plasma diode, the direct production of neutrons by fast ions and ion beam heating of the pinch plasma. We first check the validity of the parameters measured here with the model's variables.

1. Diode geometry

To be in conformity with formula (9), assuming the current of fast electrons (for $E_e \cong 100$ keV, $\beta = 0.62$, $\gamma = 1.28$) of the order of the pinch current, we shall have for our virtual diode ($r \cong 0.45$ cm) $d \cong 0.15$ mm, i.e. for 100 keV particles we have to have a vortex field $E \cong 10^7$ V cm $^{-1}$; at the same time by these estimations the validity of the demands on the magnetization of the electrons and the free-streaming of the ions within this diode is established (see equations (4)):

$$r_{Be}(\geq 5 \times 10^{-3} \text{ cm}) < d < r_{Bd}(3 \times 10^{-2} \text{ cm}). \quad (11)$$

2. Current abruption time

Estimations using the Maxwell equation: $E \cong \Delta H \times \Delta x / \Delta t$ (where dimensions are in V cm $^{-1}$, G, cm and s, respectively), gave us for the above $E \cong 10^7$ V cm $^{-1}$ $\Delta t \cong 10^{-11}$ s, which is short compared with the resolution time of our PMT channels (pulse rise-time) for the HXR pulse (the fact supported by experiments) and is of the order of the ion-sound instability increment.

3. Beam existence time

Now we can estimate the diode impedance: $R \cong U/I \cong 10^5/2 \times 10^6 = 5 \times 10^{-2} \Omega$ ($U \sim E \times d$). This is a rather poor value—in the best devices the impedance can be up to 0.3–0.5 Ω [1, 2]. According to equation (10) with the inductance of our ‘plasma magnetic storage’ equal to $\sim 10^{-8}$ H we shall have for the energy release time τ_L from the plasma inductive storage

$$\tau_L \cong L_{int}/R = 200 \text{ ns}. \quad (12)$$

The real time of the ion beam existence should be at least 2 times shorter (ions substituting electrons), i.e. $\tau_i = 100$ ns. In fact, as is seen from the oscilloscope traces of figure 10, the HXR pulse at its FWHM is 50 ns. The same should be right for the ion beam. Thus let us count $\tau_i = 50$ ns. It is noticeably shorter than the neutron emission duration in the normal regime and in particular for the case of the ‘cusp-like’ plasma configuration (see paper I [6]). Together with the fact of the vanishing plasma diode after the moment of about +250 ns in the pictures it means that the *confinement time of fast ions* within the plasma cloud is large compared with the diode existence time and in particular with the time of the energy release from plasma inductive storage.

4. Energy of the beam

Now we can estimate the overall energy of the e- and i-beams:

$$W = I \times U \times \tau \cong 2 \times 10^6 \times 10^5 \times 10^{-7} = 20 \text{ kJ}. \quad (13)$$

It gives the efficiency of the beam generation on the level of about 2.5% from the power circuit consumption. Taking into consideration the figures for the highest beam efficiency reported by the Kurchatov Institute team (20%), the Limeil Laboratory (20%), and the Lebedev Institute team (10%), measured by different methods, the figure obtained must be considered to be a very modest one.

Let us examine the possibility of the above *containment* of the fast ion beam inside the pinch. As was shown the *minimal* ion Larmor radius in the azimuth field is much smaller than the size of the pinch. We can now estimate the *minimal* average

magnetic field magnitude (for both H_ϕ and H_z) inside the pinch provided that the ion Larmor radius is about the pinch radius:

$$B \geq 204(W_\perp)^{1/2}/r_{Bd} \cong 3 \times 10^4 \text{ G}. \quad (14)$$

It is close to the above estimate. To have a rough figure for the coefficient of the fast ion ‘magnetization’ we must compare their direct-flight time through the pinch’s length l_p with the duration of the second neutron pulse τ_n . We shall have for the group of ions having energy $E_i = 100$ keV

$$k = \tau_n/(l_p/v_i) = 1.5 \times 10^{-7}[\text{s}]/\{10[\text{cm}]/3 \times 10^8[\text{cm s}^{-1}]\} = 4.5. \quad (15)$$

Now we can estimate the *concentration* of the 100 keV ions *inside the pinch* during their confinement period provided that the pinch radius is 1 cm, its height is 10 cm and that all other parameters are the same. Then the length of the fast ion bunch in a free space should be $l_b = v_i \times \tau_i = 15$ cm, the cross-section of the bunch inside the pinch is 3.14 cm ^{2} , the effective volume of the bunch (provided that it is not widened, which is true inside the pinch) $V_b = l_b \times S_b \cong 50$ cm ^{3} , the full energy in the bunch $E_b = 10^4 J = 6 \times 10^{22}$ eV, the total number of ions in the bunch $N_{tot} = E_b/E_i = 6 \times 10^{17}$ particles, the concentration in the bunch $N_{i0} = N_{tot}/V_b \cong 10^{16}$ cm $^{-3}$ and taking into consideration the above coefficient of magnetization we shall have the fast ion concentration inside the pinch:

$$N_i = N_{i0} \times k \cong 5 \times 10^{16} \text{ cm}^{-3}. \quad (16)$$

According to the track detector measurements (see above) the main part of our fast ions *abandoning* the pinch is concentrated approximately within the energy range below 200 keV. Unfortunately, it is difficult to say how representative this measured part of fast ions is. Namely, the surface covered by fast ions at a distance of about 0.5 m and giving a possibility of investigating these fast ions (*if* the ion beam preserves its contents within the cone of about 20°–30° with a stem of just a few degrees at this distance) should be, in our case, of the order of 0.1 m ^{2} . That is, this area could collect, in the unsaturated regime, only the number of ions N , which escaped the pinch:

$$N \sim 10^5[\text{d mm}^{-2}] \times 10^5[\text{mm}^2] \cong 10^{10} \text{ ions}.$$

This is about 10^{-8} from the overall number of generated fast ions. Moreover, it seems that our high-current ion beam can exist as a beam only from the rear side (behind) of the SW front, where it can be compensated by an electron back-current inside the cloud of a compressed ionized plasma. After penetrating the SW front (which is about 30 cm from the anode till this moment) and injecting into a neutral gas of low density, where the mfp of fast ions is long compared with the geometry of the main part of it, the ion beam has to be disintegrated. But even the remaining part of the ion beam will still be of much higher concentration compared with the saturation level of the track detectors. Thus, all *measured* ions are distributed at a distance of 0.55 m from the anode on the periphery of the near-axis zone in an arbitrary manner. However, in spite of the facts described here it is reasonable to suppose that the energy range of our ‘acting’ fast ions (i.e. those mainly *captured* inside the pinch and producing neutrons— not those, which *escaped* the pinch) falls within the energy spectrum interval of about 10–100 keV.

Neutron spectra, depending on the energy of the fast deuterons E_d bombarding the deuterium gas/plasma target and the angle of their registration Θ , will have a peak at an energy E_n , which can be deduced from the equation [18]

$$E_n = (3.269[\text{MeV}]) + E_d + 2\sqrt{2}(E_n \cdot E_d)^{1/2} \cos \Theta. \quad (17)$$

For $E_d = 0$ and $\Theta = 90^\circ$ we shall have, as well known, $E_n = 2.45$ MeV. At the same time for *our* data on a side-on spectrum of DPF, which demonstrates the same peak position, it only means that the energy (and velocity) distribution of fast ions is peaked at 0 MeV, i.e. we might have rotations of fast ions in both directions *around* the Z -axis—clock-wise and counter-clock-wise—equally possible. It is so, in particular, because estimations of the probable plasma temperature deduced from the FWHM ΔE of the spectra in this very direction ($\Delta E = 72.5(T_{pl})^{1/2}$) usually gave us a value of about 3–4 keV, which is impossible as we have shown above. However, it should be noted that neutron spectral measurements have not been provided in these series of experiments.

The head-on neutron spectra for the energy of fast ions of $E_d = 100$ keV moving *along* the Z -axis and *producing* these neutrons gave us the following data for its peak: $E_n = 2.85$ MeV. This is reflected in the anisotropy factor, which is connected with the energy of fast ions (neutron energy spectrum shift) according to the kinematics of the reciprocal fusion reaction [18].

The neutron yield distribution in different directions to the path of the beam of fast deuterons in the case of 100 keV deuterons will look as shown in figure 15. The angular distribution of the cross-section in the centre of the inertia system is described by the formula

$$\sigma(\Theta) = \sigma(\pi/2)(1 + A \cos^2 \Theta). \quad (18)$$

We compared the data for the experimental coefficient A measured by many authors in the 1950s and 1960s for different energy ranges of deuterons and in various geometries of experiments and took its value for our case to be equal to 0.8.

A comparison of the experimental data of figure 7 with the computed ones according to equation (17) demonstrates fairly good agreement. One has to take into account that because our chamber is made of steel with thick walls we put our activation counters at positions where the thickness is the same for all directions except 180° . But our estimations have shown that the influence of the materials in this direction (copper anode lid and stainless steel flange) on neutron penetration is about the same as in all other directions. However, the experimental curve has two distinctions: the absence of the yield's decrease ('neck', 'waist') on distribution at 90° and slightly lower values of the yield along the Z -axis (0° and 180°). However, if one takes into account rotations of the entrapped fast ions inside the pinch in contrast to the parallel beam, they both vanish, first by the disappearance of the distribution valley and subsequently the ratio of the yields.

We can now estimate two things—direct neutron production by *fusion collisions* of these *magnetized* ions with plasma and plasma *heating* because of *Coulomb interaction* of these fast magnetized ions with the pinch's plasma.

Let us examine first a relaxation of the ion beam inside the pinch. We shall be interested whether our ion beam can

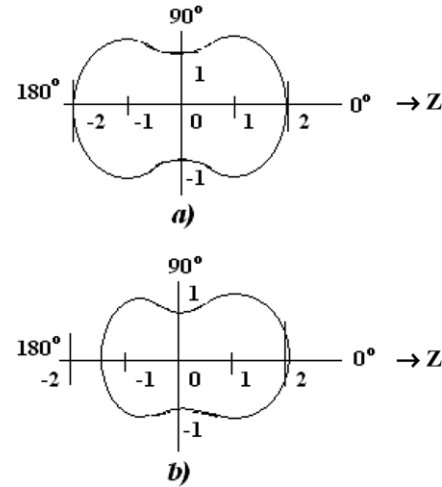


Figure 15. Theoretical angular distribution of neutron intensity produced in a thin gas target by a low-intensity parallel beam of 100 keV deuterium ions as test particles: (a) in a centre-of-mass system and (b) in a laboratory coordinate frame.

lose its energy and add it to the bulk plasma by a comparable degree in relation to the pinch's own energy content.

It is a well-known fact [15, 16, 19] that fast ions of energy E_i lose their energy to the bulk plasma via Coulomb collisions with *field* electrons and/or ions depending on the plasma temperature T_{pl} . When $E_i > (m_i/m_e)T_i$ the ions are cooled by Coulomb drag on electrons and undergo little angular scattering. Within the range $T_i < E_i < (m_i/m_e)T_i$ most collisions happen with *field* ions and a strong *scattering* of ions occurs. In our case the above latter energy interval lies in the range between 10 and 400 keV.

We shall use for these estimations the following equation valid for the so-called 'slowing-down' time $\tau_s^{i/i}$ of the deuterium i-beam relaxation on *field* ions (the energy loss time for this beam is correspondingly longer by a factor of $(m_i/m_e)^{1/2}$) [15, 16]:

$$\tau_s^{i/i} = (2)^{1/2} E_i^{3/2} / 9 \times 10^{-8} n_i \lambda, \quad (19)$$

where λ is the Coulomb logarithm, e is the electron charge and E_i is the energy of fast deuterons in eV. This formula is true if $m_i V_i^2 / 2kT_i \ll 1$ as in our case. Taking into account the pinch's radius, its height (1 cm and 10 cm, respectively) and its ion density $2 \times 10^{18} \text{ cm}^{-3}$ one can see that the 'slowing-down' time of 100 keV ions is $2.4 \times 10^{-8} \text{ s}$ (i.e. smaller compared with the neutron pulse duration time). It gives the mean-free path of 7 cm for the slowing-down time (strong scattering of ions on ions), whereas to establish the Maxwell distribution function the time will be about $1.5 \times 10^{-6} \text{ s}$. For ions within the energy interval 10–100 keV it will be noticeably shorter. On the other hand, ions in the above energy range (velocities $(1-3) \times 10^8 \text{ cm s}^{-1}$) being magnetized should make 2–10 rotations inside the pinch during the neutron pulse duration (150 ns) thus increasing their trajectory length to $\sim 50-200 \text{ cm}$ during the *plasma* confinement time.

Now let us check the slowing-down time of these ions on *field* electrons for our case $m_e V_i^2 / 2kT_e \ll 1$ (where T_e is the field electron temperature):

$$\tau_s^{i/e} = (2)^{1/2} T_e^{3/2} / 1.6 \times 10^{-9} n_e \lambda = 5 \times 10^{-8} \text{ s}. \quad (20)$$

The result is close to that obtained by equation (19). It is not surprising. The scattering process is greater for particles with smaller velocity differentials. In our case the velocity of fast ions ($3 \times 10^8 \text{ cm s}^{-1}$) is about five times higher than the field-ion velocities ($\leq 5 \times 10^7 \text{ cm s}^{-1}$) and also about five times less than the field-electron ones ($\geq 1.5 \times 10^9 \text{ cm s}^{-1}$). However, the ion slowing-down time interval is determined by two–three collisions with ions and about 100 collisions with electrons. This fact explains our estimations made using equations (19) and (20). It means that our fast ions interact with both plasma components in almost a comparable degree with a certain preference for field ions and with one important feature—the scattering character is strong for the ion component changing the direction of the decelerated fast ions, which was mentioned above. Thus, between the two parts of fast ions (approximately quantitatively equal but both small compared with the total number generated in the PF) the group *slowing down on field ions* is *isotropic* in contrast to those stopped by field electrons. Later on, because of the fact that our pinch has a shorter confinement time compared with the ion energy loss time (the time of establishment of the new Maxwellian distribution), namely, the first group of decelerated ions will be magnetized and this group will produce the main part of fusion neutrons.

It means that about $\sim 1/8$ – $1/4$ of the fast ions will lose their energy inside the plasma. Thus, $\cong 2 \text{ kJ}$ of the beam energy will be deposited inside the pinch within the field particles (mainly on the ion component). As a result, each particle of the ion component of plasma will acquire an additional portion of energy: $\sim 2 \text{ kJ} / n_i V_{pl} \geq 0.5 \text{ keV}$. It is comparable to the initial thermal one ($\sim 1.3 \text{ keV}$). At this increased temperature $T_i = 1.8 \text{ keV}$ the mean-free path for ions (equation (6)) is $< 5 \text{ cm}$ —still less compared with the pinch height. It can explain a partial neutron yield increase during the second pulse: according to equation (5) with an increase in volume by four times (radius by two times), a decrease in plasma density by four times, but with an increase in the cross-section of fusion reactions [16] by about ten times:

$$Y = (n_i^2/4)(\sigma v)_{DD} \pi R_p^2 h_p \tau \cong 4 \times 10^{10} \text{ n pulse}^{-1}. \quad (21)$$

On the other hand, a 10 kJ beam of 100 keV ions contains 6×10^{17} particles. For these particles equation (5) will look as follows:

$$Y = (n_{ipl} N_{i \text{ fast}}/4) \sigma v_{i \text{ fast}} \pi R_p^2 h_p \tau, \quad (22)$$

where $n_{ipl} = 2 \times 10^{18} \text{ cm}^{-3}$ is the plasma density, $N_{i \text{ fast}}$ the concentration of fast ions within the pinch and $v_{i \text{ fast}}$, σ the velocity of fast ions and fusion cross-section, respectively.

Taking into consideration the concentration of these fast (100 keV) ions in the bunch, their magnetization inside the pinch and using the fusion cross-section for 100 keV deuterium ions equal to $1.5 \times 10^{-26} \text{ cm}^2$ [18] one will obtain a figure for the total neutron yield of 10^{12} neutrons/shot, which is a bit high compared with the best experimental one. But this exaggeration can be easily explained by a lower real concentration of fast ions within the pinch plasma.

The overall result for the fast ion interaction with the hot dense pinch's plasma can be reformulated now in the following manner: the situation for the primary ('thermonuclear') model of neutron production presented in the first hypothesis is remarkably *improved* by the second mode described here. Namely, the above inductive (rapid in comparison with the

collisional one!) mechanism of generation of fast ions within the DPF's plasma will form the overall distribution function with an *enriched high-energy tail* that usually produces most fusion reactions.

Now we have to link these results to our data on the ion beam observations made for the *outside* part of the pinch (figures 11, 12 and 13). First let us propose that the conical structures seen in figures 13(b) and (c) are produced by a beam of fast ions, which escape the pinch and heat/ionize the residual gas/plasma in this zone on the rear side of the SW, thus becoming visible itself. Indeed this explanation of the cone-tubular structure type is quite reasonable. In fact, inside the pinch we have a superposition of longitudinal (captured by PCS and compressed) and azimuth (compressing and diffusing) magnetic fields. And in the centre, as well as outside the pinch (above it), H_z prevails whereas on the periphery *inside* the pinch the situation is in favour of H_ϕ . Above the pinch the force lines of the longitudinal (mainly presented here) magnetic field should fan out (diverge in a cone structure).

Because of this, in the close vicinity of the Z-axis fast ions are accelerated along the singularity line of the *azimuth* magnetic field (so without any influence of it) thus forming the above-mentioned stem with a slight divergence. We believe that this stem is reflected in the pictures of figure 13. At the same time the conical-tubular structure of the ions leaving the pinch can be formed by these *higher energy* ions, which made only a few (or just one incomplete) gyrations inside the pinch and were collected near its generatrix. And because the cone-like shape is specific for the upper part of the pinch (and for the longitudinal magnetic field connected with it) their stream acquires this shape. It is seen both in the direct-irradiated track detectors of figure 11 and in the pictures of figure 13.

Because the generatrix of the cone outside the pinch at a distance of about 10 cm has a regular cone shape it is clear that the *external* magnetic field (outside the pinch) cannot change the orbit of 100 keV ions. This means that its magnitude is 10^{-2} MG or less. This is the *upper* limit for H_{ext} having mainly longitudinal components and produced by the compression of an initial small field as was mentioned above. The *smallest extreme* of the field, at a distance of $\sim 10 \text{ cm}$ from the pinch, can be roughly estimated by the inverse quadratic distance law $-H_{\text{ext}} \sim 1/r^2 \sim 10^3 \text{ G}$.

The situation described above and connected with the disintegration of the ion beam after penetrating the SW front could explain why we cannot see this 'centred ring' in figure 11 but can see it with the ion pinhole chambers in figure 12. Indeed, because this scattering of the ion beam has a random character any pinhole positioned at certain distances from the SW front will form an image of the source of the ions similarly to the optical case of an image produced due to light scattering by an object. At the same time in the above weak fields the Larmor radius of ions of energy above 100 keV will be $\sim 100 \text{ cm}$ —i.e. large compared with the value for our distance from the pinch to the SW front and the two copper semi-rings with track detectors. This means that the ions, which travel first along the conical generatrix, later on scatter randomly. This is why we do not see them in figure 11.

But there is something more. The 'discs' (layers) seen in figure 12 and taken with a 1 ns time exposure reflect the temporal structure of the ion beam. Namely, the velocity of

ions is above $3 \times 10^8 \text{ cm s}^{-1}$. In 1 ns they pass a length of less than 3 mm. Namely, this is the width of the discs we saw in the picture (it should be mentioned here that only a small fraction of these fast ions interact with the heat plasma in this rear part of the SW because the collisional cross-section for these ions is low). The verification of the plasma cooling rate (and photo- and triple-particle recombination processes in deuterium gas [20] if it has already become neutral) showed that their time is short compared with 1 ns (our exposure time of the frame camera). It means that the ion beam consists in fact of a sequence of multiple pulses having duration ≤ 1 ns each and separated from one another by distances 5–10 mm, i.e. by time intervals 2–3 ns.

In connection with all the above estimations we have to make three remarks. First, examining the virtual diode configuration one can note a very small effective separation between the virtual anode and the cathode—only $150 \mu\text{m}$ with a large diode aspect ratio. It seems that such a geometry of the diode should be unstable during this unsteady-state plasma phenomenon. Probably it has an oscillating character, which is reflected in those short bunches seen in figure 13.

Second, one can see that estimations made by using equations (5) and (21) are very sensitive to plasma/beam parameters. In particular, this relates to equation (5) where density is presented to the power of 2. Moreover, a thermonuclear reaction rate averaged over the Maxwellian distribution depends on this formula on plasma temperature to the power of 4.5 within the range 1–5 keV (below 1 keV even faster). Thus, in this situation a really astonishing fact is *not* the large neutron yield *variations* in a DPF from shot to shot (usual criticism to this device), but its relatively *stable* average behaviour (within a factor of 2 in good facilities)! Probably this fairly fine performance is an additional argument in favour of the second neutron production mechanism: still fusion head-on impact cross-section on the level of fast ion's energy near 100 keV follows the ion energy change even slower than a linear law. Very likely this remark relates not only to the second neutron pulse but also to the first one: this yield looks more stable than PCS velocity changes. We know that usually when we have fast electrons ('runaways' during the first compression of plasma) we can also expect fast ions.

Our third remark is connected to the ions escaping the pinch in the Z-axis direction. According to our measurements and analysis they have slightly higher energy compared with the 'working ones'. Leaving the pinch they interact inside our large chamber (2 m to the chamber wall) with residual gas having a density of $\sim 2 \times 10^{17} \text{ cm}^{-3}$ (4 Torr D_2). We suppose here that the effective distance of interaction is equal to $l_{\text{int}} = v_i \times \tau_n = (3-4) \times 10^8 \times 1.5 \times 10^{-7} \cong 50 \text{ cm}$. We have also to take into account that the duration of the ion beam (and consequently the volume occupied by it) is three times shorter than the neutron pulse duration (i.e. 50 ns instead of 150 ns) and there is no appreciable magnetization effect. Thus, the ratio of the external neutron yield to the internal one will be

$$N_{\text{n(Ext)}}/N_{\text{n(Int)}} \sim \{(N_i n_i) V\}_{\text{Ext}}/\{(N_i n_i) V\}_{\text{Int}} < 1/20. \quad (23)$$

It means that the neutron yield from the space above the pinch is less compared with the yield from the pinch during the first neutron pulse. For simplicity, here we neglect the ionization loss, which is not low (!); thus the obtained figure has a certain

exaggeration. Our preliminary measurements with collimation of the neutron emission from the PF-1000 chamber support the result of equation (23). This is a beneficial consequence of a plasma density increase, its heating and confinement (*hot compressed target*) as well as the *magnetization* of fast ions.

Being guided by these results let us now discuss possible ways for the PF-1000 facility optimization. As was mentioned above, during our experiments we found that a *simultaneous* increase in the charging voltage and the initial gas pressure raises the device neutron yield. We observed that in the ranges of pressure 1.0–3.6 Torr and of voltage 30–36 kV the current of the device increased linearly with both parameters. Thus, in the course we have practically reached the upper limit of the operational regime of our device. At the same time, as seen from the oscilloscope traces in figure 2, the current is not changed during the formidable part of the 1st quarter of the discharge period (after the first $3.5 \mu\text{s}$). In fact, it demonstrates that at the end of this time interval the current has the character of a plateau with a tendency for a certain decrease (beginning from the 5th microsecond till the peculiarity occurs at the 8th microsecond). It means that the internal dynamical inductance of the device (its PCS) increases approximately at the same rate as the quasi-cosinusoid current is increased thus compensating it. So the first step for device optimization could be done by a decrease in the *internal* variable inductance of the DPF chamber executed by anode tube shortening. Indeed because of the coaxial configuration the overall internal inductance of the gun is expressed by the formula

$$L_{\text{int}} = 2l_{\sim} \times \ln(R/r_{\sim}), \quad (24)$$

where $l_{\sim}$ is the length of the part of the anode tube included into the electrical circuit during the PCS travelling *along* it, R is the radius of the cathode rod and $r_{\sim}$ is the radius of the pinch, which is included into the electrical circuit as the variable internal conductor of the coaxial part of the circuit's inductance during the PCS travelling *radially* to its implosion. It is seen that during the run-down phase internal inductance increases linearly with $l_{\sim}$ whereas at the implosion stage its enlargement proceeds much slower following a logarithmic law with radius $r_{\sim}$. Thus, it is seen that this re-design of the anode will give an opportunity for eliminating the 'plateau' on the current waveform and for increasing the amplitude of the discharge current as it is approximated from the current waveform by at least about 30%.

Another opportunity becomes clear from the total *external* inductance measurements. It appears that this is of the order of 20 nH at the present moment. At the same time it is known that an increase in the number of capacitors N in a battery should result in a decrease in its inductance according to a *practical* law valid for large batteries: $L_{\text{ext}} \sim N^{1/2}$ (an increase in the battery size and consequently its inductance must be *partially* compensated by the *parallel* operation of the increased number of capacitors). It means that in our case the external inductance for our bank must really be on the level

$$L_{\text{ext}} \cong \{(40 \text{ nH}) : (264)^{1/2}\} \cong 6 \text{ nH}, \quad (25)$$

i.e. three times lower in comparison with the actual one. According to our estimations the main impact in our present external inductance is produced by our current collector. Its

new design might improve the situation tremendously. We may expect an increase of the current in this situation by a few times. This hope is based on our previous experience with the PLAMYA facility [1f] where we had a total current of 2.5 MA and a neutron yield (not yet completely optimized due to restrictions implied by the outer diameter of the DPF chamber used) of the level of 2×10^{11} n pulse⁻¹ (i.e. practically the same figures as in PF-1000). This was done on the device with an energy storage of only 250 kJ based on the *same type* of capacitors but with a much more compact collector system ($\varnothing \cong 50$ cm) and at the DPF chamber having a relatively shorter anode tube.

One specific detail was very important in our experiments with the PLAMYA device [2]. Namely, inside the chamber we used a disc positioned 10 cm apart from the upper anode plate, which restricted the PCS from moving in an upward direction. The same measurement done recently and investigated in more detail with the support of numerical calculations was performed with an optimization of the PF-3 facility at the Kurchatov Institute [21, 22]. In these experiments it was found that the upper disc not only decreases the overall inductance giving rise to a *current maximum by 10%* but what is more important it also moves the current peculiarity (dip) forward in time ensuring an increase in the *pinch* current at the plasma maximal compression under identical conditions *by two times*.

6. Conclusions

According to the results of the entire set of experiments one may see that the main neutron pulse (the second one as usual) is irradiated after the phenomenon of current abruption. This event bears many features inherent to a plasma diode formed in accordance with the electron magneto-hydrodynamic model. The main mechanism of neutron generation is in tune with a GPM whereas three groups of temporal parameters rule the neutron yield.

- (1) The time of the energy release from the plasma inductive storage system (magnetic field stored around the pinch) after the moment of current abruption, when this energy is converted into streams of fast electrons and ions, as well as the efficiency of this conversion.
- (2) The confinement time of fast deuterons having medium energy (10–100 keV), which are produced at the above current abruption and gyrated in the magnetic field within the pinch.
- (3) The confinement time of the pinch plasma (the ‘hot plasma target’ bombarded by the above stream of medium-energy ions) as well as its density and volume.

The analyses of these results are in favour of the neutron emission model based on ion beam–plasma interaction with three important features: (1) *the plasma target is hot and confined during more than ten ‘inertial confinement times’*; (2) *ions of the main part of the beam are magnetized and entrapped about the pinch plasma target for a longer period than the characteristic time of the plasma inductive storage system*; and (3) *ion–ion collisions* (both *the fusion* ones due to head-on impacts and the *Coulomb* ones due to an increase in the effective temperature of the ion component of the bulk plasma) *responsible for neutron emission*. Fast ions are generated as

a sequence of pulses having a duration of less than 1 ns and separated in time by intervals of about 2–3 ns. The part of the ions leaving the pinch in the direction of the Z-axis has a conical-tubular structure. They produce neutrons in a certain volume of residual gas next to the pinch with a total yield much less than that of the main neutron pulses.

An analysis of the results has shown that one of the ways in which a future improvement in the neutron yield of the PF-1000 facility might be achieved by changing the geometry of the device. We believe that the experiments are in favour of the construction of new larger DPF devices, in particular, if based on modern high-current technology. In this case an increase in the plasma volume, the energy of fast ions as well as the longevity of the ‘target’ and the beams will give additional advantages.

Acknowledgments

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Neutron emission from a fast plasma focus of 400 Joules

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The neutron emission from a small and fast plasma focus operating in deuterium is presented. The system operates at low energy in the hundred of joules range (880 nF capacitor bank, 38 nH, 20–35 kV, 176–539 J, ~ 300 ns current rise time). The neutrons were measured by means of a silver activation counter, and the total neutron yield versus deuterium gas filling pressure was obtained. For discharges operating at 30 kV charging voltage, the maximum neutron yield was $(1.06 \pm 0.13) \times 10^6$ neutrons per shot at 9 mbar. © 2003 American Institute of Physics.
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In dynamic pinches, short-duration high-temperature and high-density plasmas are produced, which can emit x rays and intense neutron pulses (when deuterium is used in the discharge). A plasma focus (PF) is a particular pinch discharge in which a high pulsed voltage is applied to a low pressure gas between coaxial cylindrical electrodes. The central electrode is the anode partially covered with a coaxial insulator. The discharge starts over the insulator surface, and afterwards the current sheath is magnetically accelerated along the coaxial electrodes. After the current sheath runs over the ends of the electrodes the plasma is compressed in a small cylindrical column (focus). In most of the devices these three stages last a few microseconds. The pinch compression should be coincident with peak current (really with the magnetic flow) in order to achieve the best efficiency. The pinch generates beams of ions and electrons, and ultrashort x-ray pulses. Using deuterium gas, plasma focus devices produce fusion D–D reactions, generating fast-neutrons pulses (~ 2.5 MeV) and protons (leaving behind ^3He and ^3H). The neutrons burst usually lasts about tens to hundreds of nanoseconds. The emitted neutrons can be applied to perform radiographs and substance analysis, taking advantage of the penetration and activation properties of this neutral radiation. The plasma focus is a pulsed neutron source especially suited for applications because it reduces the danger of contamination of conventional isotopic radioactive sources. A passive radioactive source of fast neutrons with similar energy (for instance ^{252}Cf with similar mean energy or Am/Be with a harder spectrum) emits continuously, causing inconveniences in handling and storing. In turn, plasma-focus generators do not have activation problems for storage and handling.

During the last 30 years, substantial effort and resources have been invested in plasma focus devices.^{1–5} The studies range from small devices of around hundreds of joules, to large facilities of about 1 MJ. Specifically in the 3 kJ range^{4,5}

there are numerous results obtained by the Asian-African Association for Plasma Training Network. Repetitive plasma-focus devices for x-ray emission have been reported by Lebert *et al.*⁶ and Prasad *et al.*,⁷ both with 2–5 kJ of electrical energy stored in the capacitor bank and a repetition rate of the order of 2 Hz; and Lee *et al.*⁸ with 3 and 1.9 kJ, 3 and 16 Hz of repetition rate, respectively. In relation to neutron emitting plasma focus emissions have been found ranging from 10^7 neutrons with 1 kJ driver up to 10^{12} neutrons with 1000 kJ. If small portable PF devices were available, the value of the emissions would be substantially increased, for a number of nuclear techniques could be produced in wider domains of applications. There are few published works about devices designed to operate at hundred joules^{9,10} and they operate with slow drivers (~ 10 μF capacitor bank, ~ 100 –70 nH, ~ 7 kV, ~ 250 J, ~ 1.3 μs current rise time). In this letter we present observations of the neutron emission from a very small and fast plasma focus operating at 400 J (880 nF capacitor bank, 38 nH, 20–35 kV, 176–539 J, ~ 300 ns current rise time) in deuterium.

In spite of all the accumulated research, there are several questions still waiting for answers, particularly those concerning the sheath formation, insulator conditioning and influence of gas impurities. An area of research that is not well explored is that of the very-small low-energy plasma foci. Most of the experimental studies were focused in medium and large facilities from tens to hundreds of kilojoules, or small devices about some kilojoules. In fact, we can question if good focussing can be achieved below 1 kJ, and if so which are the appropriate design criteria in this energy region.

Experimental research with a plasma focus driven by a capacitor bank of tens to hundred of joules would allow to extend the theoretical models to the region of low energy.^{11–14} Moreover, a capacitor bank under the kilojoule has a small size in comparison with banks in the kilojoules range, thus it would be easier to operate in a repetitive regime from hertz to kilohertz, since the power requirements and the spark-gap erosion are consequently lower.

The plasma focus device used in the experiments reported here, PF-400J, consists of a capacitor bank that is discharged over the coaxial electrode through a spark gap.

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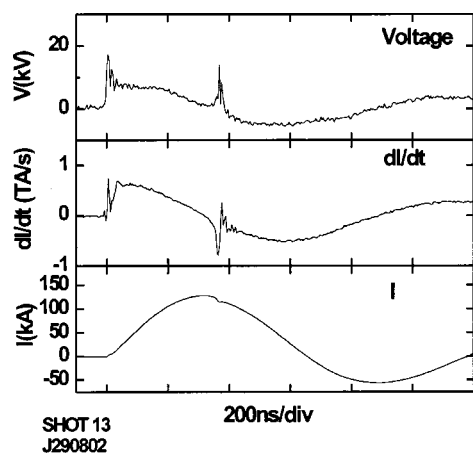


FIG. 1. Electrical signals for a shot in deuterium. Pressure=8.7 mbar, charging voltage=28 kV (345 J energy storage in the capacitor bank) in this shot. The typical dip in the signal of the current derivative associated with the formation of a pinched plasma column on the axis was observed. The voltage and current were measured with a 5% of accuracy.

The capacitor bank consists of four capacitors (220 nF, 20 nH) connected in parallel (880 nF, 5 nH). The device operates with charging voltages of 20–35 kV. In order to obtain low inductance the capacitors were connected in a compact layout. Thus, a short and coaxial spark gap was designed for the same purpose. The length of the connections between capacitor bank, spark gap, and electrodes was minimized directly connecting the capacitor bank to the spark gap and the electrodes. The measured total external inductance is 38 nH. The total impedance of the generator is of the order of 0.2 Ω . To determine the size of the electrodes the design relations suggested by Lee⁴ and a theoretical model of plasma focus for neutron production¹⁵ were considered. It is known that the pinch phase in a plasma focus is highly dependent of the current sheath formation over the insulator. Unfortunately, there are still not validated theoretical models to determine the dimensions of the insulator. Therefore, several tests with different insulator length and diameter, scanning pressure range from 1 to 12 mbar, were necessary to determine the size of the insulator in order to obtain a homogeneous initial sheath. The current sheath was studied with an image converter camera with 5 ns exposure time. Finally, structure of electrodes consists of a 28 mm long, 12 mm diameter cooper tube anode, and an outer cathode of eight 5 mm diameter cooper rods uniformly spaced on a 31 mm diameter. Anode and cathode were separated by an alumina tube of 21 mm length. Such configuration resulted from the short first quarter period of the discharge current (some 300 ns, due to the small bank capacity), which require a short effective anode (7 mm). The size of the device is of the order of 25 cm $\times$ 25 cm $\times$ 50 cm.

Voltage, total current, and current derivative are measured with usual monitors, a fast resistive divider, and a Rogowskii coil. The voltage monitor was located close to the plasma load. The Rogowskii coil monitored the current derivative signal of the capacitor bank. A silver activation counter, previously calibrated with an Am–Be source, placed at 30 cm in the side-on direction was used to record the integrated neutron signal.

Discharges were performed in deuterium at different

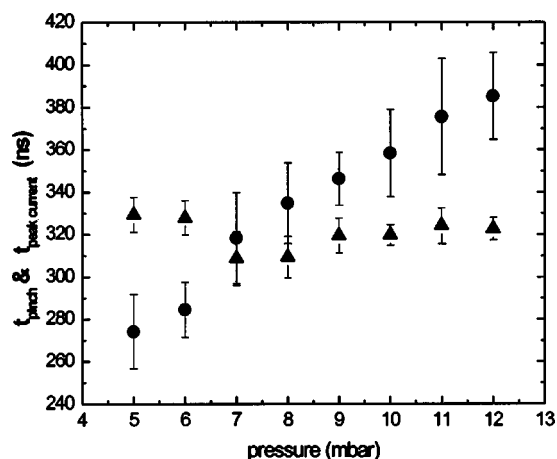


FIG. 2. Pinch time vs filling pressure for deuterium ($\bullet$). The time to peak current vs filling pressure is also shown ($\blacktriangle$). Device operating at 30 ± 2 kV charging voltage. Discharges taking place in deuterium close to 7 mbar produce compression close to the peak current in the device studied here.

pressures, 5–12 mbar, with a charging voltage of 30 ± 2 kV, i.e., ~ 400 J stored in the capacitor bank. Electrical traces for a shot in deuterium at 9 mbar pressure is shown in Fig. 1 where 127 ± 6 kA peak current is obtained at those conditions. The typical dip in the signal of the current derivative associated with the formation of a pinched plasma column on the axis was observed. From the current derivative signals the implosion time (pinch time, measured at the moment for the minimum in dI/dt) versus filling pressure was obtained and it is shown in Fig. 2. The maximum compression of the plasma occurs close to the peak current for a pressure close to 7 mbar.

The neutron yield measured by the activation counter is shown as a function of the filling gas pressure in Fig. 3. Each point is the average of ten shots and the error bars are the standard deviations. The maximum measured neutron yield was $(1.06 \pm 0.13) \times 10^6$ neutrons per shot at 9 mbar. This maximum occurs for discharges with pinch close but after the current peak. The maximum observed yield agrees roughly with the empirical scaling laws available in the literature for drivers with energy in the range 1–100 kJ,¹⁶ $Y = 10^7 E^2$ and $Y = I^{3.3}$ (the storage energy in the driver E in kilojoules and the current pinch I in kiloamperes). It is probably that the electrodes and insulators size could be opti-

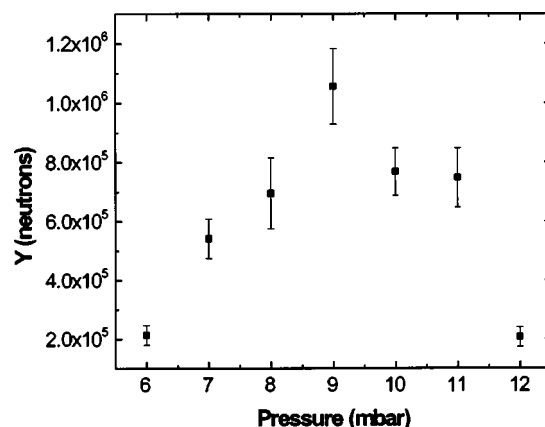


FIG. 3. Total neutron yield, Y , vs deuterium filling pressure.

mized in order to increase the neutron emission.

Most of the plasma-focus devices operate with capacitor banks that produce electrical discharges with a quarter of period, $T/4$, in the range of 1.5–4 μ s. It should be stressed then that PF-400 is much faster in comparison with conventional devices. Thus, the differences of the plasma-focus device presented here are the energy stored in the capacitor bank (hundreds of joules) and the duration of the discharge (current rise time, $T/4 \sim 300$ ns). Under these conditions we are reporting neutron yields up to $(1.06 \pm 0.13) \times 10^6$ neutrons per shot.

The device studied here is useful both for basic research and applications. Experimental research with this device would allow the extension of the existing theoretical models to the low energy region. This type of fast electric discharge instruments could provide microinstabilities and turbulent plasmas, capable of producing energetic electron and ion beams, x-ray emission, neutrons, and protons (using deuterium). Although the measured neutron yield is low in comparison with devices that operate at some kilojoules, this kind of very small device could be operated easily in a repetitive regime from hertz to kilohertz, increasing the radiation fluence, offering space for useful applications. Potential applications of small and repetitive plasma-focus devices are substance detection by transient activation analysis, x-ray imaging, and neutrography. In accordance with commercial information readily available on fast neutron radiography using a charge coupled device coupled to a Gd converter, it may be concluded that the proposed 10^6 neutron per shot source, placing the sample 5 cm from the source, a 5 cm² analysis area may be recorded with 10^3 – 10^4 shots, depending on sample nature and shape. With a 10 Hz repetition rate this fluences will attained after 100–1000 s. The device presented here conceived for laboratory purposes, is a single shot machine which can be operated only at 0.5 Hz. According to the same commercial information, sources based on generators with an accelerating tube filling with deuterium providing 10^6 – 10^8 neutrons/s at 10 kHz repetition rate are useful for prompt gamma neutron analysis. This translates

into a 10^2 – 10^4 neutrons/shot. A plasma focus device in the tens of joules range and $\sim 10^4$ neutrons/shot is currently being tested.

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Neutron yield saturation in plasma focus: A fundamental cause

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Plasma focus research in the direction of fusion energy faces the limitation of observed neutron saturation; the neutron yield Y_n falls away from $Y_n \sim E_0^2$, the scaling deteriorating as storage energy E_0 increases toward 1 MJ. Numerical experiments confirm that $Y_n \sim E_0^2$ applies at low energies and drops to $Y_n \sim E_0^{0.8}$ toward 25 MJ; deteriorating already at several hundred kilojoules. We point out that the cause is the dynamic resistance of the axial phase that is constant for all plasma foci. This dynamic resistance dominates the circuit as capacitor bank surge impedance becomes insignificant at large E_0 , causing current, hence neutron “saturation.” © 2009 American Institute of Physics. [doi:10.1063/1.3246159]

It was observed early in plasma focus research¹ that neutron yield $Y_n \sim E_0^2$ where E_0 is the capacitor storage energy. Such scaling gave hopes of possible development as a fusion energy source. Devices were scaled up to higher E_0 . It was then observed that the scaling deteriorated, with Y_n not increasing as much as suggested by the E_0^2 scaling. In fact some experiments were interpreted as evidence of a neutron saturation effect^{2,3} as E_0 approached several hundreds of kilojoules. As recently as 2006, Kraus⁴ and Scholz⁵ (November 2007) have questioned whether the neutron saturation was due to a fundamental cause or to avoidable machine effects such as incorrect formation of plasma current sheath arising from impurities or sheath instabilities.³ We should note here that the region of discussion (several hundreds of kilojoules approaching the megajoules region) is in contrast to the much higher energy region discussed by Schmidt⁶ at which there might be expected to be a decrease in the role of beam target fusion processes.³

Recent extensive numerical experiments^{7,8} also showed that whereas at energies up to tens of kilojoules the $Y_n \sim E_0^2$ scaling held, deterioration of this scaling became apparent above the low hundreds of kilojoules. This deteriorating trend worsened and tended toward $Y_n \sim E_0^{0.8}$ at tens of megajoules. The results of these numerical experiments are summarized in Fig. 1 with the solid line representing results from numerical experiments. Experimental results from 0.4 kJ to megajoules, compiled from several available published sources^{3,9–14} are also included as squares in the same figure. The combined experimental and numerical experimental results¹⁵ appear to have general agreement particularly with regards to the $Y_n \sim E_0^2$ at energies up to 100 kJ, and the deterioration of the scaling from low hundreds of kilojoules to the 1 MJ level. It is proposed here that the global data of Fig. 1 suggest that the apparently observed neutron saturation effect is overall not in significant variance with the deterioration of the scaling shown by the numerical experiments.

We wish now to provide a simple yet compelling analysis of the cause of this neutron saturation. In Fig. 2 is shown a schematic of the plasma dynamics in the axial phase of the Mather-type plasma focus.

We consider the simplest representation in which the current sheet is shown to go from the anode to the cathode perpendicularly. Observation shows that there is actually a canting of the current sheet¹⁶ and also that only a fraction (typically 0.7) of the total current participates in driving the current sheet. These points are accounted for in the modeling^{17–22} by model parameters f_m and f_c . For the moment we do not consider these two effects. The outer cathode radius is shown as b , inner anode radius as a and the moving current sheet is shown at position z in the axial phase.

By surveying published results of all Mather-type experiments we find that all deuterium plasma focus devices operate at practically the same speeds²³ and are characterized by a constancy of energy density (per unit mass) over the whole range of devices from the smallest subkilojoule to the largest megajoule devices. The time varying tube inductance is $L = (\mu/2\pi)\ln(c)z$, where $c = b/a$ and μ is the permeability of free space. The rate of change in inductance is $dL/dt = 2 \times 10^{-7}(\ln c) dz/dt$ in SI units. Typically on switching, as the capacitor discharges, the current rises toward its peak value, the current sheet is accelerated, quickly reaching nearly its peak speed, and continues accelerating slightly toward its peak speed at the end of the axial phase. Thus for most of its

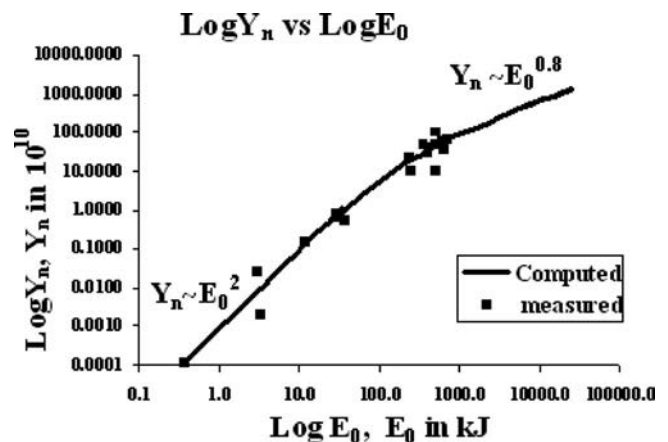


FIG. 1. Illustrating Y_n scaling deterioration observed in numerical experiments from 0.4 kJ to 25 MJ (solid line) using the Lee model code, compared to measurements compiled from publications (squares) of various machines from 0.4 kJ to 1 MJ.

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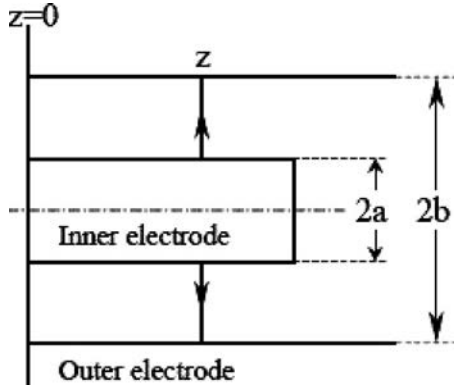


FIG. 2. Plasma focus schematic showing axial phase only.

axial distance the current sheet is traveling at a speed close to the end-axial speed. In deuterium the end-axial speed is observed to be about $10 \text{ cm}/\mu\text{s}$ over the whole range of devices. This fixes the rate of change in inductance dL/dt as $1.4 \times 10^{-2} \text{ H/s}$ for all the devices, if we take the radius ratio $c=b/a=2$. This value of dL/dt changes by at most a factor of 2, taking into account the variation in c from low values of 1.4 (generally for larger machines) to 4 (generally for smaller machines). This typical dL/dt may also be expressed as $14 \text{ m}\Omega$.

We need now to inquire into the nature of the change in the inductance $L(t)$. Consider instantaneous power P delivered to $L(t)$ by a change in $L(t)$.

Induced voltage:

$$V = d(LI)/dt = I(dL/dt) + L(dI/dt). \quad (1)$$

Hence instantaneous power into $L(t)$,

$$P = VI = I^2(dL/dt) + LI(dI/dt). \quad (2)$$

Next, consider instantaneous power associated with the inductive energy $(1/2LI^2)$

$$P_L = d(\frac{1}{2}LI^2)/dt = \frac{1}{2}I^2(dL/dt) + LI(dI/dt). \quad (3)$$

We note that P_L of Eq. (3) is not the same as P of Eq. (2).

The difference $= P - P_L = (\frac{1}{2})(dL/dt)I^2$ is not associated with the inductive energy stored in L . We conclude that whenever $L(t)$ changes with time, the instantaneous power delivered to $L(t)$ has a component that is not inductive. Hence this component of power $(\frac{1}{2})(dL/dt)I^2$ must be resistive in nature; and the quantity $(\frac{1}{2})(dL/dt)$ is identified as a resistance due to the motion associated with dL/dt , which we call the dynamic resistance.¹⁵ Note that this is a general result and is independent of the actual processes involved. In the case of the plasma focus axial phase, the motion of the current sheet imparts power to the shock wave structure with consequential shock heating, Joule heating, ionization, radiation etc. The total power imparted at any instant is just the amount $(\frac{1}{2})(dL/dt)I^2$, with this amount powering all consequential processes. We denote the dynamic resistance of the axial phase as DR_0 .

We have thus identified for the axial phase of the plasma focus a typical dynamic resistance of $7 \text{ m}\Omega$ due to the motion of the current sheet at $10 \text{ cm}/\mu\text{s}$. It should be noted here that similar ideas of the role of dL/dt as a resistance was discussed by Bernard *et al.*³ In that work the effect of dL/dt was discussed only for the radial phase. In our opinion

TABLE I. Discharge characteristics of equivalent plasma focus circuit, illustrating the saturation of I_{peak} with increase of E_0 to very large values. The last column presents results using circuit (LCR) computation, with a fixed resistance load of $7 \text{ m}\Omega$, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

E_0 (kJ)	C_0 (μF)	Z_0 ($\text{m}\Omega$)	DR_0 ($\text{m}\Omega$)	Z_{total} ($\text{m}\Omega$)	$I_{\text{peak}} = V_0/Z_{\text{total}}$ (kA)	$I_{\text{peak, LCR}}$ (kA)
0.45	1	173	7	197	152	156
4.5	10	55	7	67	447	464
45	100	17	7	26	1156	1234
135	300	10	7	18	1676	1819
450	1000	5.5	7	12.9	2321	2554
1080	2400	3.5	7	10.8	2781	3070
4500	10 000	1.7	7	8.8	3407	3722
45 000	100 000	0.55	7	7.6	4209	4250

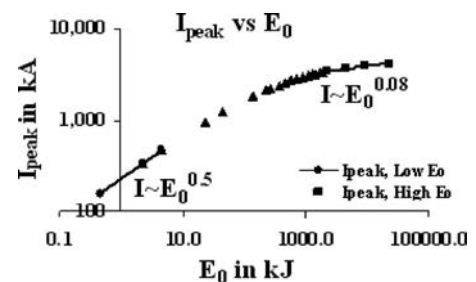
the more important phase for the purpose of neutron saturation is actually the axial phase for the Mather-type plasma focus.

We now resolve the problem into its most basic form as follows. We have a generator (the capacitor charged to 30 kV), with an impedance of $Z_0 = (L_0/C_0)^{0.5}$ driving a load with a near constant resistance of $7 \text{ m}\Omega$. We also assign a value for stray resistance of $0.1Z_0$. This situation may be shown in Table I where L_0 is given a typical value of 30 nH . We also include in the last column the results from a circuit (LCR) computation, discharging the capacitor with initial voltage 30 kV into a fixed resistance load of $7 \text{ m}\Omega$, simulating the effect of the DR_0 and a stray resistance of value $0.1Z_0$.

Plotting the peak current as a function of E_0 we obtain Fig. 3, which shows the tendency of the peak current toward saturation as E_0 reaches large values; the deterioration of the curve becoming apparent at the several hundred kilojoule level. This is the case for $I_{\text{peak}} = V_0/Z_{\text{tot}}$ and also for the LCR discharge with simulated value of the DR_0 . In both cases it is seen clearly that a capacitor bank of voltage V_0 discharging into a constant resistance such as DR_0 will have a peak current I_{peak} approaching an asymptotic value of $I_{\text{peak}} = V_0/DR_0$ when the bank capacitance C_0 is increased to such large values that the value of $Z_0 = (L_0/C_0)^{0.5} \ll DR_0$. Thus DR_0 causes current saturation.

Recent numerical experiments^{7,8} have shown agreement with accumulated laboratory data in deriving the relationship between Y_n and I_{peak} and I_{pinch} as follows:

$$Y_n \sim I_{\text{pinch}}^{4.5},$$

FIG. 3. I_{peak} vs E_0 on log-log scale, illustrating I_{peak} saturation at large E_0 .

$$Y_n \sim I_{\text{peak}}^{3.8}.$$

Hence saturation of I_{peak} will lead to saturation of Y_n .

At this point we note that if we consider that only 0.7 of the total current takes part in driving the current sheet, as typically agreed upon from experimental observations, then there is a correction factor which reduces the axial dynamic resistance by some 40%. That would raise the asymptotic value of the current by some 40%, nevertheless there would still be saturation.

In this paper we have shown that current saturation is inevitable as E_0 is increased to very large values by an increase in C_0 , simply due to the dominance of the axial phase dynamic resistance. This makes the total circuit impedance tend toward an asymptotic value which approaches the dynamic resistance at infinite values of E_0 . The saturation of current inevitably leads to a saturation of neutron yield. Thus the apparently observed neutron “saturation” which is more accurately represented as a neutron scaling deterioration is inevitable because of the dynamic resistance. In line with current plasma focus terminology we will continue to refer to this scaling deterioration as saturation. The above analysis applies to the Mather-type plasma focus. The Filippov-type plasma focus does not have a clearly defined axial phase. Instead it has a liftoff phase and an extended prepinch radial phase which determine the value of I_{peak} . During these phases the inductance of the Filippov discharge is changing, and the changing $L(t)$ will develop a dynamic resistance which will also have the same current saturation effect as the Filippov bank capacitance becomes big enough.

Moreover the saturation as observed in presently available data is due also to the fact that all tabulated machines operate in a narrow range of voltages of 15–50 kV. Only the SPEED machines, most notably SPEED II (Ref. 24) operated at low hundreds of kilovolts. No extensive data have been published from the SPEED machines. Moreover SPEED II, using Marx technology, has a large bank surge impedance of 50 mΩ, which itself would limit the current. If we operate a range of such high voltage machines at a fixed high voltage, say 300 kV, with ever larger E_0 until the surge impedance becomes negligible due to the very large value of C_0 , then the saturation effect would still be there, but the level of saturation would be proportional to the voltage. In this way we can go far above presently observed levels of neutron

saturation; moving the research, as it were into presently beyond-saturation regimes.

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Neutron Scaling Laws from Numerical Experiments

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Abstract

Experimental data of neutron yield Y_n against pinch current I_{pinch} is assembled to produce a more global scaling law than available. From the data a mid-range point is obtained to calibrate the neutron production mechanism of the Lee Model code. This code is then used for numerical experiments on a range of focus devices to derive neutron scaling laws. The results are the following: $Y_n = 2 \times 10^{11} I_{pinch}^{4.7}$ and $Y_n = 9 \times 10^9 I_{peak}^{3.9}$. It is felt that the scaling law with respect to I_{pinch} is rigorously obtained by these numerical experiments when compared with that obtained from measured data, which suffers from inadequacies in the measurements of I_{pinch} .

Keywords: Plasma Focus Neutron Scaling Pinch Current Focus modelling Lee Model

Introduction

A major feature of the plasma focus is its fusion neutron yield. Even a simple trolley mounted 3kJ device such as the UNU/ICTP PFF routinely produces¹ a yield of $Y_n = 10^8$ neutrons, operating in deuterium. A big machine such as the PF1000 typically produces 10^{11} neutrons per shot². Moreover since the neutrons are produced in a short pulse of the order of 10ns, the rate of neutron production is 10^{16} neutrons/s even for a small machine and can go up to 10^{20} for a large machine.

From a compilation of experimental data over a wide range of energies a scaling law of $Y_n \sim I_{pinch}^{3.3}$ was presented by Bernard³, where I_{pinch} is the current flowing through the dense pinch in the focused plasma. Kies⁴ presented another compilation showing $Y_n \sim I_{pinch}^4$ whilst Herold⁵ had results showing $Y_n \sim I_{pinch}^{3.2}$. Gribkov has recently² suggested that the experimental data can be interpreted with the power law as high as 5 in particular when dealing with the same device.

One significant uncertainty in compiling such a scaling law is the interpretation of I_{pinch} . The current most conveniently measured in most experiments is the total current flowing into the tube (usually measured with a Rogowski coil placed at the collector plate

just outside the tube). This total current has a maximum value I_{peak} . If one estimates I_{pinch} from the total current measurement there are two difficulties: 1. it is difficult to determine the point on the current waveform where the plasma has gone into the pinch phase, and 2. even after estimating this point, it still remains to estimate the fraction of total current that in fact flows into the pinch. One way is to use small magnetic coils to probe the pinch region. For small machines this method is not suitable because of the amount of space available and the small size of the pinch so that the probes inevitably interfere with the pinching current sheet. For large machines, results have been obtained⁵ but with large errors quoted as 20%. Moreover the shot-to-shot variability of focus performance means that the final presentation of results relies greatly on how the particular research group chooses to present the results. For example the yield may be presented as a range, with some shots considered not representative discarded, and perhaps the biggest values of observed yield also presented. It is quite remarkable that despite all these difficulties there is a consensus of opinion that the index in this power scaling law has the value in the range of 3 to 5.

Compilation of experimental results

In this paper we have combined the laboratory data that we have¹⁻⁷, which includes recent results from some smaller machines e.g. Soto's⁶ PF400 and the large² PF1000 as well as a high performance repetitive device⁷, the NX2. This gives a good fit of $Y_n = 9 \times 10^{10} I_{\text{pinch}}^{3.8}$. The main reason for this compilation of experimental results is to provide a calibration point for setting the neutron yield mechanism of the Lee Model code, described below. A calibration point is chosen at around the middle of the current range at $I_{\text{pinch}} = 0.5 \text{ MA}$, $Y_n = 6 \times 10^9$ neutrons. This point is close to the PF1000's machine parameters with properly adjusted dimensions if it could be fired at 13.5kV.

The results of the compilation are shown in Fig 1.

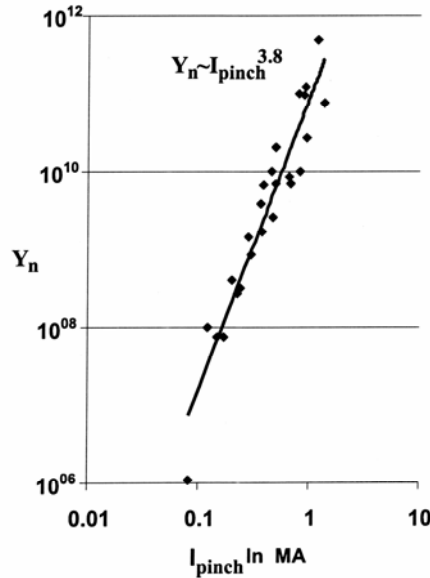


Fig 1. Y_n scaling with I_{pinch} from laboratory data

The Model used for the numerical experiments

The Lee Model has been widely used to simulate axial and radial phase dynamics, temperatures and thermodynamic properties and radiation yields. To realistically simulate any plasma focus all that is needed is a measured current trace of that plasma focus. Recently the model code⁸ has been extended to include a phenomenological beam-target mechanism based partially on that proposed by Gribkov².

The main mechanism producing the neutrons is a beam of fast deuteron ions interacting with the hot dense plasma of the focus pinch column. The fast ion beam is produced by diode action in a thin layer close to the anode with plasma disruptions generating the necessary high voltages. This mechanism, described in some details in a recent paper⁹, results in the following expression used for the model code:

$$Y_{b-t} = \text{calibration constant} \times n_i I_{\text{pinch}}^2 z_p^2 (\ln(b/r_p)) \sigma / V_{\text{max}}^{0.5}$$

where I_{pinch} is the current at the start of the slow compression phase, r_p and z_p are the pinch radius and pinch length at the end of the slow compression phase, V_{max} is the maximum value attained by the inductively induced voltage, σ is the D-D fusion cross section (n branch)¹⁰ corresponding to the beam ion energy **and n_i is the pinch ion density**. The D-D cross section σ is obtained by using beam energy equal to 3 times V_{max} , to conform to experimental observations.

Scaling Laws derived from the numerical experiments

This paper applies the code to several machines including the PF400, UNU/ICTP PFF, the NX2 and Poseidon. The PF1000 which has a current curve published at 27kV and Y_n published at 35kV provided an important point. Moreover using parameters for the PF1000 established at 27 kV and 35 kV, additional points were taken at different voltages ranging from 13.5kV upwards to 40kV.

These machines were chosen because each has a published current trace and hence the current curve computed by the model code could be fitted to the measured current trace. Once this fitting is done our experience is that the other computed properties including dynamics, energy distributions and radiation are all realistic. This gives confidence that the computed Y_n for each case is also realistic. Moreover since each chosen machine also has measured Y_n corresponding to the current trace, the computed Y_n could also be compared with the measured to ensure that the computed results are not incompatible with the measured values.

The results are shown in Table 1 and Fig 2.

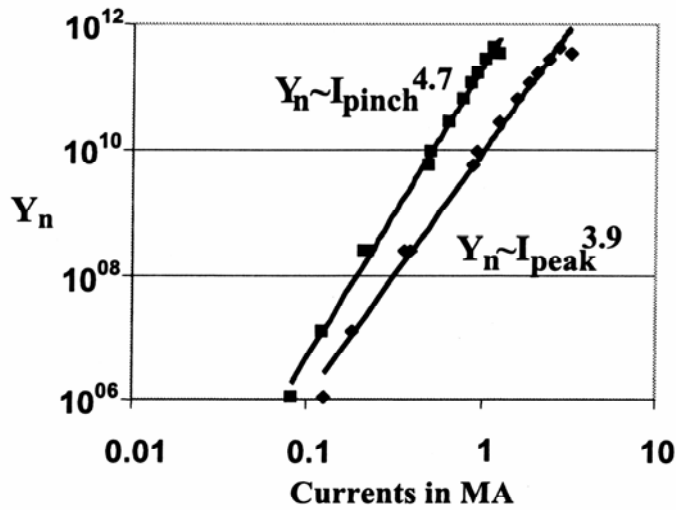
In Table 1, corresponding to each laboratory device, the operating voltage V_o and pressure P_o are typical of the device, as is the capacitance C_o . It was found that the static inductance L_o usually needed to be adjusted from the value provided by the laboratory. This is because the value provided could be for short circuit conditions, or an estimate including the input flanges and hence that value may not be sufficiently close to L_o . The

dimensions b (outer radius), a (anode radius) and z_0 (anode length) are also the typical dimensions for the specific device. The speed factor¹¹ S is also included. All devices except Poseidon have typical S values. Poseidon is the exceptional high speed device in this respect. The minimum pinch radius is also tabulated as $k_{\min}=r_p/a$. It is noted that this parameter increases from 0.14 for the smaller machines towards 0.2 for the biggest machines. The ratio $I_{\text{pinch}}/I_{\text{peak}}$ is also tabulated showing a trend of decreasing from 0.65 for small machines to 0.4 for the biggest machines.

Table 1. Computed values of I_{peak} , I_{pinch} and Y_n for a range of Plasma Focus Machines

Machine	V_0 (kV)	P_0 (torr)	L_0 (nH)	C_0 (μ F)	b (cm)	a (cm)	Z_0 (cm)	I_{peak} (MA)	I_{pinch} (MA)	S	Y_n	$k_{\min}$	$I_{\text{pinch}}/I_{\text{peak}}$
PF400	28	6.6	40	0.95	1.55	0.60	1.7	0.126	0.082	82	1.1×10^6	0.14	0.65
UNU	15	4	110	30	3.2	0.95	16	0.182	0.123	96	1.2×10^7	0.14	0.68
NX2 T	15	5	20	28	5	2	7	0.386	0.225	86	2.5×10^8	0.16	0.58
Calibration	16	5	24	308	7	4	30	0.889	0.496	99	5.6×10^9	0.17	0.56
NX2 T-2	12.5	10.6	19	28	3.8	1.55	4	0.357	0.211	71	2.4×10^8	0.16	0.59
PF1000	13.5	3.5	33	1332	8.00	5.78	60	0.924	0.507	89	9.6×10^9	0.17	0.55
	18	3.5	33	1332	10.67	7.70	60	1.231	0.636	89	2.9×10^{10}	0.18	0.52
	23	3.5	33	1332	13.63	9.84	60	1.574	0.766	89	6.8×10^{10}	0.19	0.49
	27	3.5	33	1332	16	11.60	60	1.847	0.862	89	1.2×10^{11}	0.19	0.47
	30	3.5	33	1332	17.77	12.80	60	2.049	0.929	89	1.6×10^{11}	0.20	0.45
	35	3.5	33	1332	20.74	15.00	60	2.399	1.037	89	2.7×10^{11}	0.20	0.43
	40	3.5	33	1332	23.70	17.10	60	2.736	1.137	89	4.1×10^{11}	0.21	0.42
Poseidon	60	3.8	18	156	9.50	6.55	30	3.200	1.260	251	3.3×10^{11}	0.20	0.39

Fig 2. Y_n scaling with I_{pinch} and I_{peak} from numerical experiments



The resultant data with improved optimization yield more up to date scaling laws: $Y_n \sim I_{\text{pinch}}^{4.7}$ and $Y_n \sim I_{\text{peak}}^{3.9}$. It is necessary to emphasize again that the I_{pinch} may be considered to be computed rigorously especially for those cases where an experimental current curve is available. Once the computed current curve is fitted accurately to the experimental current curve, the resultant pinch position is pinpointed as well as the fraction of current going into the pinch.

This is in contrast to the laboratory data where I_{pinch} is usually only estimated and if measured is subject to large errors. A study of the data suggests that in most cases I_{pinch} is overestimated by experimentalists. With all these considerations it would appear that the scaling laws arising from the code are not inconsistent with experimental observations and may complement the more conventionally compiled scaling laws to provide comprehensive database for experiments.

Conclusion

Neutron scaling laws have been derived from computation using the Lee Model code. These are: $Y_n \sim I_{\text{pinch}}^{4.7}$ and $Y_n \sim I_{\text{peak}}^{3.9}$. In these numerical experiments I_{pinch} is rigorously computed whereas in compilation of laboratory results I_{pinch} is usually just guessed at or at best estimated. These numerically derived scaling laws are not inconsistent with compilation from laboratory experiments. The numerically derived scaling law against I_{pinch} has an index of 4.7 which is higher than the usually accepted scaling law with index of 3.2 to 4. The indications are that the numerically derived scaling laws being more rigorous and consistent in derivation may actually be more realistic and more reliable for use in interpreting, designing or planning experiments.

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Erratum

This version of the paper contains two additions to the published paper on pg 3.
The paragraph containing the additions is reproduced here in parenthesis, with the
additions highlighted in bold red:

$$Y_{b-t} = \text{calibration constant} \times \mathbf{n_i} I_{\text{pinch}}^2 z_p^2 (\ln(b/r_p)) \sigma / V_{\text{max}}^{0.5}$$

where I_{pinch} is the current at the start of the slow compression phase, r_p and z_p are the pinch radius and pinch length at the end of the slow compression phase, V_{max} is the maximum value attained by the inductively induced voltage and σ is the D-D fusion cross section (n branch)¹⁰ corresponding to the beam ion energy **and n_i is the pinch ion density**. The D-D cross section σ is obtained by using beam energy equal to 3 times V_{max} , to conform to experimental observations."

Current and neutron scaling for megajoule plasma focus machines

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Abstract

In a 2007 paper Nukulin and Polukhin surmised from electrodynamical considerations that, for megajoule dense plasma focus devices, focus currents and neutron yield Y_n saturate as the capacitor energy E_0 is increased by increasing the capacitance C_0 . In contrast, our numerical experiments show no saturation; both pinch currents and Y_n continue to rise with C_0 although at a slower rate than at lower energies. The difference in results is explained. The Nukulin and Polukhin assumption that the tube inductance and length are proportional to C_0 is contrary to laboratory as well as numerical experiments. Conditions to achieve Y_n of 10^{13} in a deuterium plasma focus are found from our numerical experiments, at a storage energy of 3 MJ with a circuit peak current of 7.6 MA and focus pinch current of 2.5 MA.

1. Introduction

In a 2007 paper Nukulin and Polukhin [1] surmised that the peak discharge current I_{peak} in a plasma focus reaches a limiting value when the storage energy of its capacitor bank is increased to the megajoule level by increasing the bank capacitance C_0 at a fixed charging voltage V_0 . The crux of their argument is that for such large banks, increasing C_0 increases the discharge current risetime which then requires an increase in the length of the focus tube in order for the axial transit time to match the current risetime. According to their reasoning the axial tube inductance $L_a = 2 \times 10^{-7} \ln(b/a) z_0$ (their equation (5)) where b and a are the outer and inner radii, respectively, and the length of the coaxial section is $z_0 = (\pi/2)(L_a C_0)^{0.5} v_a$ (their equation (4)). We rewrite their equations in SI units throughout except where stated otherwise. Here v_a is the average axial speed in the rundown stage which in experimental situations is known to be best kept at a value around 10^5 (or $10 \text{ cm } \mu\text{s}^{-1}$). This argument leads

to $L_a = (10^{-7} \pi v_a \ln(b/a))^2 C_0$. That is, L_a is proportional to C_0 , resulting in, for fixed V_0 , a saturated $I_{\text{peak}} = V_0/(L_a/C_0)^{0.5}$ for megajoule banks, where L_a is so large as to make the static bank inductance insignificant. We shall refer to this chain of argument as the Nukulin and Polukhin (N&P) scenario. Saturation of Y_n then follows in that scenario.

A careful consideration of the above argument reveals two factors that need to be re-examined. Firstly, matching the transit time to the ‘rise time’ of $(L_a C_0)^{0.5}$ (as required by their equation (4)) is a hypothetical situation assuming the circuit inductance has the value of L_a from the beginning of the discharge. In actual fact the circuit starts with a much smaller value of L_0 and only attains the value of L_a towards the end of the axial transit. Secondly, the dynamic resistance loading the circuit due to current sheet motion at instantaneous speed v_z is $(1/2)(dL/dt) = 10^{-7} \ln(b/a) v_z$ and has the same value, $3.3 \text{ m}\Omega$, for $v_z = 10^5$ and $b/a = 1.39$, independent of the value of C_0 . This dynamic resistance becomes increasingly dominant and controlling in the early stage of the discharge for larger and larger C_0 , since at the early stage of the discharge the tube inductance has not grown to large values yet.

Because of these two factors, for large devices with large C_0 , we will show that the current peaks early in the discharge and then exhibits a slight drooping, nearly flat-top behavior as seen in the published discharge current waveform of the PF1000 [2, 3]. This early peaking changes the situation from the N&P scenario, resulting in much smaller optimized L_a with correspondingly shorter z_0 . This invalidates their equation (4). Laboratory and also numerical experiments are not carried out with the values of L_a and z_0 envisaged by the N&P scenario, simply because these N&P values are far too large for optimum conditions. Using optimized values of L_a and z_0 , in contrast to the saturation envisaged by the N&P scenario, the optimized I_{pinch} and Y_n continue to rise with E_0 , as C_0 is increased, although the rates of increase indeed slow down. In the case of Y_n the scaling is $Y_n \sim E_0^2$ at small E_0 and becomes $Y_n \sim E_0$ in the higher energy ranges.

We would like to state here that we are not disputing the experimental observations [1, 4, 5] that have led to the idea of a neutron saturation effect in plasma focus operation. What we dispute in this paper is the N&P scenario, which is erroneous in its conclusion that the cause of neutron saturation is electrodynamical (electrotechnical in their words) in nature. Our numerical experiments show that from electrodynamical considerations, the currents I_{peak} and I_{pinch} do not saturate, nor does the neutron yield. The cause of saturation needs to be looked for elsewhere, beyond electrodynamical considerations, which is outside the scope of this paper. This paper continues to present our numerical experiments.

Although the analytic and intuitive approach is useful in attempts to understand this electrodynamic problem it could also lead to oversimplified, indeed erroneous, conclusions. The underlying physics is simple, requiring only the charge and energy conservation conditions imposed into the time-varying circuit equations, for example, in the form often expressed by Kirchhoff’s current and voltage rules, and an equation of motion for the axial phase. These equations are coupled to reflect the physics that the plasma current I_p drives the motion, and the resistive and inductive loading of the motion in turn affect the magnitude and temporal behavior of the total discharge current, I_{total} . The solution of such a coupled set of equations will take into account all of the subtle interplay of current drive and motional impedances and the temporal relationships among early and late discharge characteristics imposed by a large capacitance C_0 , coupled to a static inductance L_0 and a growing tube inductance L_z . This electrodynamic situation is very well handled by the Lee model code [6] which after the axial phase goes on to compute the radial, including the pinch phase. This paper describes numerical experiments carried out with the code to uncover the scaling of I_{pinch} and Y_n up to tens of megajoules.

2. The Lee model code

The Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics and radiation, enabling realistic simulation of all gross focus properties.

The basic model, described in 1984 [7], was successfully used to assist several projects [8–11]. An improved 5-phase model and code incorporating a small disturbance speed [12] and radiation coupling with dynamics assisted other research projects [13–15] and was web-published in 2000 [16] and 2005 [17]. Plasma self-absorption was included in 2007 [16] improving soft x-ray yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF [8, 11, 13, 14, 18], NX2 [14, 15], NX1 [14] and adapted for the Filippov-type plasma focus DENA [19]. A recent development is the inclusion of the neutron yield, Y_n , using a beam–target mechanism [3, 20, 22], incorporated in the present version [6] of the code RADPFV5.13, resulting in realistic Y_n scaling with I_{pinch} [20]. The versatility and utility of the model is demonstrated in its clear distinction of I_{pinch} from I_{peak} [21] and the recent uncovering of a plasma focus pinch current limitation effect [3, 22]. The description, theory, code and a broad range of results of this ‘Universal Plasma Focus Laboratory Facility’ is available for download from [6].

The last sections of this paper discuss the scaling of the neutron yield with increasing voltage. In that discussion it is found that there is little advantage for D–D beam–target fusion, and indeed a disadvantage for D–T beam–target fusion, to exceed 90 kV charging voltage. To understand that situation it is necessary to revisit the neutron production mechanism used in the model. The neutron yield is computed using a phenomenological beam–target neutron generating mechanism [2]. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. The beam–target yield is derived [3] as

$$Y_{\text{b-t}} = C_n n_i I_{\text{pinch}}^2 z_p^2 (\ln b/r_p) \sigma / V_{\text{max}}^{0.5},$$

where n_i is the ion density, r_p is the radius of the plasma pinch with length z_p , σ the cross-section of the D–D fusion reaction, n -branch and V_{max} the maximum voltage induced by the current sheet collapsing radially towards the axis. C_n is treated as a calibration constant combining various constants in the derivation process. The model uses a value of C_n obtained by calibrating the yield [3, 20] at an experimental point of 0.5 MA.

The D–D cross-section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute σ . The code computes V_{max} of the order of 20–50 kV. However, it is known from experiments that the ion energy responsible for the beam–target neutrons is in the range 50–150 keV [2], and for smaller lower-voltage machines the relevant energy [18] could be lower at 30–60 keV. Thus, to align with experimental observations the D–D cross section σ is reasonably fitted by using beam energy equal to three times V_{max} . With this fitting it is found [20] that the computed neutron yield agrees with experimental measurements over a wide range of plasma focus machines from the small (sub-kJ) PF400 to the large (MJ) PF1000.

3. Procedures for the numerical experiments

The Lee code is configured to work as any plasma focus by inputting the bank parameters L_0 , C_0 and stray circuit resistance r_0 , the tube parameters b , a and z_0 and operational parameters V_0 and P_0 and the fill gas. The standard practice is to fit the computed total current waveform to an experimentally measured total current waveform [3, 16, 17, 20–22] using four model

parameters representing the mass swept-up factor f_m , the plasma current factor f_c for the axial phase and factors f_{mr} and f_{cr} for the radial phases.

From experience it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace.

The exact time profile of the total current trace is governed by the bank parameters, by the focus tube geometry and the operational parameters. It also depends on the fraction of the mass swept up and the fraction of sheath current and the variation of these fractions through the axial and radial phases. These parameters determine the axial and radial dynamics, specifically the axial and radial speeds which in turn affect the profile and magnitudes of the discharge current. The detailed profile of the discharge current during the pinch phase also reflects the Joule heating and radiative yields. At the end of the pinch phase the total current profile also reflects the sudden transition of the current flow from a constricted pinch to a large column flow. Thus, the discharge current powers all dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus. Conversely all the dynamic, electrodynamic, thermodynamic and radiation processes in the various phases of the plasma focus affect the discharge current. It is then no exaggeration to say that the discharge current waveform contains information on all the dynamic, electrodynamic, thermodynamic and radiation processes that occur in the various phases of the plasma focus. This explains the importance attached to matching the computed current trace to the measured current trace in the procedure adopted by the Lee model code.

A measured current trace of PF1000 with $C_0 = 1332 \mu\text{F}$, operated at 27 kV, 3.5 Torr deuterium, has been published [2], with cathode/anode radii $b = 16 \text{ cm}$, $a = 11.55 \text{ cm}$ and anode length $z_0 = 60 \text{ cm}$. In the numerical experiments we fitted the external (or static) inductance $L_0 = 33.5 \text{ nH}$ and the stray resistance $r_0 = 6.1 \text{ m}\Omega$ (damping factor $\text{RESF} = \text{stray resistance}/(L_0/C_0)^{0.5} = 1.22$). The fitted model parameters are $f_m = 0.13$, $f_c = 0.7$, $f_{mr} = 0.35$ and $f_{cr} = 0.65$. The computed current trace [20, 22] agrees very well with the measured trace through all the phases, axial and radial, right down to the bottom of the current dip indicating the end of the pinch phase. This agreement confirms the model parameters for PF1000. Once the model parameters have been fitted to a machine for a given gas, these model parameters may be used with some degree of confidence when operating parameters such as the voltage are varied [6]. With no measured current waveforms available for the higher megajoule numerical experiments, it is reasonable to keep the model parameters that we have got from the PF1000 fitting.

4. Discrepancies between the N&P scenario and our numerical experiments

We now examine the case of PF1000 at $C_0 = 1332 \mu\text{F}$, which has an E_0 of 0.82 MJ at 35 kV. According to the N&P scenario, for this case with $b/a = 1.39$ and $v_a = 10^5 \text{ m s}^{-1}$, the final tube inductance works out at $L_a = (10^{-7} \pi v_a \ln(b/a))^2 C_0 = 144 \text{ nH}$, and since the coaxial section with $b/a = 1.39$ has an inductance per unit length of $2 \times 10^{-7} \ln(b/a) = 0.66 \times 10^{-7} \text{ H m}^{-1}$ or 0.66 nH cm^{-1} , then $z_0 = 218 \text{ cm}$ using the N&P scenario. In the actual case PF1000 is operated in the laboratory at a typical experimentally optimized length of 60 cm [2].

Our numerical experiments show an optimum length of $z_0 = 50 \text{ cm}$, in near agreement with the laboratory operation. In the numerical experiments if z_0 is taken to be the N&P scenario value of 218 cm, both the pinch current and the Y_n are far below optimum. The difference becomes even clearer in the next example.

Table 1. Numerical experiments to optimize Y_n by varying z_0 for fixed $C_0 = 39960 \mu\text{F}$.

z_0 (cm)	a (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_n (10^{10})
6480	11.95	4227	933	53.4
1000	21.50	5463	2208	1102
800	22.29	5548	2282	1211
600	23.12	5634	2345	1298
500	23.55	5678	2368	1320
400	23.91	5715	2378	1307
350	24.05	5729	2375	1280
274	24.15	5739	2355	1206

We look at another case of even larger $C_0 = 39960 \mu\text{F}$, 30 times bigger than PF1000, with an E_0 of 24.5 MJ at 35 kV. According to the N&P scenario $L_a = 4278 \text{ nH}$ and $z_0 = 64.8 \text{ m}$. We note that these figures for L_a and z_0 are 30 times bigger than for PF1000, since the crux of the N&P scenario is simply that L_a is proportional to C_0 .

We carried out numerical experiments which show that the matching conditions proposed by Nukulin and Polukhin give very poor results. We compute that the length for optimum Y_n is $z_0 = 500 \text{ cm}$, which practically corresponds to the optimum length for I_{pinch} . Table 1 shows the results of this series of experiments with $C_0 = 39960 \mu\text{F}$, varying z_0 to find the optimum. For each z_0 , ' a ' is varied so that the end axial speed is $10 \text{ cm } \mu\text{s}^{-1}$. It is clear that one would not operate at the N&P scenario $z_0 = 6480 \text{ cm}$, for which case the current has dropped so low that I_{pinch} only attains 933 kA with Y_n of only 5.3×10^{11} , compared with the numerically computed optimum Y_n of 1.32×10^{13} at $z_0 = 500 \text{ cm}$ with I_{pinch} of 2.37 MA.

5. Explaining the discrepancy

We look for the explanation of the discrepancy between the N&P scenario and our numerical experiments. To do this we plot in figure 1 Case 1 which depicts the time scale for the case in which a discharge occurs with $C_0 = 39960 \mu\text{F}$ and a constant inductance $L = 4260 \text{ nH}$ according to the N&P scenario. In the same figure we plot Case 2 which is the discharge current computed from our model code with $C_0 = 39960 \mu\text{F}$ and a length of $z_0 = 6480 \text{ cm}$, the required matching length as envisaged by the N&P scenario. Case 3 is the computed discharge current for $z_0 = 500 \text{ cm}$, corresponding to line 5 of table 1, which is the optimum length, producing maximum Y_n of 1.3×10^{13} and a nearly optimum I_{pinch} of 2.37 MA. In both Case 2 and Case 3 the anode radius ' a ' has been adjusted to give a final axial speed (end axial phase) of $10 \text{ cm } \mu\text{s}^{-1}$.

If the discharge current were to have the time profile as shown in Case 1 of figure 1, then an axial rundown time of $600 \mu\text{s}$ would be appropriate, with a corresponding z_0 of around 6480 cm, reaching the radial phase just when the current was peaking. Such a situation would justify the N&P scenario. However, computation using the model code shows that the actual current profile using a matching $z_0 = 6480 \text{ cm}$ as envisaged by the N&P scenario is that of Case 2 with the current peaking at 4.2 MA at just $19 \mu\text{s}$; thereafter the current profile droops, dropping to below 2 MA as the current sheet moves into the radial phase. Because of the severe drop in the total current, I_{pinch} is only 0.93 MA producing Y_n of 5.3×10^{11} . With such a current profile it is clearly better to have a shorter z_0 , so that the pinch could be allowed to occur much earlier before the current has dropped too much. As seen in the results of table 1, the optimum z_0 is in fact found to be 500 cm with $Y_n = 1.3 \times 10^{13}$. The current profile corresponding to

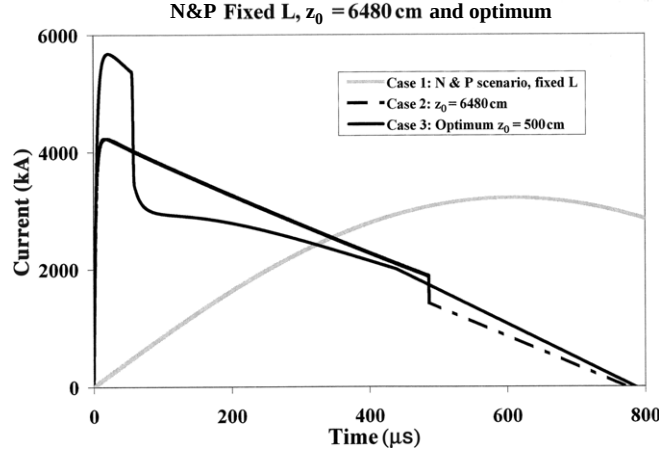


Figure 1. Current waveform for the N&P scenario (Case 1) compared with computed waveform using N&P matching $z_0 = 6480$ cm (Case 2). Also shown is the computed current waveform for optimum $z_0 = 500$ cm (Case 3).

this optimum is shown in Case 3 of figure 1. Thus, figure 1 shows that the conclusion of the N&P scenario that the tube inductance and tube length should grow proportionately with C_0 , for large C_0 , is not correct. This effectively invalidates their argument for I_{peak} saturation and hence also Y_n saturation.

Looking more closely at the numerical results we note that the risetime to I_{peak} is only $19 \mu\text{s}$, which is less than the short circuit rise time of $(\pi/2)(L_0 C_0)^{0.5} \sim 58 \mu\text{s}$. At this time of $19 \mu\text{s}$, the axial speed has already reached 9.9 cm us^{-1} . At that speed, the dynamic resistance $0.5(dL/dt) = 10^{-7} \ln(b/a)v_z = 3.3 \text{ m}\Omega$, which is dominant when compared with the bank stray resistance of $1.1 \text{ m}\Omega$ and short circuit surge impedance of $0.9 \text{ m}\Omega$, even if we consider that at this time the current sheet has traveled 140 cm adding another 92 nH to the circuit, so that at this time the effective surge impedance is $1.7 \text{ m}\Omega$. It can then be seen that the dynamic resistance is the controlling factor and it is the small initial inductance coupled with the rapid increase in dynamic resistance which causes this early peaking and subsequent flattening and droop of the discharge current. We also note that this dominance of the dynamic resistance occurs only at large C_0 ; and the larger the C_0 , the more the dominance. At small C_0 , for example, at $100 \mu\text{F}$, the short circuit impedance is $18 \text{ m}\Omega$, whilst the dynamic resistance is unchanged at $3.3 \text{ m}\Omega$. In those cases of lower C_0 , no early peaking followed by subsequent drooping flat-top is observed.

This early peaking and subsequent current droop invalidate the N&P scenario.

We now describe the numerical experiments which show how I_{peak} , I_{pinch} and Y_n vary with C_0 .

6. Numerical experiments at 35 kV, 10 Torr, $L_0 = 33.5 \text{ nH}$, RESF = 1.22 and $b/a = 1.39$, varying C_0 - No saturation observed

The numerical experiments are then carried out for a range of C_0 . The pressure is fixed at $P_0 = 10 \text{ Torr}$ deuterium. The results are shown in figures 2–5. From these figures we see that as E_0 is increased by increasing C_0 , from 8.5 kJ to 25 MJ , there is no saturation in I_{peak} , I_{pinch} or Y_n as functions of C_0 or E_0 .

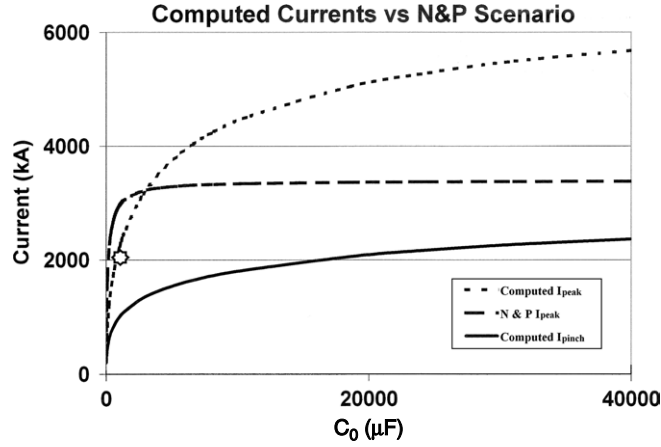


Figure 2. I_{peak} (top trace) computed from numerical experiments as a function of C_0 , compared to I_{peak} envisaged by N&P scenario (middle trace). Also shown is the I_{pinch} curve (lower trace). The single point at the 2 MA level is an experimental PF1000 point [23].

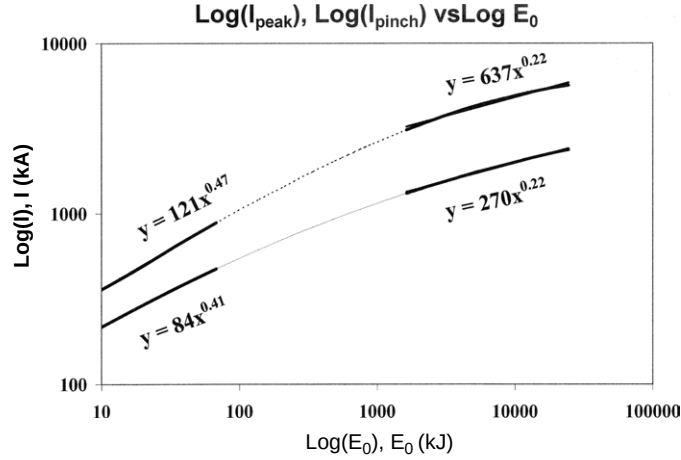


Figure 3. $\text{Log } I_{\text{peak}}$ (top curve) and $\text{Log } I_{\text{pinch}}$ versus $\text{Log } E_0$, showing no saturation for E_0 up to 25 MJ.

Figure 2 shows the computed I_{peak} as a function of C_0 , from our numerical experiments compared with that postulated by the N&P scenario. The important difference is that whereas the N&P scenario shows I_{peak} saturation, our numerical experiments show no saturation; although there is a scaling shift from $I_{\text{peak}} \sim E_0^{0.47}$ to $I_{\text{peak}} \sim E_0^{0.22}$ which is seen when plotted on log–log scale (see figure 3).

More importantly, the I_{pinch} scaling with E_0 shows a similar slowdown from $I_{\text{pinch}} \sim E_0^{0.41}$ to $I_{\text{pinch}} \sim E_0^{0.22}$ (see figure 3), but again no saturation. As was shown in earlier papers [3, 20–22] it is the I_{pinch} scaling, rather than I_{peak} , which directly affects the neutron yield scaling.

For this series of experiments we find that the Y_n scaling changes from $Y_n \sim E_0^{2.0}$ at tens of kJ to $Y_n \sim E_0^{0.84}$ at the highest energies (up to 25 MJ) investigated in this series. This is shown in figure 4(a). Figure 4(b) shows the values of z_0 , optimized for the neutron yield and

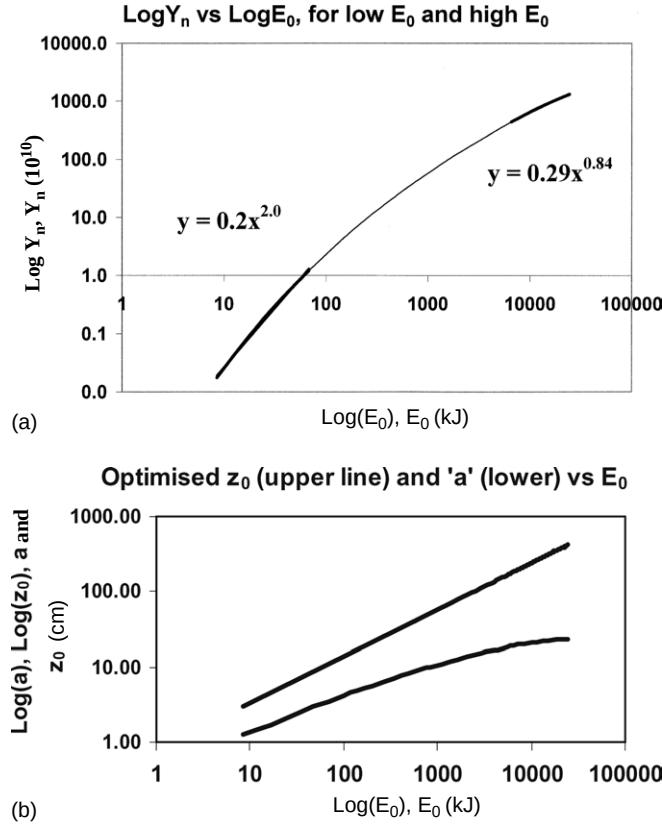


Figure 4. (a) Y_n plotted as a function of E_0 in log–log scale, showing no saturation of the neutron yield up to 25 MJ, the highest energy investigated. (b). Optimized z_0 and ‘a’ versus E_0 for the numerical experiments of (a).

the corresponding value of ‘a’ for an end axial speed of $10 \text{ cm } \mu\text{s}^{-1}$. These anode dimensions are used in the numerical experiments recorded in figure 4(a).

Because of the way Y_n versus E_0 scaling slows down at the megajoule level and the corresponding way I_{peak} and I_{pinch} scaling also slow down, the scaling of Y_n with I_{peak} and I_{pinch} over the whole range of energies investigated up to 25 MJ (figure 5) is as follows:

$$Y_n = 3.2 \times 10^{11} I_{\text{pinch}}^{4.5}; \quad Y_n = 1.8 \times 10^{10} I_{\text{peak}}^{3.8} \text{ where } I_{\text{peak}} \text{ and } I_{\text{pinch}} \text{ are in MA.}$$

In this scaling, I_{peak} ranges from 0.3 to 5.7 MA and I_{pinch} ranges from 0.2 to 2.4 MA.

7. Numerical experiments to attain $Y_n = 10^{13}$ D–D neutrons per shot, using a less resistive bank of RESF = 0.12

Gribkov *et al* [24] had pointed out that $Y_n = 10^{13}$ in deuterium is a desired landmark to achieve in a plasma focus device, from the point of view of possible exploitation as a powerful source of *fusion* neutrons for testing of prospective materials for the first wall components and construction elements in magnetic confinement fusion and, especially, in inertial confinement fusion reactors. Converting such a plasma focus yield to operation in D–T with $Y_n = 10^{15}$ could produce, during a one-year run, an overall fluence of the order of 0.1–1.0 dpa for such

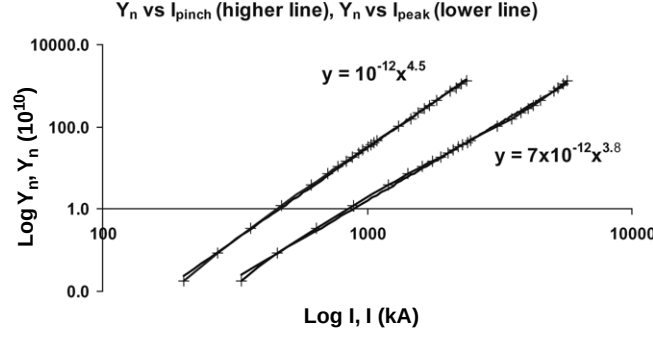


Figure 5. $\text{Log}(Y_n)$ scaling with $\text{Log}(I_{\text{peak}})$ and $\text{Log}(I_{\text{pinch}})$, for the range of energies investigated, up to 25 MJ.

testing purposes, at a very low cost relative to other methods currently being considered. We now examine the requirements to reach this landmark.

In the above series of numerical experiments we have shown that Y_n does not saturate with increasing E_0 at the megajoule level. The scaling does deteriorate from $Y_n \sim E^2$ to a relationship closer to $Y_n \sim E_0$. Nevertheless, because of the non-saturation, $Y_n = 10^{13}$ is achieved at 18–19 MJ (see figure 4(a)) with I_{peak} and I_{pinch} of 5.5 MA and 2.3 MA, respectively.

However, in the above experiments the capacitor bank was assigned a relatively large resistance r_0 with $\text{RESF} = r_0/(L_0/C_0)^{0.5}$ of 1.22, which is an unusually high damping factor associated with PF1000. With a modern bank we should be able to have a less highly damped bank with an RESF of say 0.12.

We repeat the above experiments with the RESF changed to 0.12, representative of a higher performance modern capacitor bank. We keep $c = b/a = 1.39$ and $P_0 = 10$ Torr Deuterium. We obtain results which are summarized in figure 6(a).

These results show that using a less resistive modern bank reduces the E_0 required to reach $Y_n = 10^{13}$ in deuterium to some 8 MJ with I_{peak} and I_{pinch} of 6 MA and 2.3 MA, respectively. Figure 6(b) shows the optimized geometry required for the numerical experiments of figure 6(a).

8. Investigating the role of pressure, electrode ratios and static inductance L_0

We want to investigate the effect of increase in V_0 [1, 2]. A preliminary run at $C_0 = 1332 \mu\text{F}$ under the conditions of figure 6(a) shows that as V_0 is increased from 35 to 90 kV, Y_n increases substantially to above 2×10^{13} . The indications are that at 90 kV, C_0 in the region 700–800 μF would be sufficient to produce $Y_n = 10^{13}$ in deuterium. However, before we finalise these numerical experiments, varying V_0 , we need to fix practical optimum conditions in pressure, radius ratio and static inductance L_0 .

We vary the pressure from 1 Torr upwards in suitable steps, adjusting z_0 and ‘ a ’ for optimum Y_n at each P_0 , with the requirement that the end axial speed is maintained at 10 cm us^{-1} . Then we look for the optimum over the range of pressures. We find the following. At $E_0 = 1332 \mu\text{F}$, Y_n peaks at 10 Torr. As E_0 is increased, the optimum P_0 increases. At the highest energy investigated there is a factor of 3 in Y_n between 10 and 60 Torr, with Y_n still increasing above 60 Torr. However, at this point we consider the technical situation [25] regarding the current per unit radius, I_{peak}/a . The factor controlling speed is $S = (I_{\text{peak}}/a)/P_0^{0.5}$ [11]. Hence, at any I_{peak} , as P_0 is increased, to maintain the end axial speed

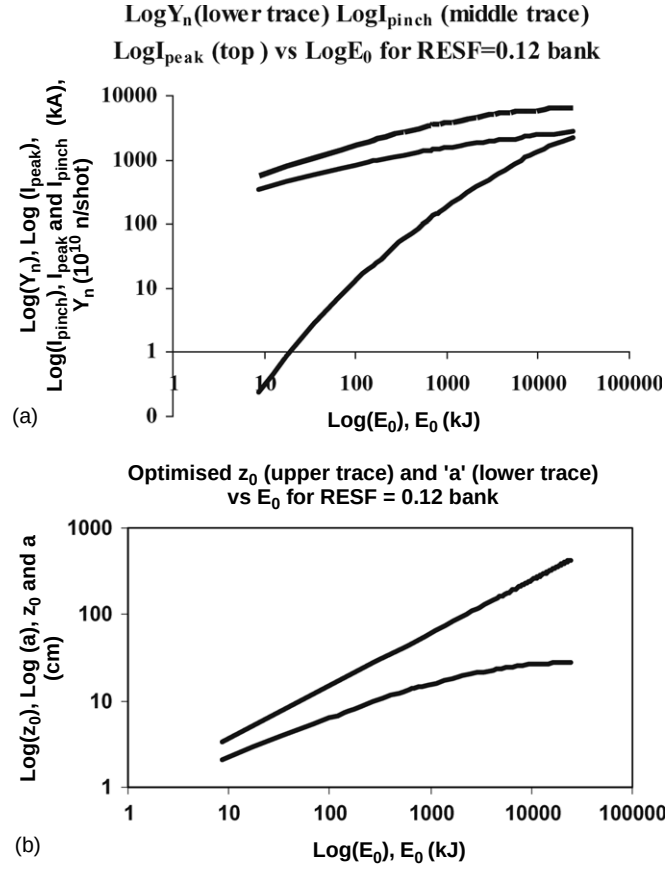


Figure 6. (a) Log-log plots of Y_n (lower trace), I_{pinch} (middle trace) and I_{peak} (top trace) versus E_0 for a high performance bank up to 25 MJ; computed from numerical experiments. (b) Optimised electrode geometry used in numerical experiments of (a).

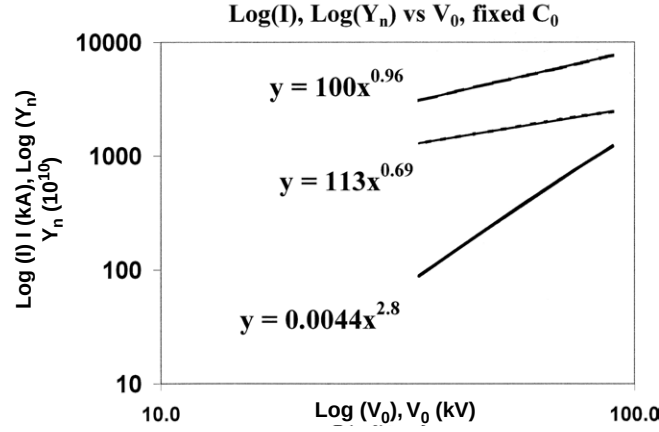
of 10 cm us^{-1} , (I_{peak}/a) has to be increased by reducing ‘ a ’. At 10 Torr, (I_{peak}/a) is in the region $250\text{--}300 \text{ kA cm}^{-1}$ over the range of energies investigated. At $P_0 = 60$ Torr, (I_{peak}/a) needs to be increased by a factor nearly 2.5. From this technical aspect, for this exercise, we set a limit of 300 kA cm^{-1} . Hence, from this point of view we keep the pressure at 10 Torr for all our higher E_0 experiments, knowing that to go lower in P_0 would move the operational point further from optimum and sacrificing the move closer to optimum at higher P_0 in order not to exceed (I_{peak}/a) of 300 kA cm^{-1} . We make a note here that if we can improve anode materials technology to withstand (I_{peak}/a) greater than 300 kA cm^{-1} , then, in that case, the following results would be conservative and may be upgraded accordingly.

We next vary the radius ratio $c = b/a$. We start with the optimum condition which we have found for $C_0 = 1332 \mu\text{F}$. At each value of ‘ c ’, we adjusted the values of ‘ a ’ and z_0 for optimum. We vary ‘ c ’ from 1.2 to 1.6 and find that 1.39 is at the optimum. It appears that the radius ratio $c = 1.39$ used in PF1000 [2] had already been very well chosen.

We next examine the choice of L_0 . It had been shown [3, 22] that for a fixed C_0 , if L_0 is reduced, there is a range of L_0 at which I_{pinch} reaches a flat maximum. There is no advantage lowering L_0 below this range; indeed I_{pinch} would suffer a slight decrease, due to

Table 2. Numerical experiments on effect of increasing V_0 , at fixed C_0 of 777 μF .

V_0 (kV)	E_0 (kJ)	b (cm)	a (cm)	z_0 (cm)	I_{peak} (kA)	I_{pinch} (kA)	Y_n (10^{10})
90	3147	39.92	27.65	25	7580	2483	1228
70	1904	31.14	22.40	30	5955	2091	631
50	971	23.44	16.86	35	4365	1652	246
35	476	16.69	12.01	37	3074	1286	88

**Figure 7.** Scaling of currents and Y_n as functions of operating voltage V_0 . Top curve: $\text{Log}(I_{\text{peak}})$, middle curve: $\text{Log}(I_{\text{pinch}})$ and bottom curve: $\text{Log}(Y_n)$.

this focus pinch current limitation. Looking at the range of large E_0 we are dealing with in these experiments we find that a good compromise value of L_0 is 36 nH which ensures optimum I_{pinch} .

In consideration of the above we fixed optimum values of $L_0 = 36$ nH, $c = b/a = 1.39$ and settled on $P_0 = 10$ Torr (for the highest pressure whilst keeping the technical condition of not exceeding 300 kA cm $^{-1}$). We consider these as the practical optimum conditions.

9. Investigating the effect on Y_n as operating voltage is increased from 35 to 90 kV, at $C_0 = 777 \mu\text{F}$

We next run numerical experiments at practical optimum conditions $c = b/a = 1.39$, $L_0 = 36$ nH, $P_0 = 10$ Torr. We keep C_0 at 777 μF and vary V_0 from 35 to 90 kV. The results are summarized in table 2. The results are also plotted in figure 7 in log-log scale.

Figure 7 shows that $Y_n \sim V_0^{2.8}$ over the range of voltages examined from 35 to 90 kV.

Looking at this scaling, it may at first sight be tempting to think in terms of increasing the voltage further. However, it is then necessary to look more closely at that prospect. An examination of the computed results shows that the computed effective beam energy [3,20,22] for 90 kV is already at the 330 keV level. Looking at data for the D–D cross-section [26] as a function of the beam energy, it is seen that above 300 keV, the rise in the D–D fusion cross-section is very slow. Hence, there is little advantage operating above 90 kV. In fact, the situation is actually disadvantageous to increasing the operating voltage if one considers changing to D–T operation. The D–T fusion cross-section [26] has already peaked at 120 keV,

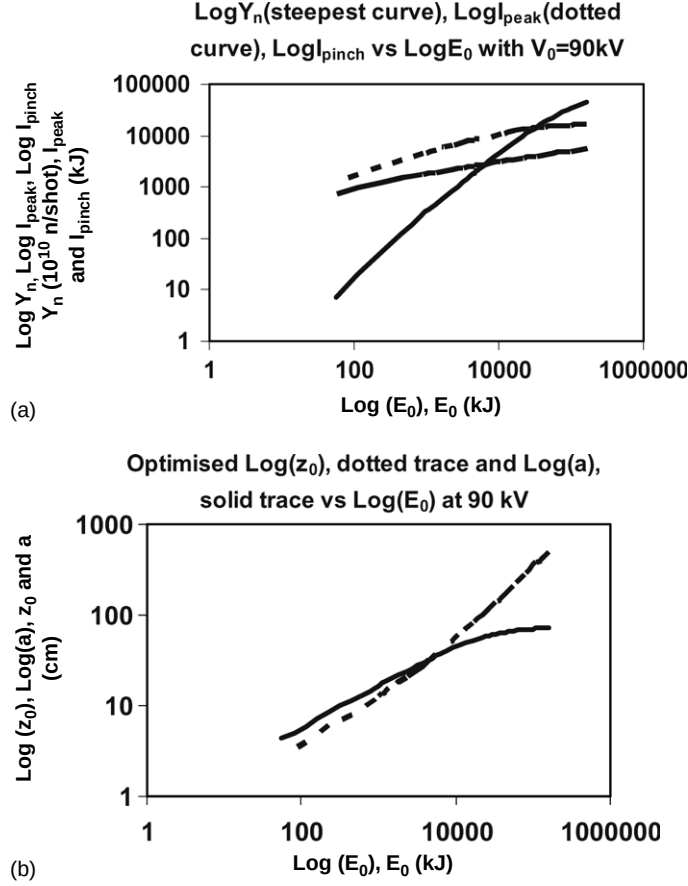


Figure 8. (a) Numerical experiments at 90 kV, varying C_0 , to obtain scaling of I_{peak} , I_{pinch} and Y_n with E_0 . $\text{Log}(Y_n)$: steepest curve; $\text{Log}(I_{\text{peak}})$: dotted curve; $\text{Log}(I_{\text{pinch}})$: other curve. Y_n in units of 10^{10} D–D neutrons/shot; I_{peak} and I_{pinch} in kA. (b) Optimized geometry corresponding to numerical experiments for (a).

and operating at 90 kV with the beam energy at 330 keV, the beam energy is already too high, the D–T cross-section having dropped by a factor of around 3.6 from its peak. It seems then that from this point of view there is no advantage in operating a plasma focus at higher than 90 kV. For conversion to D–T operation it would probably be better to operate at a lower voltage. It would then be necessary to increase C_0 until 10^{15} D–T neutrons is reached.

10. Investigating operation at 90 kV, varying E_0 , by varying C_0 ; at 10 Torr, $L_0 = 36$ nH and $b/a = 1.39$; RESF = 0.12

We consider the effect of operating at 90 kV. We run experiments at 90 kV with increasing E_0 (by increasing C_0) to obtain the energy required to reach $Y_n = 10^{13}$ D–D neutrons per shot. At each C_0 , z_0 is varied whilst adjusting ‘ a ’ for an end axial speed of 10 cm us^{-1} . The optimum z_0 is thus found for each C_0 (E_0). The results are shown in figure 8(a). Again at this higher voltage, no saturation is found for I_{peak} , I_{pinch} or Y_n . At 90 kV we confirm we are able to reduce the E_0 required for $Y_n = 10^{13}$ D–D fusion neutrons per shot to 3 MJ, with $C_0 = 777 \mu\text{F}$ as

shown in figure 8(a). The values of I_{peak} and I_{pinch} are, respectively, 7.6 MA and 2.5 MA. The required anode geometry is also shown in figure 8(b).

Furthermore, at 90 kV with the highest value of C_0 investigated as 39960 μF , the storage energy is 162 MJ. At that storage energy, optimized Y_n is 4.5×10^{14} D–D neutrons/shot with $I_{\text{peak}} = 17.3$ MA and $I_{\text{pinch}} = 5.7$ MA.

11. Conclusion

This paper shows that the N&P scenario is erroneous in its conclusion regarding the saturation of Y_n at megajoule energies as E_0 is increased by the increase in C_0 . The N&P scenario contends that this saturation is due to electrodynamic effects. Our numerical experiments show that the scaling of L_a and z_0 envisaged by the N&P scenario is far from the optimum. Laboratory experiments at the 1 MJ level as reported in the literature have been carried out close to the optimum as confirmed by our numerical experiments. The numerical experiments show no saturation in I_{peak} , I_{pinch} or Y_n that may be traced to the electrostatics governing the system, although there is a slowing down of scaling at high E_0 , e.g. $Y_n \sim E_0^2$ at low energies and $Y_n \sim E_0^{0.84}$ at high megajoule levels. Thus, any saturation of Y_n with E_0 (as C_0 is increased) cannot be ascribed to the physics governing the electrostatics of the system. Other, possibly machine-related, effects outside the scope of this paper may have to be investigated to account for the apparently observed saturation effects. In connection with this it may be pointed out that the drop in scaling for Y_n below E_0 is a significant disappointment from the point of view of scaling for fusion energy production purposes.

This paper finds that scaling up from a PF1000-like capacitor bank requires close to 19 MJ to reach a target D–D neutron yield of 10^{13} per shot. However, the numerical experiments also find that a modern bank with typical lower damping may achieve the same target D–D neutron yield of 10^{13} at 8 MJ operating at a typical voltage of 35 kV. The energy requirement is further reduced to 3 MJ by increasing the operational voltage to 90 kV. Because of the high effective beam energy already at 90 kV, there is little advantage in operating at voltages above 90 kV for the D–D neutron yield.

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Soft x-ray yield from NX2 plasma focus

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The Lee model code is used to compute neon soft x-ray yield Y_{sxr} for the NX2 plasma focus as a function of pressure. Comparison with measured Y_{sxr} shows reasonable agreement in the Y_{sxr} versus pressure curve, the absolute maximum yield as well as the optimum pressure. This gives confidence that the code gives a good representation of the neon plasma focus in terms of gross properties including speeds and trajectories and soft x-ray yields, despite its lack of modeling localized regions of higher densities and temperatures. Computed current curves versus pressure are presented and discussed particularly in terms of the dynamic resistance of the axial phase. Computed gross properties of the plasma focus including peak discharge current I_{peak} , pinch current I_{pinch} , minimum pinch radius r_{min} , plasma density at the middle duration of pinch n_{pinch} , and plasma temperature at middle duration of pinch T_{pinch} are presented and the trends in variation of these are discussed to explain the peaking of Y_{sxr} at optimum pressure. © 2009 American Institute of Physics.
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I. INTRODUCTION

Plasma focus has been demonstrated as potential x-ray source for various medicobiological and industrial applications such as lithography^{1–4} (using ~ 0.9 – 1.5 keV photons), radiography,^{5,6} microscopy^{7,8} (using ~ 0.25 – 2.5 keV radiations), and micromachining⁹ (using ~ 4 keV photos). This has led to an increasing interest in exploiting the plasma focus device as a viable intense x-ray source due to some clear advantages such as being relatively cheap, compact, and ease of construction. The x-ray emissions from plasma focus devices have been explored over the wide range of capacitor bank energies ranging from large megajoule and few hundred kilojoule banks¹⁰ to medium sized kilojoule banks^{4,11–14} to subkilojoule banks of miniature sized focus devices.^{15,16} In the past few years various efforts have been made for enhancing the x-ray yield by changing various experimental parameters such as bank energy,¹⁷ discharge current, electrode configuration (shape and material),^{11,13} insulator material and dimensions,¹¹ gas composition, and filling gas pressure.⁵ Thus, soft x-ray yield optimization studies on the plasma focus devices operating over the wide range of bank energies have been one of the actively pursued fields of plasma focus research owing to their vast possible applications. Currently used systematic trial and error experimental procedure to obtain the optimized conditions for maximum radiation yield is highly time-consuming. Hence, the quicker optimization of plasma focus device is highly desirable, which can be achieved if the reliable focus model and corresponding simulation code to predict the x-ray yields from plasma focus device can be developed and used. Obviously the computed yields need to be checked against corresponding measured yields. Further, if the computed soft x-ray

yields are consistently reliable against measured values; then it is reasonable to use the computed gross plasma properties as indicative of what we can expect when these plasma properties are measured. In this way, a reliable model code cannot only be used to compute radiation yields, but also be used as a good indicative diagnostic tool for multiple gross plasma properties of the plasma focus.

In the present paper, we used the Lee model code version 13.6b to carry out the numerical experiments on NX2 plasma focus device to compute its neon soft x-ray yield Y_{sxr} as a function of filling gas pressure. The NX2 is a 3 kJ plasma focus originally designed to operate as a neon soft x-ray source with 20 J per shot at 16 shots/s with burst durations of several minutes.⁴ Its performance in repetitive mode has been extensively studied, especially in regards to its discharge currents and soft x-ray yield Y_{sxr} . In this paper, we have simulated the operation of NX2 focus device in numerical experiments which are designed to compare its currents, dynamics, and some plasma pinch gross properties at various pressures so as to examine the role played by various relevant plasma properties on the way the Y_{sxr} peaks at the optimum pressure.

II. THE MODEL CODE USED FOR NUMERICAL EXPERIMENTS

The Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation, enabling realistic simulation of all gross focus properties. The basic model, described in 1984,¹⁸ was successfully used to assist several projects.^{14,19–21} Radiation-coupled dynamics was included in the five-phase code leading to numerical experiments on radiation cooling.²² The vital role of a finite small disturbance speed discussed by Potter²³ in a Z-pinch situation was incorporated together with real gas thermodynamics and radiation-yield terms;²⁴ this version of the code assisted

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other research projects^{4,25,26} and was web-published in 2000²⁷ and 2005.²⁸ Plasma self-absorption was included in 2007 (Ref. 27) improving soft x-ray yield simulation. The code has been used extensively in several machines including UNU/ICTP PFF,^{4,14,21,25,29} NX2,^{4,26} NX1,⁴ and adapted for the Filippov-type plasma focus DENA.³⁰ A recent development is the inclusion of the neutron yield Y_n using a beam-target mechanism,^{31–34} incorporated in the present version³⁵ of the code RADPFV5.13, resulting in realistic Y_n scaling with pinch current I_{pinch} .^{31,32} The versatility and utility of the model is demonstrated in its clear distinction of pinch current I_{pinch} from peak discharge current I_{peak} (Ref. 36) and the recent uncovering of a plasma focus pinch current limitation effect^{31,33} as well as elucidation of neutron scaling laws to multimega-Joule facilities.³⁴ The description, theory, code and a broad range of results of this “Universal Plasma Focus Laboratory Facility” is available for download from world wide web.³⁵

A brief description, however, of the five phases incorporated in the Lee model code is as follows.

- (1) Axial phase: the axial phase is described by a snowplow model with an equation of motion which is coupled to a circuit equation. The equation of motion incorporates the axial phase model parameters: mass and current factors f_m and f_c . The mass swept-up factor f_m accounts for not only the porosity of the current sheath but also for the inclination of the moving current sheath-shock front structure and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor f_c accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall, current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.
- (2) Radial inward shock phase: it is described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, also called the magnetic piston, allowing the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock front and current sheath due to the finite small disturbance speed is crucially implemented in this phase. The model parameters, radial phase mass swept up, and current factors f_{mr} and f_{cr} are incorporated in all three radial phases. The mass swept-up factor f_{mr} accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase not least of which

could be axial ejection of mass. The current factor f_{cr} accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

- (3) Radial reflected shock (RS) phase: when the shock front hits the axis, because the focus plasma is collisional, a RS develops which moves radially outwards, while the radial current sheath piston continues to move inwards. Four coupled equations are also used to describe this phase, these being for the RS moving radially outwards, the piston moving radially inwards, the elongation of the annular column and the circuit equation. The same model parameters f_{mr} and f_{cr} are used as in the previous radial phase. The plasma temperature behind the RS undergoes a jump by a factor nearly 2.
- (4) Slow compression (quiescent) or pinch phase: when the outgoing RS hits the ingoing piston the compression enters a radiative phase in which for gases such as neon, the radiation emission may actually enhance the compression where we have included energy loss/gain terms from Joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects are incorporated into this phase. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration.
- (5) Expanded column phase: to simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration. In this final phase the snow plow model is used and two coupled equations are used similar to the axial phase above. This phase is not considered important as it occurs after the focus pinch.

We note that in radial phases 2, 3, and 4, axial acceleration and ejection of mass caused by necking curvatures of the pinching current sheath result in time dependent strongly center-peaked density distributions. Moreover the transition from phase 4 to phase 5 is observed in laboratory measurements to occur in an extremely short time with plasma/current disruptions resulting in localized regions of high densities and temperatures. These center-peaking density effects and localized regions are not modeled in the code, which consequently computes only an average uniform density and an average uniform temperature which are considerably lower than measured peak density and temperature (we thank a Reviewer for his comments regarding this point). However, because the four model parameters are obtained by fitting the computed total current waveform to the measured total current waveform, the model incorporates the energy and mass balances equivalent, at least in the gross sense to all the processes, which are not even specifically modeled. Hence

the computed gross features such as speeds and trajectories and integrated soft x-ray yields have been extensively tested in numerical experiments for several machines and are found to be comparable with measured values.

III. X-RAY EMISSIONS IN PLASMA FOCUS AND ITS INCORPORATION IN MODEL CODE

The focused plasma, with electron temperature of a few hundreds of eV to about keV and high enough electron density, is a copious source of x rays. The plasma focus emits both soft (thermal) as well as hard (nonthermal) x rays but for the scope of this paper, we will concentrate only on soft thermal x rays. The plasma focus emits soft thermal x rays by three processes,^{37,38} namely: bremsstrahlung (free-free transition) from the Coulomb interactions between electrons and ions; recombination radiation (free-bound transition) emitted by an initially free electron as it loses energy on recombination with an ion; and de-excitation radiation (bound-bound transition) when a bound electron loses energy by falling to a lower ionic energy state. The first two processes give rise to the continuum of the x-ray spectrum, while the third process produces the characteristic line radiation of the plasma. The relative strengths of the continuum and line emissions depend on how the plasma was formed; typically, for a plasma formed from a high-Z material continuum emission dominates, while for a low-Z material line emission can be stronger. The calculation of the power emitted by processes within the plasma depends on assumptions made about the state of the plasma. Following the spectral data obtained by Mahe²⁴ and Liu *et al.*²⁵ for the soft x rays from neon operated 3.3 kJ UNU-ICTP plasma focus device, it was found that 64% of soft x-ray emission can be attributed to line radiations at 922 eV (Ly- α) and 1022 eV (He- α) and the remaining 36% by the rest, mainly recombination radiation, for optimized operations. For NX2 plasma focus device, Zhang³⁹ reported the contribution of line radiation rising to about 80%. It is for these reasons, and also for the temperatures of interest in our numerical experiments on NX2 device we take the neon soft x-ray yield to be equivalent to line radiation yield, i.e., $Y_{\text{SXR}} = Q_L$.

In the code in phase 4, pinch phase neon line radiation Q_L is calculated using the relation

$$\frac{dQ_L}{dt} = -4.6 \times 10^{-31} n_i^2 Z Z_n^4 (\pi r_p^2) z_f T,$$

after being integrated over the pinch duration. Hence the SXR energy generated within the plasma pinch depends on the properties: number density n_i , effective charge number Z , atomic number of gas Z_n , pinch radius r_p , pinch length z_f , plasma temperature T , and the pinch duration.

This generated energy is then reduced by the plasma self-absorption, which depends primarily on density and temperature; the reduced quantity of energy is then emitted as the SXR yield. It was first pointed by Mahe²⁴ that a temperature around 300 eV is optimum for SXR production from neon operated plasma. Bing's²⁶ subsequent work and our subsequent experience through numerical experiments suggest that around 2×10^6 K (below 200 eV) seems to be

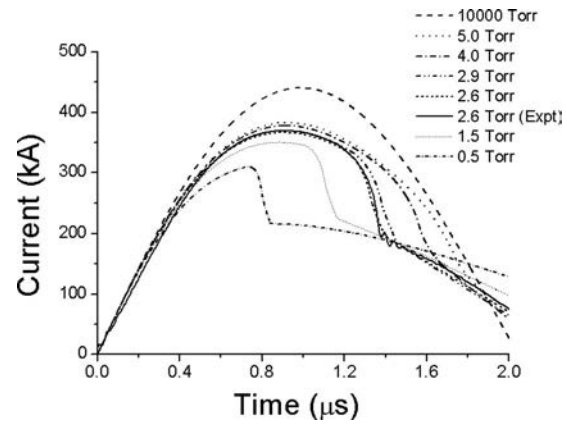


FIG. 1. Fine tuning of Lee model parameter by fitting of computed total current waveform of numerical experiment conducted at 2.6 Torr to that of experimentally measured waveform at same 2.6 Torr of neon. Plots of discharge current waveforms from numerical experiments performed over wide range of neon filling gas pressures are also shown for comparison.

better. Hence unlike the case of neutron scaling, for neon SXR scaling there is an optimum small range of temperatures (T window) to operate.

IV. NUMERICAL EXPERIMENTS AND COMPARISON WITH EXPERIMENTAL RESULTS

To start the numerical experiments we select a discharge current trace of the NX2 taken with a Rogowski coil. The selected measured waveform is of a shot at 2.6 Torr neon, near optimum Y_{SXR} yield. The following bank, tube, and operation parameters are used; bank: static inductance $L_0 = 15$ nH, $C_0 = 28$ μ F, stray resistance $r_0 = 2.2$ m Ω ; tube: cathode radius $b = 4.1$ cm, anode radius $a = 1.9$ cm, anode length $z_0 = 5$ cm; and operation: voltage $V_0 = 11$ kV, pressure $P_0 = 2.6$ Torr.

The computed total current waveform is fitted to the measured waveform by varying model parameters f_m , f_c , f_{mr} , and f_{cr} one by one until the computed waveform agrees with the measured waveform. First, the axial model factors f_m and f_c are adjusted (fitted) until the computed rising slope of the total current trace and the rounding off of the peak current as well as the peak current itself are in reasonable (typically very good) fit with the measured total current trace (see Fig. 1, e.g., 2.6 Torr measured trace and computed trace). Then we proceed to adjust (fit) the radial phase model factors f_{mr} and f_{cr} until the computed slope and depth of the dip agree with the measured. In this case, the following fitted model parameters are obtained: $f_m = 0.1$, $f_c = 0.7$, $f_{mr} = 0.12$, and $f_{cr} = 0.68$. These fitted values of the model parameters are then used for the computation of all the discharges at various pressures.

The code is used for each pressure, starting at high pressure (about 10 000 Torr, which is not an issue in numerical experiments although we would not use such pressures in “hardware” experiments) so that the discharge current stayed at the backwall with hardly any motion and hence can be treated as short circuit discharge. The discharge current then resembles that of a simple L - C - R discharge, which is a damped sinusoid. The pressure is then lowered for another

TABLE I. Computed plasma dynamics and pinch plasma parameters for different neon filling gas pressures by numerical experiments conducted on NX2 device using Lee model code. [Parameters used in the table are: I_{peak} is the peak value of the total discharge current; I_{pinch} is the pinch current, taking its value at the start of the pinch phase; peak v_a =peak axial speed, typically end axial speed; S =speed parameter (in kA/cm/Torr^{1/2}); peak v_s , v_p =peak radial shock and piston speeds, respectively; r_{min} =minimum radius or focus pinch radius at maximum compression; z_{max} =maximum length of focus pinch at time of maximum compression (note that the anode is hollow); T_{pinch} =plasma temperature at middle of pinch duration; n_i pinch=ion density at the middle of pinch duration; Z =effective charge of the neon plasma at middle of pinch duration; and EINP=work done by the dynamic resistance during radial phase expressed as % of E_0 .]

P_0 (Torr)	I_{peak} (kA)	I_{pinch} (kA)	Peak v_a (cm/ μ s)	S	Peak v_s (cm/ μ s)	Peak v_p (cm/ μ s)	r_{min} (cm)	z_{max} (cm)	Pinch duration (ns)	T_{pinch} 10 ⁶ K	n_i pinch (10 ²³ /m ³)	Z	EINP (%)	Y_{sxr} (J)
High	440										Middle of pinch			
5	383	76	4.6	90	11.1	8.6	0.86	2.84	100	0.3	1.1	5.5	6.3	0
4.5	381	99	4.8	94	12.2	9.5	0.42	2.7	60	0.47	2.4	7.7	8.6	0
4	378	114	5	99	14.9	11.6	0.29	2.7	46	0.7	3.2	8	10.7	0
3.5	374	128	5.3	105	17	12.8	0.22	2.75	37	1.03	3.9	8	12.9	4.5
3.2	372	135	5.6	109	18.8	13.7	0.19	2.79	34	1.23	4.1	8	14.4	14.6
3	370	140	5.7	113	20	14.1	0.18	2.8	32	1.4	4.1	8	15.2	19.9
2.9	369	142	5.8	114	20.6	14.5	0.17	2.79	30.6	1.51	4	8	15.5	20.8
2.8	369	144	5.9	116	21.1	14.8	0.17	2.79	29.6	1.61	3.8	8	15.7	20
2.7	368	146	6	118	21.8	15	0.18	2.78	28.8	1.72	3.5	8	15.8	17.9
2.6	367	148	6.1	120	22.5	15.3	0.19	2.75	27.3	1.86	3	8	15.8	14.4
2.4	364	152	6.3	124	24.4	15.4	0.22	2.7	23.5	2.18	2.3	8	15.6	8
2	359	159	6.8	134	25.2	16.7	0.25	2.73	23.6	2.8	1.6	8.2	16.2	3.9
1.5	350	164	7.6	151	27.6	18.8	0.26	2.77	22.4	3.9	1.1	8.7	16.7	1.5
1	338	165	8.8	178	32	22.7	0.26	2.77	19.3	5.5	0.7	9.3	16.5	0.4
0.5	310	157	11.1	230	41	28.6	0.26	2.78	15.5	9.4	0.35	10	14.4	0.05

run. This is repeated each time lowering the filling neon pressure. Figure 1 records the discharge current waveforms for some of the selected pressures covering a wide range of neon operating pressures from 5 Torr down to 0.5 Torr. The Fig. 1 also includes the simulated waveform for high pressure shot and measured waveform at 2.6 Torr. It may be noticed that computed total current waveform at 2.6 Torr numerical experiment is almost identical to the measured total current waveform for the 2.6 Torr actual experiment conducted by Zhang indicating an extremely good fine tuning of Lee model parameters, i.e., f_m , f_c , f_{m^2} and f_{cr} [0.1, 0.7, 0.12, and 0.68, respectively, for this shot] and hence provide confidence in simulated results of the gross properties. Figure 1 shows that the unloaded (dynamically) high pressure discharge waveform peaks at about 440 kA just before 1.1 μ s. At 5 Torr, the peak of the total current I_{peak} is 380 kA and a small current dip is seen at 1.8 μ s which is well after peak current with the total discharge current having dropped to 150 kA at the start of the dip. At successive lower pressure, I_{peak} reduces progressively while the current dip appears at progressively earlier times. At 1.5 Torr, I_{peak} has dropped to 350 kA and the dip starts at about the time of peak current of the high pressure shot. It is reasonable to correlate the current dip with the radial phase, so the shifting of the current dip earlier and earlier at lower and lower pressures is consistent with higher and higher axial speeds. The higher speeds lead to correspondingly higher dynamic resistance (which is numerically half the rate of change of inductance; thus is proportional to the axial speed for an axial run-down tube of constant cross-sectional dimensions). We also tabulate some properties of the dynamics and the pinch plasma as a function of the pressure as computed by numerical experiments. This is shown in Table I.

From the Table I it is seen that optimum Y_{sxr} is computed at $P_0=2.9$ Torr from the numerical experiments. In order to plot all the properties in one figure each quantity is normalized to its value at optimum, i.e., the value obtained for 2.9 Torr operation. The normalized pinch plasma parameters and absolute Y_{sxr} are then plotted as a function of filling gas pressure of neon (P_0) in Figs. 2(a) and 2(b), respectively. The experimentally measured Y_{sxr} of NX2 operated under similar conditions is also included for comparison. The experimental data in Fig. 2(b) is taken from Fig. 6b of Ref. 4 and also from Fig. 6.7b on page 206 of Ref. 37, and hence the numerical experiments were performed for NX2 device with 5 cm long anode with the device being operated at 11.5 kV. It is evident from Fig. 2(b) (also from Table I) that the Y_{sxr} values from numerical experiments fit the experimentally measured yields reasonably well. It is also necessary to point out here that our computed n_i (being an averaged uniform value) is considerably lower than values measured experimentally. From shock theory we compute for this case (2.6 Torr neon in NX2) a peak on-axis RS value of 2.63×10^{24} ions/m³. Similarly we compute a peak on-axis RS temperature of 2.7×10^6 K. This illustrates that consideration of density and temperature distributions can allow more realistic estimation of these quantities and even their spatial and temporal distributions. Hence, though our model gives only mean values of the key plasma parameters (such as that of n_i and T) and is unable to trace their evolution with an accuracy that probably can be achieved by modern diagnostics technique, but at the same time we also point out that our average methods allow us to compute realistic gross quantities such as trajectories, speeds, and soft x-ray yields.

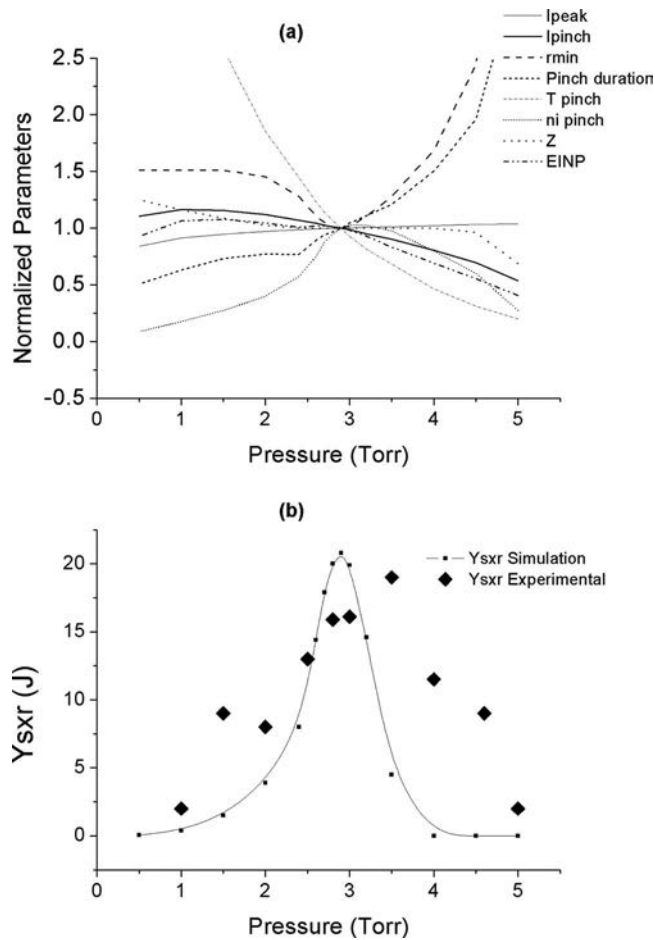


FIG. 2. Effect of operating gas pressure on (a) some key pinch plasma parameters (all normalized using value at optimum operating pressure of 2.9 Torr) and (b) Y_{sxr} , as estimated by numerical experiments. The experimentally measured Y_{sxr} of NX2 operated under similar conditions is also included in (b) for comparison.

V. DISCUSSION OF RESULTS

It is evident from Fig. 2(a) that the peak value of total discharge current I_{peak} decreases with decreasing pressure. This is attributed to increasing dynamic resistance (i.e., increasing rate of change of plasma inductance, dL/dt) due to the increasing current sheath speed as pressure is decreased. We note that, on the contrary, the current I_{pinch} that flows through the pinched plasma column, increases with decreasing pressure. This is due to the shifting of the pinch time toward the time of peak current until the pressure nears 1.2 Torr. As the pressure is decreased below 1.2 Torr, the I_{pinch} starts to decrease as the pinch time now occurs before current peak time. The T_{pinch} , which is the temperature at the middle of the pinch, keeps increasing as pressure is decreased. The n_{pinch} , which is the ion density at middle of the pinch, increases as pressure decreases peaking around 3 Torr and then dropping at lower pressures. The r_{min} , which is the minimum radius of the pinch, has a complementary trend with a minimum at around 3 Torr. This shows that as the operating pressure is reduced toward 3 Torr, the increasing I_{pinch} increases the compression sufficiently so that despite the drop in ambient number density, the pinch n_i is still able to reach a higher value at 3 Torr. As the operating pressure is reduced

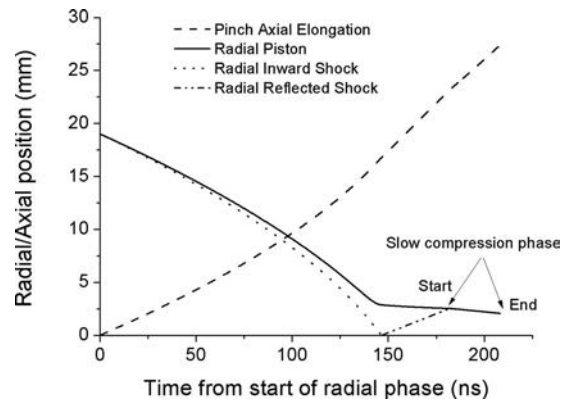


FIG. 3. The currents sheath (radial piston) continues to move in slow compression phase after the radial RS hits it and reaches minimum pinch radius r_{min} at the end of slow compression phase.

below 3 Torr, the increase in I_{pinch} does not appear to be sufficient to further increase n_i or indeed even to compress the pinch to a smaller radius than at 3 Torr. To clarify this situation we briefly explain the plasma dynamics during the radial collapse phase.

The radial phase uses a slug model with an imploding cylindrical shock wave forming the front of the slug, driven by a cylindrical magnetically driven current sheath piston at the rear of the slug. Between the shock wave and the current sheath is the shock heated plasma. When the shock front implodes onto the tube axis, because the plasma is collisional, a RS develops. The RS front moves radially outwards into the inwardly streaming particles of the plasma slug, leaving behind it a stationary doubly shocked plasma with a higher temperature and density than the singly shocked plasma ahead of it. When the RS reaches the incoming current sheath, typically the magnetic pressure exceeds the doubly shocked plasma pressure, in which case the current sheath continues inwards in a further slow compression, until the end of this quasiequilibrium phase. The duration of this slow compression phase may be defined by the transit time of small disturbances. For a well-designed and operated plasma focus there is a slow compression throughout this whole duration and the pinch radius reaches its minimum r_{min} at the end of the phase. These various phases/phenomena can be seen in Fig. 3. The radiation yield depends on: (a) the absolute density (which depends on the ambient density and the compression of which r_{min} is a measure, the smaller r_{min}/a where a is the anode radius, the greater the compression), (b) the temperature (which depends on the imploding speeds [the lower the operating pressure, the higher the imploding speeds, noting that shocked temperatures depends on the square of the shock speeds] and the further compression), (c) the duration of the slow compression phase (which scales inversely as the square root of the pinch temperature), and (d) the volume of the pinched plasma during the slow compression phase (which predominantly scales as a). Thus, in this particular example, as the operating pressure is reduced below 3 Torr, although I_{pinch} still increases, speeds also increase, increasing the temperature, which tends to oppose the severity of the compression during the slow compression phase, although the decreased ambient number density tends

to work in the opposite direction. The interaction of all these factors are taken care of in the code and manifests in the peaking of n_i at 3.1 Torr and the minimum value of $r_{\min}$ at 2.9 Torr. Moreover, as can be seen in Table I, the pinch duration progressively reduces, as the temperature increases with lowering pressure; while the radiating plasma volume reaches a minimum around 2.9 Torr. The interactions of all the behavior of $r_{\min}$, n_i , and T_{pinch} , pinch duration and plasma volume all contribute to the peak in Y_{srx} as a function of operating pressure. Looking at the Table I and Fig. 2(a) it does appear that the peaking of n_{pinch} at 3.1 Torr is a notable factor for the peaking of Y_{srx} at 2.9 Torr.

The Fig. 2(b) shows reasonable agreement the results of numerical experiments and experimentally measured; in terms of absolute value of Y_{srx} at optimum pressure (about 20.8 J by numerical experiment, refer Table I, and about 16.1 J as experimentally measured^{4,39}) as well as the optimum pressure value itself. The computed curve falls off more sharply on both sides of the optimum pressure. This agreement validates our views that the fitting of the computed total current waveform with the measured waveform enables the model to be energetically correct in all the gross properties of the radial dynamics including speeds and trajectories and soft x-ray yields despite the lack of fine features in the modeling.

VI. CONCLUSIONS

To conclude, the Lee model code has been successfully used to perform numerical experiments to compute neon soft x-ray yield for the NX2 as a function of pressure with reasonable degree of agreement in (i) the Y_{srx} versus pressure curve trends, (ii) the absolute maximum yield, and (iii) the optimum pressure value. The only input required is a measured total current waveform. This reasonably good agreement, against the background of an extremely complicated situation to model, moreover the difficulties in measuring Y_{srx} , gives confidence that the model is sufficiently realistic in describing the plasma focus dynamics and soft x-ray emission for NX2 operating in Neon. This encourages us to present Table I and to present the above views regarding the factors contributing to the peaking of Y_{srx} at an optimum pressure.

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Pinch current limitation effect in plasma focus

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The Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiation. It is used to design and simulate experiments. A beam-target mechanism is incorporated, resulting in realistic neutron yield scaling with pinch current and increasing its versatility for investigating all Mather-type machines. Recent runs indicate a previously unsuspected “pinch current limitation” effect. The pinch current does not increase beyond a certain value however low the static inductance is reduced to. The results indicate that decreasing the present static inductance of the PF1000 machine will neither increase the pinch current nor the neutron yield, contrary to expectations. © 2008 American Institute of Physics. [DOI: 10.1063/1.2827579]

This model in its two-phase form was described in 1984.¹ It was used to assist in the design and interpretation of several experiments.^{2–4} An improved five-phase model and code incorporating finite small disturbance speed,⁵ radiation and radiation coupling with dynamics assisted several projects,^{6–8} and was web published⁹ in 2000 and in 2005.¹⁰ Plasma self-absorption was included⁹ in 2007. It has been used extensively as a complementary facility in several machines, for example, UNU/ICTP PFF,^{2,6} the NX2,^{7,8} NX1,⁷ and DENA.¹¹ It has also been used¹² in other machines for design and interpretation including Soto’s subkilojoule plasma focus machines,¹³ FNII,¹⁴ and the UBA hard x-ray source.¹⁵ Information obtained from the model includes axial and radial velocities and dynamics,^{1,7,12,11} soft x-ray (SXR) emission characteristics and yield,^{5,7,8,16} design of machines,^{13,16} optimization of machines, and adaptation to other machine types such as the Filippov-type DENA.¹¹ A study of speed-enhanced neutron yield^{4,13} was also assisted by the model code.

A detailed description of the model is already available on the internet.^{9,10} A recent development in the code is the inclusion of neutron yield using a phenomenological beam-target neutron generating mechanism,¹⁷ incorporated in the present RADPFV5.13. A beam of fast deuteron ions is produced by diode action in a thin layer close to the anode, with plasma disruptions generating the necessary high voltages. The beam interacts with the hot dense plasma of the focus pinch column to produce the fusion neutrons. In this modeling, each factor contributing to the yield is estimated as a proportional quantity and the yield is obtained as an expression with proportionality constant. The yield is then calibrated against a known experimental point.

The beam-target yield is written in the form $Y_{b-t} \sim n_b n_i (r_p^2 z_p) (\sigma v_b) \tau$ where n_b is the number of beam ions per unit plasma volume, n_i is the ion density, r_p is the radius of the plasma pinch with length z_p , σ is the cross section of the D–D fusion reaction, n branch,¹⁸ v_b is the beam ion speed, and τ is the beam-target interaction time assumed proportional to the confinement time of the plasma column.

Total beam energy is estimated¹⁷ as proportional to $L_p I_{\text{pinch}}^2$, a measure of the pinch inductance energy, L_p being the focus pinch inductance. Thus, the number of beam ions is $N_b \sim L_p I_{\text{pinch}}^2 / v_b^2$ and n_b is N_b divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$ where U is the disruption-caused diode voltage.¹⁷ Here, b is the cathode radius. We also assume reasonably that U is proportional to V_{max} , the maximum voltage induced by the current sheet collapsing radially toward the axis.

$$\text{Hence, we derive } Y_{b-t} = C_n I_{\text{pinch}}^2 z_p^2 [(\ln b/r_p)] \sigma / V_{\text{max}}^{1/2}, \quad (1)$$

where I_{pinch} is the current flowing through the pinch at start of the slow compression phase; r_p and z_p are the pinch dimensions at end of that phase. Here, C_n is a constant which, in practice, we will calibrate with an experimental point.

The D–D cross section is highly sensitive to the beam energy so it is necessary to use the appropriate range of beam energy to compute σ . The code computes V_{max} of the order of 20–50 kV. However, it is known¹⁷ from experiments that the ion energy responsible for the beam-target neutrons is in the range of 50–150 keV,¹⁷ and for smaller lower-voltage machines the relevant energy¹⁹ could be lower at 30–60 keV. Thus, to align with experimental observations the D–D cross section σ is reasonably obtained by using beam energy equal to three times V_{max} .

A plot of experimentally measured neutron yield Y_n vs I_{pinch} was made combining all available experimental data.^{2,4,11,13,17,19–22} This gave a fit of $Y_n = 9 \times 10^{10} I_{\text{pinch}}^{3.8}$ for I_{pinch} in the range 0.1–1 MA. From this plot, a calibration point was chosen at 0.5 MA, $Y_n = 7 \times 10^9$ neutrons. The model code²³ RADPFV5.13 was thus calibrated to compute Y_{b-t} which in our model is the same as Y_n .

From experience, it is known that the current trace of the focus is one of the best indicators of gross performance. The axial and radial phase dynamics and the crucial energy transfer into the focus pinch are among the important information that is quickly apparent from the current trace. Numerical experiments were carried out for machines for which reliable current traces and neutron yields are available. Figure 1 shows a comparison of the computed total current trace

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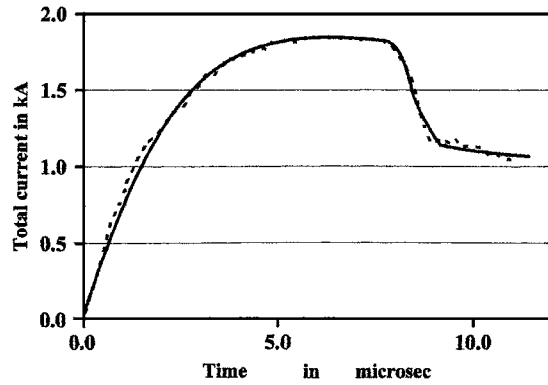


FIG. 1. PF1000 at 27 kV measured (dashed line) vs computed (smooth line) current traces.

(solid smooth line) with the experimental trace (dotted line) of the PF1000 at 27 kV (Ref. 17), 3.5 torr deuterium, with outer/inner radii $b=16$ cm, $a=11.55$ cm, and anode length $z_o=60$ cm. In the numerical experiments we fitted external (or static) inductance $L_o=33$ nH and stray resistance $r_o=6$ m Ω with model parameters mass factor, current factor, and radial mass factor as $f_m=0.14$, $f_c=0.7$, and $f_{mr}=0.35$. The computed current trace agrees very well with the experiment, a typical performance of this code.

Each numerical experiment is considered satisfactory when the computed current trace matches the experiment in current rise profile and peak current, in time position of the current dip, in slope, and absolute value of the dip (see Fig. 1). The results were obtained for the PF400, the UNU/ICTP PFF, the NX2, and PF1000 at 35 kV; for which current traces and neutron yields are available. We thus established these reliable points for our computed Y_n data. To make the results less sketchy, additional points were obtained for the PF1000 from 13.5 to 40 kV though these additional points are not supported by published results. More work will need to be done. However, even with the results obtained, it is clear that the model code is producing a scaling of $Y_n \sim I_{pinch}^{4.7}$; and $Y_n \sim I_{peak}^{3.9}$. These computed scaling laws are in reasonable agreement with those put up from time to time by experimental compilations,^{20,21} considering that in the experimental results, I_{pinch} is seldom properly measured, in many cases, only estimated from I_{peak} . Such estimates are dicey since the relationship between the peak total current I_{peak} (measured in the external circuit) and the pinch current I_{pinch} flowing in the tube is variable. Our code is consistent in that I_{pinch} is rigorously computed by fitting the total current trace. This gives confidence in the scaling ability of the code for Y_n as well.

An important question is how to improve the neutron yields of experiments. One obvious strategy is to increase I_{pinch} by reducing L_o . For example, the 30 μ F, 110 nH UNU/ICTPPFF (Refs. 2, 4, 12, and 19) had its L_o reduced to 20 nH evolving, as it were, into the NX2.^{7,16,22} I_{peak} more than doubled. More importantly, though less than doubled, I_{pinch} increased from 120 to 220 kA. Neutron yields increased three to five times, as did SXR yields.

What about a bank such as the PF1000? With C_o at 1332 μ F, its L_o of 30 nH (fitted by the code) is already low relative to its huge C_o . We have run the code using the machine and model parameters determined from Fig. 1, modified by information about values of I_{peak} at 35 kV. Operating the PF1000 at 35 kV and 3.5 torr, we varied the anode radius

a (with corresponding adjustment to b to maintain a constant $c=b/a$) to keep the peak axial speed at 10 cm/ μ s. The anode length z_o was also adjusted to maximize I_{pinch} .

L_o was decreased from 100 nH progressively to 5 nH. As expected, I_{peak} increased from 1.66 to 4.4 MA. As L_o was reduced from 100 to 35 nH, I_{pinch} also increased, from 0.96 to 1.05 MA. However, then unexpectedly on further reduction from 35 to 5 nH, I_{pinch} stopped increasing, instead decreasing slightly to 1.03 MA at 20 nH, to 1.0 MA at 10 nH, and to 0.97 MA at 5 nH. Y_n also had a maximum value of 3.2×10^{11} at 35 nH.

To explain this unexpected result, we examine the energy distribution in the system at the end of the axial phase (see Fig. 1) just before the current drops from peak value I_{peak} and then again near the bottom of the almost linear drop to I_{pinch} . The energy equation describing this current drop is written as follows:

$$0.5I_{peak}^2(L_o + L_a f_c^2) = 0.5I_{pinch}^2(L_o/f_c^2 + L_a + L_p) + \delta_{cap} + \delta_{plasma}, \quad (2)$$

where L_a is the inductance of the tube at full axial length z_o . δ_{plasma} is the energy imparted to the plasma as the current sheet moves to the pinch position and is the integral of $0.5(dL/dt)I^2$. We approximate this as $0.5L_p I_{pinch}^2$ (which is an underestimate) for this case. δ_{cap} is the energy flow into or out of the capacitor during this period of current drop. If the duration of the radial phase is short compared to the capacitor time constant, the capacitor is effectively decoupled and δ_{cap} may be put as zero. From this consideration we obtain

$$I_{pinch}^2 = I_{peak}^2(L_o + 0.5L_a)/(2L_o + L_a + 2L_p), \quad (3)$$

where we have taken $f_c=0.7$ and approximated f_c^2 as 0.5.

Taking the example of PF1000 at 35 kV we obtain for each L_o the corresponding L_a (~ 0.65 nH/cm of z_o) and L_p [~ 3.8 nH/cm of (Ref. 4) $z_p \sim a$]. For example, at $L_o=100$ nH, $L_a=52$ nH, and $L_p=29$ nH giving I_{pinch}/I_{peak} as 0.63. This ratio drops progressively as L_o decreases. For $L_o=5$ nH, $L_a=13$ nH, and $L_p=77$ nH giving the ratio as 0.25. The results show that as L_o is reduced from 100 nH, at first, the increase in I_{peak} more than compensates for the drop in I_{pinch}/I_{peak} and I_{pinch} increases from $L_o=100$ nH to $L_o=40$ nH. Below $L_o=40$ nH, the drop in I_{pinch}/I_{peak} catches up with the increase in I_{peak} leading to the numerically observed flat maximum of I_{pinch} . Y_n also has a flat maximum of 3.2×10^{11} at $L_o=40$ –30 nH.

The current limitation can now be seen as firstly a consequence of Eq. (3). Generally, as L_o is reduced, I_{peak} increases; a is necessarily increased leading (Ref. 4) to a longer pinch length z_p , hence a bigger L_p . Lowering L_o also results in a shorter rise time, hence a necessary decrease in z_o , reducing L_a . Thus, from Eq. (3), lowering L_o decreases the fraction I_{pinch}/I_{peak} . Secondly, this situation is compounded by another mechanism. As L_o is reduced, the L - C interaction time of the capacitor bank reduces while the duration of the current drop increases due to an increasing a . This means that as L_o is reduced, the capacitor bank is more and more coupled to the inductive energy transfer processes with the accompanying induced large voltages that arise from the radial compression. Looking again at the derivation of Eq. (3) from Eq. (2) a nonzero δ_{cap} , in this case, of positive value, will act to decrease I_{pinch} further. The lower L_o the more pronounced is this effect.

Summarizing this discussion, the pinch current limitation is not a simple effect, but is a combination of the two complex effects described above, namely, the interplay of the various inductances involved in the plasma focus processes abetted by the increasing coupling of C_o to the inductive energetic processes, as L_o is reduced.

We carried out several sets of experiments on the PF1000, each set with a different damping factor. In every case, an optimum inductance was found around 30–60 nH with I_{pinch} decreasing as L_o was reduced below the optimum value. We also carried out another set of experiments with a planned focus with C_o of 300 μF . For that device, optimum L_o was found to be 20 nH. More sets of experiments need to be run to gain further experience and insight to understand better the complex interactions of the several parameters that conspire to determine the optimum L_o . The results of these ongoing studies will be published in more detail in due course.

In the meantime, enough information has been obtained from the numerical experiments to enable a statement that for PF1000, reducing L_o from its present 20–30 nH will increase neither the observed I_{pinch} , nor the neutron yield.

The prevailing thinking seems to be that the lower L_o is made, the higher performance a plasma focus would have in terms of driving current and Y_n . This paper shows that, on the contrary, given a fixed C_o powering a plasma focus, there exists an optimum L_o for maximum I_{pinch} . Reducing L_o further will increase neither I_{pinch} nor Y_n . Plasma focus research now has to meet the challenges posed by this “pinch current limitation” effect.

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Erratum

The published paper contains 2 errors on page 1 which are corrected by this note. The relevant paragraph is reproduced here in parenthesis with the corrections highlighted in bold red:

"Total beam energy is estimated¹⁷ as proportional to $L_p I_{\text{pinch}}^2$, a measure of the pinch inductance energy, L_p being the focus pinch inductance. Thus the number of beam ions is $N_b \sim L_p I_{\text{pinch}}^2 / v_b^2$ and n_b is **N_b** divided by the focus pinch volume. Note that $L_p \sim \ln(b/r_p) z_p$, that⁴ $\tau \sim r_p \sim z_p$, and that $v_b \sim U^{1/2}$ where U is the disruption-caused diode voltage¹⁷. Here 'b' is the cathode radius. We also assume reasonably that U is proportional to V_{max} , the maximum voltage induced by the current sheet collapsing radially towards the axis.

$$\text{Hence we derive: } Y_{b-t} = C_n \text{ **n}_i \text{ } I_{\text{pinch}}^2 z_p^2 ((\ln b/r_p)) \sigma / V_{\text{max}}^{1/2} \quad (1)**$$

where I_{pinch} is the current flowing through the pinch at start of the slow compression phase; r_p and z_p are the pinch dimensions at end of that phase. Here C_n is a constant which in practice we will calibrate with an experimental point."

There is another error on page 2, Fig 1. The vertical axis should be labeled 'Total Current in MA'.

Computing plasma focus pinch current from total current measurement

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The total current I_{total} waveform in a plasma focus discharge is the most commonly measured quantity, contrasting with the difficult measurement of I_{pinch} . However, yield laws should be scaled to focus pinch current I_{pinch} rather than the peak I_{total} . This paper describes how I_{pinch} may be computed from the I_{total} trace by fitting a computed current trace to the measured current trace using the Lee model. The method is applied to an experiment in which both the I_{total} trace and the plasma sheath current trace were measured. The result shows good agreement between the values of computed and measured I_{pinch} . © 2008 American Institute of Physics. [DOI: 10.1063/1.2899632]

The total current I_{total} waveform in a plasma focus discharge is easily measured using a Rogowski coil. The peak value I_{peak} of this trace is commonly taken as a measure of the drive efficacy and is often used to scale the yield performance of the plasma focus.^{1,2} This is despite the fact that yields^{3–5} should more consistently be scaled to focus pinch current I_{pinch} , since it is I_{pinch} which directly powers the emission processes. The reason many researchers use I_{peak} instead of I_{pinch} for scaling is simply that while I_{peak} is easily measured, I_{pinch} , which is the value of the plasma sheath current I_p at time of pinch, is very difficult to measure even in large devices where it is possible to place magnetic probes near the pinch.^{3–5} This measurement is also inaccurate and perturbs the pinch. In a small device, there is no space for such a measurement. A simpler method was tried to compute the I_p waveform using measured waveforms of I_{total} and tube voltage.^{6,7} This was achieved only up to the start of the radial phase thereby missing the crucial I_{pinch} . To date, I_{pinch} is still one of the least measured and often misunderstood quantities. In this connection, an attempt was made⁸ to compute the time of pinch. However, in that work, I_{pinch} was assumed to be I_{total} at pinch time.

The relationship between I_{pinch} and I_{peak} is not simple and has only been recently elaborated.⁹ It primarily depends on the value of the static inductance L_0 compared to the dynamic inductances of the plasma focus. As L_0 is reduced, the ratio $I_{\text{pinch}}/I_{\text{peak}}$ drops. Thus, yield laws scaled to I_{peak} will not consistently apply when comparing two devices with all parameters equal but differing significantly in L_0 . Better consistency is achieved when yield laws are scaled to I_{pinch} .

In this paper, we propose a numerical method to consistently deduce I_{pinch} from any measured trace of I_{total} . This method will improve the formulation and interpretation of focus scaling laws. Specifically, we define I_{pinch} as the value of I_p at the start of the quiescent (or pinch) phase of the plasma focus radial dynamics. We now discuss the distinction between I_{total} and the plasma sheath current I_p .

A measured trace of I_{total} is commonly obtained with a Rogowski coil wrapped around the plasma focus flange¹⁰ through which is fed I_{total} discharged from the capacitor bank between the coaxial electrodes across the back wall. A part of

I_{total} , being the plasma sheath current I_p , lifts off the back-wall insulator and drives a shock wave axially down the coaxial space. At the end of the anode, the plasma sheath turns from axial into radial motion. The previously axially moving I_p becomes a radial inward moving cylindrical sheath, driving a radially collapsing cylindrical shock front. When this shock front arrives on axis, because the plasma is collisional, a reflected shock (RS) moves radially outwards¹¹ until it meets the incoming driving current sheath. The increased pressure of the RS region then rapidly slows down the sheath. This is the start of the pinch phase. All the dynamics dominating the axial and radial phases is determined by I_p . A proportion of the current, the difference between I_{total} and I_p , does not take part in the dynamics. This leakage current stays at the back wall,^{4–7,12} but parts of it may be diffusely distributed.

We define for the axial phase f_c as I_p/I_{total} and distinguish it from f_{cr} for the radial phase. Likewise, it had been shown that only a fraction of the mass^{6,12} encountered by the axial sheath is swept up. This fraction we call f_m , distinguishing the radial phase fraction as f_{mr} . The rest of the mass either leaks through the sheath or is swept outwards due to the canting of the sheath.

The exact time profile of the I_{total} trace is governed by the bank, tube, the operational parameters, and by the mass and current fractions and variation of these fractions through the axial and radial phases. Although we may expect these fractions to vary, for simplicity, we average these model parameters as f_m , f_c and f_{mr} and f_{cr} .

The Lee model couples the electrical circuit with plasma focus dynamics, thermodynamics, and radiations enabling realistic simulation of all gross focus properties. The basic model was described in 1984 (Ref. 13) and used to assist projects.^{6,7,10,11,14–16} An improved five-phase code crucially incorporating small disturbance speed,¹⁷ and radiation coupling with dynamics, assisted further projects,^{8,18–23} and was published in the internet in 2000 (Ref. 24) and 2005.²⁵ Plasma self-absorption was included²⁴ in 2007. It has been used in machines including UNU/ICTP PFF,^{10,11,15,16,21} NX2,^{18–20} and NX1,¹⁸ and has been adapted to the Filippov-type DENA.^{8,22,23} Neutron yield Y_n using a beam-target mechanism,¹ is included in the present version RADPFV5.13, (Ref. 26) resulting in realistic Y_n scaling²⁷ with I_{pinch} . Since

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the detailed theory of the model and the code are given in the websites,²⁴⁻²⁶ we proceed to the proposed method to compute I_{pinch} .

The method requires a measured I_{total} waveform from a discharge in which the bank parameters, the tube geometry, and operating parameters are known. The Lee model code²⁶ is used to simulate this discharge using the model parameters for fitting. The model parameters are varied until the simulated I_{total} trace agrees with the measured I_{total} trace. The start of the quiescent or pinch phase is pinpointed from the computation and the computed value of I_p at this time is obtained as I_{pinch} .

For the actual fitting process, the bank parameters L_0 , C_0 (capacitance), and r_0 (resistance) are put into the active sheet of the EXCEL code. If r_0 is not available, a trial value of $0.1(L_0/C_0)^{1/2}$ is used. Next, the tube parameters b (cathode radius), a (anode radius), and z_0 (anode length) and the operational parameters V_0 (voltage) and P_0 (pressure) are entered. The fill gas is indicated by its atomic weight and number in the cells provided. Trials values of f_m , f_c , f_{mr} , and f_{cr} are then entered, e.g., 0.08, 0.7, 0.1, and 0.7, respectively. The code is then run. The computed I_{total} trace which is one of the graphical outputs is transferred onto a comparison active sheet and plotted onto a graph together with the pre-loaded measured I_{total} trace. Detailed comparison, feature by feature, of the traces is made.

The first step is fitting the axial phase. This involves variation of f_m and f_c while observing the changes that appear on the resulting computed I_{total} trace in respect to the rise time, rising shape, and I_{peak} and how these features compare with the corresponding features of the measured I_{total} trace. During this fitting an increase in f_c increases axial speed which increases dynamic resistance, thus, lowering current magnitude on the rising slope. The greater rate of increase of tube inductance flattens out the rising slope. A decrease in f_m has almost the same effect. However, a change in f_c has an additional subtle effect of changing the relative effect of the tube inductance. This means that increasing the speed by a certain amount by increasing f_c , then reducing it by exactly the same amount by a corresponding increase in f_m will not bring the I_{total} shape and magnitude back to the shape and value before either change is made. Thus, one has to get each of f_m and f_c separately correct to get both the current shape and magnitude correct in the rising current profile.

The value of r_0 may need to be adjusted. An increase of r_0 lowers the current trace at all points proportionately. Adjustment to nominally given values of L_0 , sometimes even C_0 , may need to be made before a good fit is achieved. When all values are properly adjusted and when f_m and f_c are correctly fitted, the measured rising profile of the computed I_{total} , usually up to the peak value I_{peak} , is found to fit the measured rising profile well in both shape and magnitude.

Two other points need to be noted.^{6,7} The measured I_{total} profile usually has a starting portion which seems to rise more slowly than the computed trace. This is due to the switching process during which, until fully switched, the spark gap presents additional resistance. It could also be compounded by the lift-off delay.²¹ Practically, this effect is compensated by shifting the whole computed trace forward in time, usually by a small amount around 50 ns. A related note is that z_0 may need to be reduced to account for the shape of the back-wall insulator.

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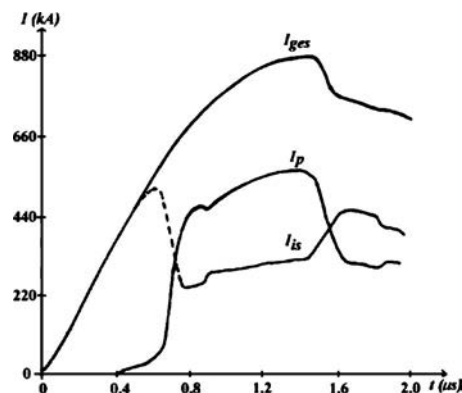


FIG. 1. DPF78 measured I_{total} (labeled as I_{ges}) and measured I_p waveforms. The third trace I_{is} is the difference of I_{total} and I_p .

The next step is fitting the radial phases. We need to understand the transition from the axial to the radial phase. For a plasma focus to work well, it is usually operated with a speed such that its axial run-down time is about equal to the rise time of the circuit with the device short circuited across its back wall. With the focus tube connected, the current rise time will be larger. At the same time, the current trace is flattened out. In most cases this increased rise time will be cut short by the start of the radial phase. As this phase starts, the current trace starts to roll over, at first imperceptibly, then clearly dipping and then sharply dips as the focus dynamics enters the severe pinch phase which absorbs a significant portion of the energy from the driving magnetic field. Thus, the second step in the fitting consists of adjusting f_{mr} and f_{cr} so that the computed current roll over and the dip agree in shape, slope, and extent of dip with the measured waveform.

We now describe how we tested the validity of this method. In an experiment in Stuttgart using the DPF78,^{4,5} a Rogowski coil measured the I_{total} trace, and magnetic probes measured the I_p waveform. The bank parameters were $C_0 = 15.6 \mu\text{F}$ (nominal) and $L_0 = 45 \text{ nH}$ (nominal), tube parameters were $b = 50 \text{ mm}$, $a = 25 \text{ mm}$, and $z_0 = 150 \text{ mm}$, and operating parameters were $V_0 = 60 \text{ kV}$, and $P_0 = 7.6 \text{ Torr}$ deuterium. Figure 1 shows these measured I_{total} (labeled as I_{ges} in Fig. 1) and I_p waveforms. The third trace is the difference of I_{total} and I_p .

These parameters were put into the code. The best fit for the computed I_{total} with the measured I_{total} waveform was obtained with the following: bank parameters were $C_0 = 17.2 \mu\text{F}$, $L_0 = 55 \text{ nH}$, and $r_0 = 3.5 \text{ m}\Omega$; tube parameters were $b = 50 \text{ mm}$, $a = 25 \text{ mm}$, and $z_0 = 137 \text{ mm}$; and operating parameters were $V_0 = 60 \text{ kV}$ and $P_0 = 7.6 \text{ Torr}$ deuterium. Model parameters of $f_m = 0.06$, $f_c = 0.57$, $f_{mr} = 0.08$, and $f_{cr} = 0.51$ were fitted.

With these parameters, the computed I_{total} trace compared well with the measured I_{total} trace, as shown in Fig. 2. The computed dynamics, currents, and other properties of this plasma focus discharge were deemed to be correctly simulated.

From the computation results the start of the pinch phase was obtained as $1.551 \mu\text{s}$. At this time I_{pinch} was computed as $0.51 \times 778 = 396.8 \text{ kA}$. The value of I_{pinch} from the measured I_p trace was not immediately obvious since there was no striking feature that marked this moment on the measured I_p trace. We used the following procedure to obtain it, at the

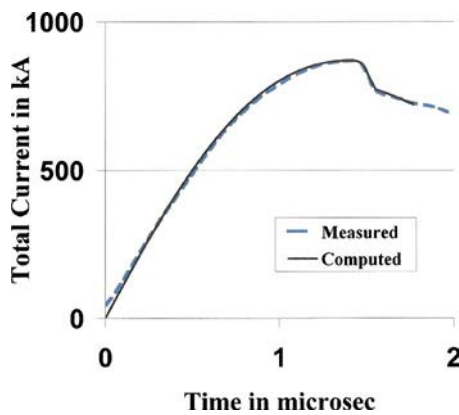


FIG. 2. (Color online) Comparison of computed (solid line) and measured (dashed line) I_{total} waveforms.

same time to get further insight into f_c and f_{cr} .

The ratio I_p/I_{total} (digitized from Fig. 1) was plotted as a function of time and shown in Fig. 3. At time = 1.551 μs , the ratio was found to be 0.49, and I_{total} was measured to be 778 kA. Hence, $I_{\text{pinch}} = 381.2$ kA was measured in the Stuttgart DPF78 experiment. The computed I_{pinch} was 4% larger than the measured I_{pinch} . This difference was to be expected considering that the modeled f_{cr} was an average value of 0.51; while the laboratory measurement showed (Fig. 3) that in the radial phase I_p/I_{total} varied from 0.63 to 0.4, and at the start of the pinch phase this ratio was 0.49 and rapidly dropping. Thus, one would expect the computed value of I_{pinch} to be somewhat higher than the measured, which turned out to be the case. Nevertheless, the difference of 4% is better than the typical error of 20% estimated for I_{pinch} measurements using magnetic probes.³

The conclusion is that the numerical method is a good alternative, being more accurate and convenient and only needing a commonly measured I_{total} waveform.

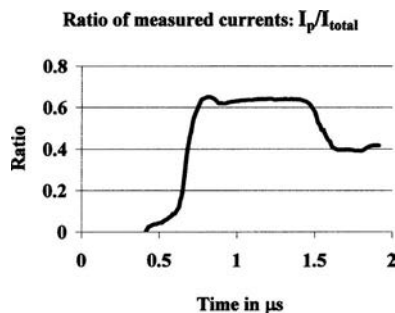


FIG. 3. Ratio of measured I_p to I_{total} as a function of time.

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Dimensions and Lifetime of the Plasma Focus Pinch

Sing Lee and Adrian Serban

Abstract—In spite of intensive studies of the plasma focus, scaling laws, with the exception of those for neutron yield, are not well known. This paper points out that Mather-type plasma focus devices operated in the neutron-optimized regime have remarkably little variation in a drive parameter, $(I_p/a)/\rho_o^{1/2}$, the peak drive current per unit anode radius divided by the square root of the fill density; this quantity having the value of 89 kA/cm per torr deuterium equivalent of fill gas with a standard deviation of less than 10%. This parameter controls the speed of the plasma in both the axial and radial phases; and its constancy over a wide range of plasma focus devices really indicates that these devices all operate at the same axial and radial speeds, and hence by inference they all have the same temperatures in the axial and radial phases. Using a simple dynamical model the linear dimensions and time scales of the gross plasma focus pinch are shown to be related to the anode radius of the plasma focus device. The results of experiments performed with a 3-kJ device using different anodes are in good agreement with the theoretical predictions.

I. INTRODUCTION

THE PLASMA focus is an important device for the generation of intense radiation including neutrons, X-rays, and particle beams. The physics underlying the mechanisms for the generation of these radiations in the plasma focus is not known, although there has been intensive investigations for the past three decades. Experimental and theoretical work on the focus has reached quite high levels. For example, detailed simulation work on the plasma focus had been carried out since 1971 [1] and a large range of devices has been constructed from 100 J small focus to 1 MJ large focus. Advanced experiments have been carried out on the dynamics, radiation, instabilities, and nonlinear phenomena. Yet despite all these intensive studies, very little regarding scaling appears to be known or at least documented, with the exception of the scaling law for neutron yield. In order to acquire a better overall understanding of the plasma focus it may be useful to start by looking at the scaling of simple quantities. For example, it needs to be pointed out that neutron-optimized Mather-type plasma focus, from small to large devices, all have the same drive parameter $(I_p/a)/\rho_o^{1/2}$, where I_p is the peak current driving the plasma sheath, a the anode radius, and ρ_o is the ambient gas density. Strictly speaking, the drive parameter should include a mass sweeping factor in the density as it is known that the mass swept up by the current sheath is not 100% of the mass encountered so that the “effective” gas density is actually less. Also, a current factor should be included in the drive parameter. Not including the mass and current factors in the

TABLE I
ACTUAL-SHOT DATA OF SOME NEUTRON-OPTIMIZED MATHER-TYPE PLASMA FOCUS DEVICES AND THEIR CALCULATED DRIVE PARAMETER. THE MEAN VALUE AND THE CORRESPONDING STANDARD DEVIATION OF THE DRIVE PARAMETER ARE 89.0 AND 7.7 kA/CM/(TORR D_2)^{1/2}, RESPECTIVELY

Device/Author [Reference]	V (kV)	W (kJ)	p (torr D_2)	I_p (kA)	a (cm)	$I_p/a/p^{1/2}$ (kA/cm/(torr D_2) ^{1/2})
DPF-78 [16]	60	28	5	550	2.5	98.4
SPEED 2 [17]	183	70	4	1000	6	83.3
PF-360 [18]	21	60	3	750	5	86.6
Mather [19]	17.3	13.5	6.5	537	2.5	84.3
UMPF [20]	14	6	6.5	300	1.25	94.1
UNU/ICTP-PFF [10,15]	14	2.9	3.5	172	0.95	96.8
Bernstein [21]		27	8	820	3.9	74.3
POSEIDON [22]	60	280	3.8	1200	6.55	94.0

drive parameter is equivalent to assuming that these factors remain constant over the range of machines. Data of the drive parameter are compiled in Table I. In this compilation care is taken that the data represent actual neutron-optimized shots as published. Actual-shot data need to be distinguished from machine data which may give maximum current capability of a machine or design current which when combined with the pressure of an actual shot may give a misleading drive parameter since the actual shot is usually carried out not at the maximum or designed current. Thus much published data of machines could not be used in this compilation because of uncertainty over actual shot data. In Table I the value of the peak current I_p flowing into the focus tube is used as this value is more usually quoted than the plasma current and thus using the peak current more actual shot data points are available.

Table I shows that the drive parameter for a range of neutron-optimized plasma focus devices is 89 ± 8 kA/cm per (torr)^{1/2} of deuterium. The standard deviation of less than 10% is quite remarkable since the machines in the compilation range from 3 to 280 kJ and include early machines of Mather and Bernstein, advanced machines such as SPEED 2, and small training machines.

Any reasonable modeling will show that the drive parameter determines the speed in both the axial and radial phases [2], [3]. This will be shown in the following section. Having the same drive parameter indicates that the range of machines surveyed all operate with the same axial and radial speeds. This is not contradictory to the often reported speeds of just under 10 cm/ μ s for the axial phase just before the radial phase and a radial speed of about 25 cm/ μ s when the imploding shock is nearing the axis. The on-axis radial shock speed is seldom given as it is difficult to determine experimentally. Since the energetics of the plasma focus is determined by the shock speed up to the time the radial shock goes on axis,

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constancy of the drive parameter gives a strong indication that the temperature is also the same over the range of plasma focus surveyed. The density also varies remarkably little, a range of two, considering the large range of energies encompassed in the survey. Thus the plasma energy density of the machines has also relatively little variation. The mechanisms for radiation of energy and particles depend ultimately on the energy density and/or the temperature. The small variations observed in these quantities over the large range of machines while on the one hand is an advantage for comparing experiments may perhaps on the other hand act as a limitation. For example, in terms of fusion neutrons, the thermonuclear fusion cross section scales as temperature $T^{4.5}$ in the region of 1 keV which is the temperature regime of the plasma focus. Would it not be an advantage to increase the drive parameter while still maintaining the strong focusing characteristics since the shocked plasma temperature scales as the square of the drive parameter [2]? By increasing the value of the drive parameter, the shocked plasma temperature is increased, and hence the thermonuclear yield. The scaling in the limit can be $Y \sim$ (drive parameter) to the power of nine.

The above discussion centers around the drive parameter and its impact on plasma focus energetics. The identification of the constancy of the drive parameter as a possible limitation on focus performance may be useful in considerations of scaling. Other factors such as geometrical and temporal characteristics may also be useful for scaling of radiation performance; for example, the size of the focused plasma and the lifetime. Reviews in the literature [4]–[8] give a range of plasma dimensions of 1–15 mm in the diameter and 10–70 mm in the length and the pinch lifetime (defined as the time from the first compression to a minimum radius to the time of violent breakup of the plasma column) of tens of nanoseconds to 100–200 ns. However, no attempt has been made to discuss how these dimensions and times scale with, for example, stored energy or current. Yet by considering any simple model of the plasma focus dynamics the dimensions and time scales of the gross plasma focus pinch are easily related to the parameters of the plasma focus device.

II. THEORY

Using a snowplow model for the axial phase and a slug model for the radial phase (see Fig. 1), characteristic axial and radial transit times for the plasma shock front are, respectively, [9]–[11]

$$t_a = \left[\frac{4\pi^2(c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{z_o \rho_o^{1/2}}{\left(\frac{I_o}{a} \right)} \quad (1)$$

and

$$t_r = \left[\frac{16\pi^2}{\mu(\gamma + 1)} \right]^{1/2} \frac{a \rho_o^{1/2}}{\left(\frac{I_o}{a} \right)} \quad (2)$$

where $c = b/a$, b is outer radius of coaxial electrodes, a is inner radius, μ is permeability, z_o is length of inner electrode, I_o is characteristic current, ρ_o is ambient density, and γ is the specific heat ratio.

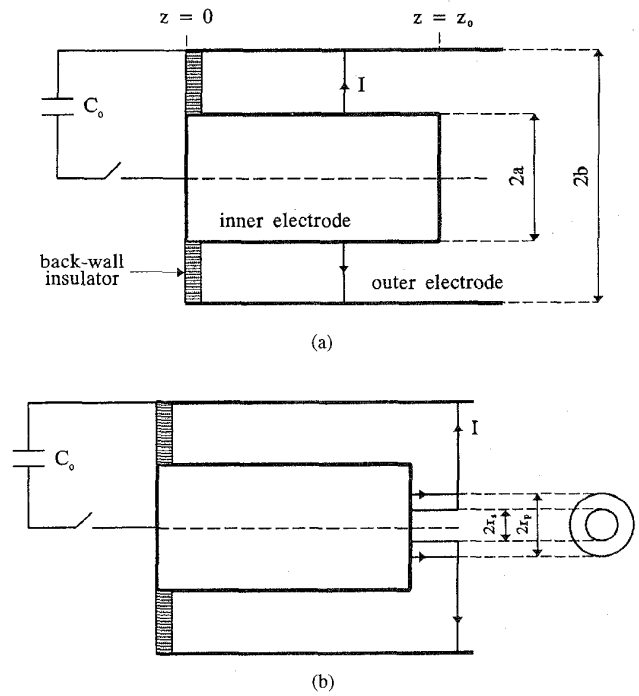


Fig. 1. Schematic of the dynamics of the plasma focus for the one-dimensional computational model: (a) axial phase—snowplow model, (b) radial phase—slug model.

From these we may write the characteristic speeds for the axial and radial shock, respectively, as

$$c_a = \frac{z_o}{t_a} = \frac{\left(\frac{I_o}{a} \right)}{\rho_o^{1/2}} \left[\frac{\mu \ln c}{4\pi^2(c^2 - 1)} \right]^{1/2} = \frac{\left(\frac{I_o}{a} \right)}{\rho_o^{1/2}} g_a \quad (3)$$

and

$$c_r = \frac{a}{t_r} = \frac{\left(\frac{I_o}{a} \right)}{\rho_o^{1/2}} \left[\frac{\mu(\gamma + 1)}{16\pi^2} \right]^{1/2} = \frac{\left(\frac{I_o}{a} \right)}{\rho_o^{1/2}} g_r \quad (4)$$

where g_a and g_r are the effective geometrical constants governing the axial and radial phases, respectively; g_r is a constant and g_a varies by less than 10% from a mean value of 0.52 over the range of $c = 1.5$ to 2 for the machines of Table I. Hence the characteristic speeds c_a and c_r depend primarily on the characteristic drive parameter $(I_o/a)/\rho_o^{1/2}$; and the operational speeds in the axial and radial phases depend only on the operational drive parameter $(I_p/a)/\rho_o^{1/2}$, where I_p is the peak current entering the focus tube. A similar parameter has been discussed earlier [12].

A radial collapse model which has proven useful for comparison with experimental work is shown in Fig. 1(b). Essentially, a zero length pinch starts at $r = a$ and elongates as it pinches inwards. The collapsing magnetic piston at position r_p pushes ahead of it a radially converging structure at position r_s . Using this model and having shown that the characteristic speed of radial transit is the same over the range of devices, it follows that the compression time $t = t_{\text{comp}}$ (see Fig. 2) depends on the radius a of the anode. As the shock converges on the axis we may define the position of the piston at this time as r_c .

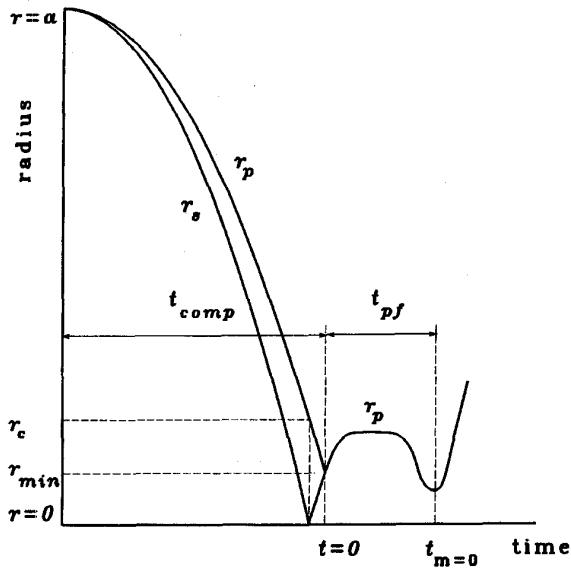


Fig. 2. Radial trajectories of shock (r_s) and piston (r_p) as a function of time.

After the shock hits the axis a reflected shock propagates outward. In the meantime the piston (see Fig. 2) continues inwards until it is hit by the reflected shock at which point the radially inward motion of the piston is reversed at position $r_p = r_{\min}$. Experimentally it is observed that the plasma column usually pinches in a second time sometimes to an even smaller radius before finally exploding outward. This ends the pinch phase and defines the quantity t_{pf} as shown in Fig. 2.

A numerical computation of this model [9]–[11], [13] shows that both r_c and $r_{\min}$ are proportional to the anode radius a .

The ending of the pinch phase is observed to be caused by instabilities. Thus we may relate the period t_{pf} to the transit time of small disturbances across the compressed plasma column. The speed of these small disturbances is practically a constant over the range of devices because of the near constancy of plasma temperature, density, and magnetic flux. Thus in such a scenario t_{pf} also depends on the radius a .

We have carried out a numerical computation with the endpoint fixed by an energy-balance method [14] and found the following for the deuterium focus: $r_{\min} \approx 0.12a$; $z_{\max} \approx 0.8a$; $t_{\text{comp}} \approx 4.5a$; $t_{pf} \approx 2a$, with a in millimeters, t_{comp} and t_{pf} in nanoseconds. Here, $z_{\max}$ is the length of the focus column measured from the face of the anode.

Similar results are applicable to the gross pinch of the plasma focus in other gases such as Argon, Neon, and Xenon. Generally for these other gases $r_{\min} \leq 0.1a$. The difference in the computation involves the value of γ . Scaling these parameters of the focus to the anode radius a is extremely useful for understanding and for applications. We have carried out experiments to confirm these theoretical results.

III. EXPERIMENTS

Experiments were carried out using a 3 kJ plasma focus device [15] operated at 14 kV in deuterium at the neutron optimized pressure of 4.5 mbar. The inner electrode was the

TABLE II
MEAN VALUES OF THE PINCH LIFETIME (t_{pf}) AND OF THE COMPRESSION TIME (t_{comp}) AS FUNCTIONS OF THE ANODE RADIUS (a)

a (mm)	t_{comp} (ns)	t_{pf} (ns)
9.5	45.1	19.5
8.75	41.1	17.6
8.5	39.2	17.1
7.5	31.4	15.2
7	29.1	14.3

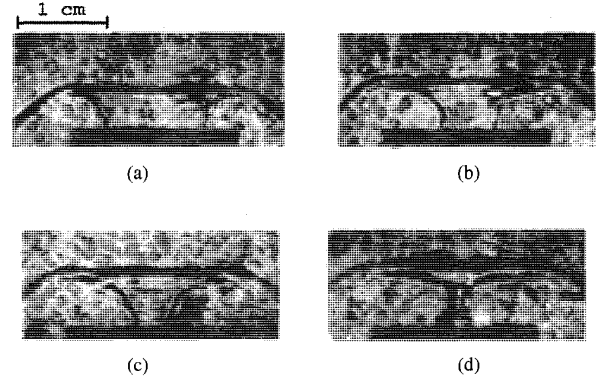


Fig. 3. Examples of shadowgraphs of plasma focus during the radial phase for an anode of 9.5 mm radius.

anode. All anodes tested were made of copper and hollow at the open end.

The diagnostics employed included [15]: shadowgraphy using a TEA nitrogen laser, current (I) and current derivative (dI/dt) using current transformer and Rogowski coil, respectively, voltage (V) using a resistive voltage probe, soft X-ray emission rate (dY_{SX}/dt) using filtered PIN diodes, and hard X-ray and neutron time-resolved measurements (dY_n/dt and dY_{HX}/dt , respectively) using two fast response plastic scintillator-photomultiplier systems in a time-of-flight arrangement. The field of view of the shadowgraph technique is above the open end of the anode. The X-ray and neutron emission rates were recorded in a radial plane with respect to the axis of the focus tube.

The dimensions of the pinch were determined from the time sequences of shadowgraphs. The timing of the shadowgraphs as well as the timing of the important periods of the evolution of the plasma focus were performed with better than 1 ns accuracy. The time reference was taken at the instant when the plasma column reached the minimum radius ($t = 0$, $r = r_{\min}$). The instant $t = 0$ was identified from the current derivative signal (the first sharp negative spike) and from the soft X-ray signal (the first spike). The pinch lifetime (t_{pf}) was defined as the duration from $t = 0$ to the instant corresponding to the development of the $m = 0$ instability ($t_m = 0$). The latter is associated with the beginning of the hard X-ray pulse.

Experiments were performed with several anodes of radii of 9.5, 8.75, 8.5, 7.5, and 7 mm, respectively [15]. For each electrode configuration a large number of discharges (more than 50) was analyzed. Our statistical analysis was performed

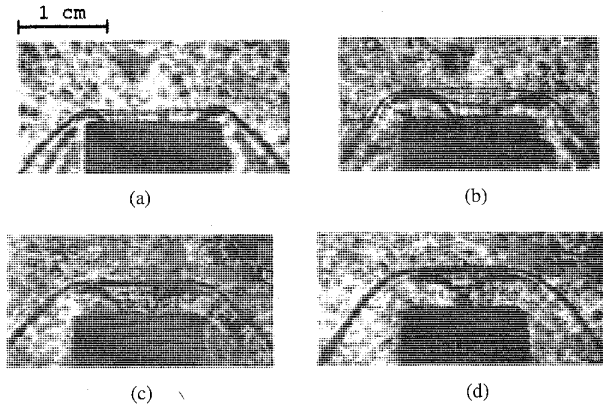


Fig. 4. Examples of shadowgraphs of plasma focus during the radial phase for an anode of 7.5 mm radius.

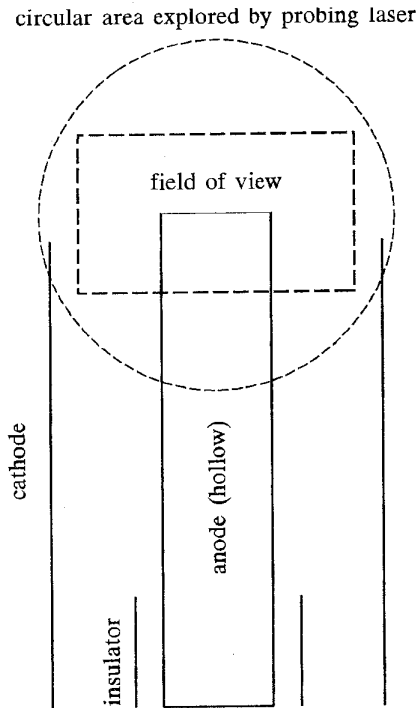


Fig. 5. Field of view of shadowgraphs at the open end of the electrode system.

on series of shots without excluding the discharges with very low neutron yield. Detailed discussions and correlations of the discharge parameters (e.g., peak current, peak axial velocity, axial transit time, radial compression time, and pinch lifetime) and neutron and X-ray emission were presented elsewhere [15]. We would like to mention only that the standard deviation of the peak discharge current was less than 2%, the standard deviation of the peak axial speed was less than 7% while the standard deviation of the neutron yield reached 60% for some anode configurations. The mean values of the pinch lifetime and of the compression time are illustrated in Table II. They show that the pinch lifetime (t_{pf}) as well as the compression time (t_{comp}) are scaled to the anode radius in agreement with the theoretical predictions.

The evolution of the plasma during the radial compression phase and pinch phase was analyzed from shadowgraphs. Some images of the plasma during the final stages of the compression phase and pinch phase are illustrated in Figs. 3 and 4 for two anodes of radii 9.5 mm and 7.5 mm, respectively. The field of view of the shadowgraphs is illustrated in Fig. 5. From the shadowgraphs, r_p is estimated with 10% accuracy.

Measurements of the axial elongation and radius of the plasma column during the radial collapse phase are in good agreement with the theoretical model. At the end of the radial implosion the following experimental values were determined:

when $a = 9.5$ mm it was found that:

$$\begin{cases} r_{\min} \approx 1.2 \text{ mm} \approx 0.13a \\ z_{\max} \approx 7.5 \text{ mm} \approx 0.8a \end{cases}$$

and

when $a = 7.5$ mm it was found that:

$$\begin{cases} r_{\min} \approx 1 \text{ mm} \approx 0.13a \\ z_{\max} \approx 6 \text{ mm} \approx 0.8a. \end{cases}$$

Here, $z_{\max}$ is the length of the focus column measured from the face of the anode. These results confirm our theoretical predictions.

IV. CONCLUSION

Numerical computations on a simple model yield the following dimensions and times for the gross pinch phase of deuterium plasma focus:

$$\begin{aligned} r_{\min} &\approx 0.12a & z_{\max} &\approx 0.8a \\ t_{comp} &\approx 4.5a & t_{pf} &\approx 2a \quad (t \text{ in ns, } a \text{ in mm}). \end{aligned}$$

Experimental results are presented which verify the predictions of the model.

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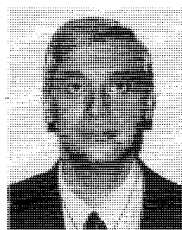
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Scaling of the Plasma Focus – Viewpoint from Dynamics

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Abstract

The gross dynamics of the plasma focus is discussed in terms of phases. The dynamics of the axial and radial phases is computed using respectively a snowplow and an elongating slug model. A reflected shock phase follows, giving the maximum compression configuration of the plasma focus pinch. An expanded column phase is used to complete the post-focus electric current computation. Parameters of the gross focus pinch obtained from the computation, supplemented by experiments are summarised as follows:

		Deuterium	Neon (for SXR)
minimum radius	$r_{\min}$	0.13a	0.04a
maximum length	z	0.7a	0.8a
radial shock transit	t_{comp}	$5 \times 10^{-6}a$	$4 \times 10^{-6}a$
pinch lifetime	t_p	$2 \times 10^{-6}a$	$1 \times 10^{-6}a$

where, for the times in sec, the value of anode radius, a, is in m. For the neon calculations radiative terms are included. The scaling suggests a speed enhancement effect on neutron yield, enhancing from the conventional I^4 to a superior I^8 scaling law.

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1. Introduction

This paper starts with a simple axial-radial model of the Mathers type plasma focus¹ to show that the elongating plasma focus pinch achieves a gross compressed configuration in which its minimum radius and its maximum length are dependent on its anode radius 'a'. Moreover for a given gas there appears to be experimental mechanisms that require the drive magnetic energy density to be constant over a range of the devices from small (kJ) to big (hundreds of kJ). From this it follows that the lifetime of the plasma focus pinch is also dependent on anode radius 'a'. Thus the bigger the anode radius, the bigger are the radius, length and lifetime of the compressed pinch. This dependence on 'a' is alone sufficient to derive the general rule that radiation yield is proportional to I^4 . It also suggests a speed enhancement effect on neutron yield, which however does not appear to apply to soft x-ray (SXR) yield for microelectronics lithography.

2. Model

We use a snowplow model for the axial phase and a slug model for the radial pinch phase²⁻⁴. The end-point of compressions is fixed when the radially out-going reflected shock hits the in-going compressing piston.

2a. Axial phase:

We consider the rate of change of momentum of the current sheath sweeping up mass and equate this to the driving electromagnetic force (see Fig 1a).

$$\frac{d}{dt} \left[\pi(b^2 - a^2) \rho f_m \frac{dz}{dt} \right] = \frac{\mu f_c^2 I^2}{4\pi} \ln c \quad (1)$$

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where b is the outer electrode radius, a the inner electrode radius, z the position of the 'snowplow' current sheath, ρ the ambient density, I the driving current, $c=b/a$, and μ the permeability of free space. The two parameters f_m and f_s are the mass swept-up factor and current factor used to account for the experimentally observed mass loss and current loss respectively⁵.

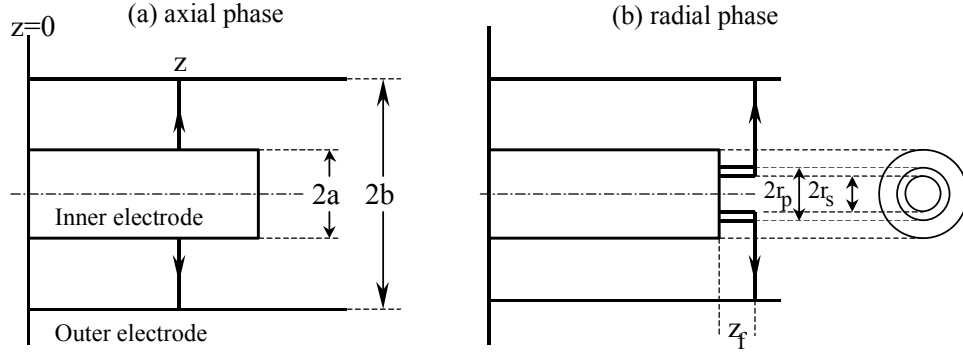


Fig 1: Schematic of (a) axial phase and (b) radial phase

The circuit equation which captures the effect of the changing inductance L_p , due to current sheath motion, on the current I is:

$$\frac{d}{dt} [(L_o + f_c L_p) I] = V_o - \frac{1}{C_o} \int_o^t I dt - r_o I \quad (2)$$

where L_o , C_o and r_o are the fixed circuit inductance, capacitance and stray resistance, and L_p is:

$$L_p = \frac{\mu}{2\pi} z \ln c \quad (3)$$

Equations (1) & (2) may be integrated step-by-step for the instantaneous values of I and z . Appropriate values used for f_m and f_c are found to be $f_m = 0.1$, $f_c = 0.7$.

2b. Radial Phase

In the radial phase (see Fig 1b), because of the compression configuration, less mass loss is observed so a radial mass swept-up factor f_{mr} is introduced. The current factor remains as f_c . We derive the shock speed from the driving magnetic pressure and use the same speed (except for a thermodynamic factor) for the elongation since the same magnetic pressure drives both the radial compression and axial elongation. We allow the current sheath (piston) to separate from the shock front by applying an adiabatic approximation⁶

relating a change in pressure P to a change in volume V , to a fixed mass of gas at any given instant of time t . These 3 equations are then closed with a fourth equation, this being the circuit equation. These equations are written as follows:

$$\frac{dr_s}{dt} = - \left\{ \frac{\mu(\gamma+1)}{\rho f_{mr}} \right\}^{\frac{1}{2}} \frac{f_c I}{4\pi r_p} \quad (4)$$

$$\frac{dz_f}{dt} = - \frac{2}{\gamma+1} \frac{dr_s}{dt} \quad (5)$$

$$\frac{dr_p}{dt} = \frac{\frac{2}{\gamma+1} \left(\frac{r_s}{r_p} \right) \frac{dr_s}{dt} - \frac{r_p}{\gamma I} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dI}{dt} - \frac{1}{\gamma+1} \frac{r_p}{z_f} \left(1 - \frac{r_s^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma} \frac{r_s^r}{r_p^2}} \quad (6)$$

$$\frac{d}{dt} [(L_o + f_c L_p) I] = V_o - \frac{1}{C_o} \int I dt - r_o I \quad (7)$$

where in the radial phase

$$L_p = \frac{\mu}{2\pi} z_o \ln c + \frac{\mu}{2\pi} z_f \ln \left(\frac{b}{r_p} \right) \quad (7a)$$

The four equations (4) – (7) may be integrated step-by-step for the instantaneous values of r_s , r_p , z_f & I . For deuterium, γ is taken as 5/3. This phase is completed when the shock front hits the axis i.e. $r_s = 0$ with velocity $(dr_s/dt)_{\text{on-axis}}$.

2c. Reflected Shock Phase

When the shock front hits the axis, because the focus plasma is collisional, a reflected shock develops (position r_r) which moves radially outwards, whilst the radial current sheath piston continues to move inwards (see Fig 2).

This phase is simulated with the following equations:

$$\frac{dr_r}{dt} = f_{rs} \left(\frac{dr_s}{dt} \right)_{\text{on-axis}} \quad (8)$$

where we usually take f_{rs} as 0.3 empirically.

$$\frac{dz_f}{dt} = -\frac{2}{\gamma+1} \left(\frac{dr_s}{dt} \right)_{\text{on-axis}} \quad (9)$$

$$\frac{dr_p}{dt} = -\frac{r_p \left(1 - \frac{r_r^2}{r_p^2} \right) \frac{dI}{dt} - \frac{1}{\gamma+1} \frac{r_p}{z_f} \left(1 - \frac{r_r^2}{r_p^2} \right) \frac{dz_f}{dt}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma} \left(\frac{r_r}{r_p} \right)^2} \quad (10)$$

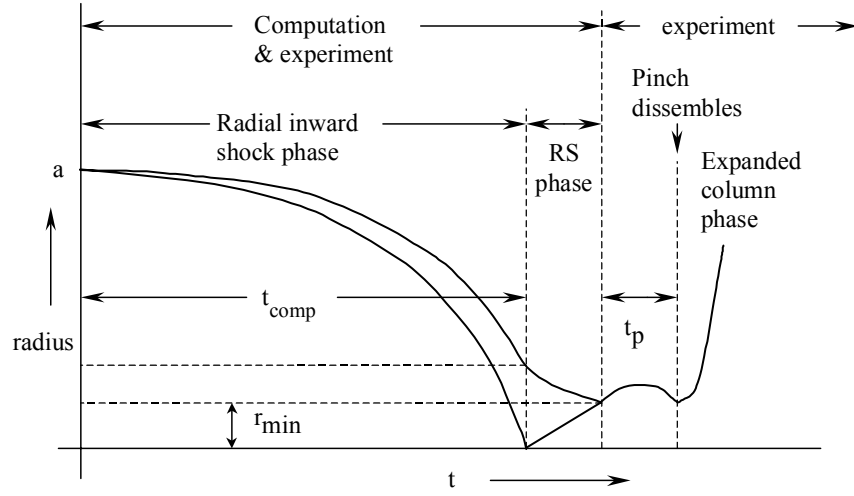


Fig 2. Schematic of radial phases

The circuit equations may be used unchanged from (7) supplemented by (7a). Equations (8), (9), (10) and (7) may be integrated step-by-step for r_r , z_f , r_p and I .

When the out-going reflected shock hits the in-going piston the compression enters a radiative phase in which for gases such as neon e.g. radiation cooling may actually enhance the compression. For deuterium we assume the radiative phase is not significant to the dynamics. We treat this point as the point of maximum compression with minimum gross radius $r_{\min}$ and maximum plasma pinch length z_p . If we just want the gross parameters of the plasma focus pinch the calculations may end here.

2d. Expanded Column Phase

To simulate the current trace beyond this point we allow the column to suddenly attain the radius of the anode, and use the expanded column inductance for further integration.

3. Normalization:

The equations are normalised in the following manner²⁻⁴:

$$\tau = \frac{t}{t_o}, \quad \zeta = \frac{z}{z_o}, \quad \iota = \frac{I}{I_o}, \quad \delta = \frac{r_o}{Z_o} \quad \text{where } Z_o = \sqrt{\frac{L_o}{C_o}}$$

$$\kappa_s = \frac{r_s}{a}, \quad \kappa_p = \frac{r_p}{a}, \quad \zeta_f = \frac{z_f}{a}$$

$$t_o = \sqrt{L_o C_o} \quad \text{and} \quad I_o = V_o / Z_o$$

Axial Phase:

The normalized axial phase equations are:

$$\frac{d^2 \zeta}{d\tau^2} = \frac{\alpha^2 \iota^2 - \left(\frac{d\zeta}{d\tau} \right)^2}{\zeta} \quad (11)$$

$$\frac{d\iota}{d\tau} = \frac{1 - \int \iota d\tau - f_c \beta \frac{d\zeta}{d\tau} \iota - \delta \iota}{1 + f_c \beta \zeta} \quad (12)$$

to compute ϕ and ι ; with two scaling parameters:

$$\beta = \left[\frac{\mu}{2\pi} z_o \ln c \right] / L_o$$

which is the ratio of the full axial phase tube inductance to the external fixed inductance. Ratio of characteristic electrical discharge time to characteristic axial transit times is:

$$\alpha = t_o / t_a$$

$$\text{where } t_a = \left[\frac{4\pi^2 (c^2 - 1)}{\mu \ln c} \right]^{1/2} \frac{z_o \sqrt{f_m}}{f_c} \frac{\sqrt{\rho}}{(I_o / a)}$$

giving a characteristic axial transit speed of $v_a = z_o / t_a$ of

$$v_a = \left[\frac{\mu \ln c}{4\pi^2 (c^2 - 1)} \right]^{1/2} \frac{f_c}{\sqrt{f_m}} \frac{(I_o / a)}{\sqrt{\rho}} \quad (13)$$

We note the dependence of v_a on $(I_o/a)/\sqrt{\rho}$ which is designated as the drive parameter S.

Radial Phase

Normalised radial phase equations are:

$$\frac{d\kappa_s}{d\tau} = -\alpha\alpha_1 \frac{1}{\kappa_p} \quad (14)$$

$$\frac{d\zeta_f}{d\tau} = -\frac{2}{\gamma+1} \frac{d\kappa_s}{d\tau} \quad (15)$$

$$\frac{d\kappa_p}{d\tau} = \frac{\frac{2}{\gamma+1} \frac{\kappa_s}{\kappa_p} \frac{d\kappa_s}{d\tau} - \frac{\kappa_p}{\gamma} \left(1 - \frac{\kappa_s^2}{\kappa_p^2}\right) \frac{d1}{d\tau} - \frac{1}{\gamma+1} \frac{\kappa_p}{\zeta_f} \left(1 - \frac{\kappa_s^2}{\kappa_p^2}\right) \frac{d\zeta_f}{d\tau}}{\frac{\gamma-1}{\gamma} + \frac{1}{\gamma} \frac{\kappa_s^2}{\kappa_p^2}} \quad (16)$$

$$\frac{d1}{d\tau} = \frac{1 - \int 1 d\tau + \frac{\beta_1}{F} f_c \frac{\zeta_f}{\kappa_p} \frac{d\kappa_p}{d\tau} + \frac{\beta_1}{F} f_c \ln\left(\frac{\kappa_p}{c}\right) \frac{d\zeta_f}{d\tau} - \delta 1}{1 + f_c \beta - f_c \frac{\beta_1}{F} \zeta_f \ln\left(\frac{\kappa_p}{c}\right)} \quad (17)$$

$$\text{where } \beta_1 = \frac{\beta}{\ln c}, F = \frac{z_o}{a}$$

and the ratio of the characteristic axial transit time to characteristic pinch time

$$\alpha_1 = \frac{t_a}{t_p} = \left[(\gamma+1)(c^2 - 1) \right]^{1/2} \frac{F \sqrt{f_m}}{2\sqrt{\ln c}}$$

$$\text{Hence } t_p = \frac{4\pi}{[\mu(\gamma+1)]^{1/2}} \frac{\sqrt{f_{mr}}}{f_c} \frac{\sqrt{\rho}}{(I_0/a)} a$$

and characteristic pinch time a/t_p is

$$v_p = \frac{[\mu(\gamma+1)]}{4\pi} f_c \frac{I_0/a}{\sqrt{\rho}} \quad (18)$$

We note that v_p has the same dependence on drive parameter S as V_a .

4. Parameters and Scaling

The UNU/ICTP PFF is a 3 kJ plasma focus ⁷ designated as the United National University/International Centre for Theoretical Physics Plasma Focus Facility. This device was developed during UNU/ICTP training programmes and is now established in

6 countries for postgraduate training and research. To compute the above model for the UNU/ICTP PFF we note its operational parameters:

$$\begin{aligned}
 C_o &= 3 \times 10^{-5} \text{F} & L_o &= 1.1 \times 10^{-7} \text{H} \\
 a &= 0.95 \times 10^{-2} \text{m} & b &= 3.2 \times 10^{-2} \text{m} & z_o &= 0.16 \text{m} \\
 c &= 3.37 & & & F &= 16.84 \\
 \text{Hence } t_o &= \sqrt{L_o C_o} = 1.82 \times 10^{-6} \text{s} & Z_o &= \sqrt{L_o / C_o} = 0.06 \Omega
 \end{aligned}$$

Typical operation is at 14 kV with ambient pressure of 3.5 torr deuterium giving:

$$\begin{aligned}
 \rho &= 8.2 \times 10^{-4} \text{kgm}^{-3} & V_o &= 1.4 \times 10^4 & \text{Also } r_o &= 0.012 \Omega \\
 \text{Hence } I_o &= V_o / Z_o = 2.33 \times 10^5 \text{A} & \delta &= r_o / Z_o = 0.2 \\
 \text{we take } f_c &= 0.7 & f_m &= 0.1 \\
 f_{mr} &= 0.3 & & & v_a &= 1.15 \times 10^5 \text{m/s} \\
 \text{Hence } t_a &= 1.38 \times 10^{-6} \text{s} & & & v_p &= 1.6 \times 10^5 \text{m/s} \\
 t_p &= 6.0 \times 10^{-8} \text{s} & & & & \\
 \text{Hence} & & & & & \\
 \alpha &= 1.31 & \alpha_1 &= 23.2 \\
 \beta &= 0.35 & \beta_1 &= 0.29
 \end{aligned}$$

We also use $\gamma = 5/3$ (specific heat ratio for fully ionized deuterium)

The above values of α , β , δ , f_c , f_m are used with equations (11) and (12) to compute the axial phase dynamics for τ and ς . For the radial phase dynamics additional parameters from the above list namely, α_1 , γ , β_1 , F , c are used together with the 4 equations 14 – 17 to compute the radial phase dynamics for κ_s , κ_p , ζ_f and ι . In the reflected shock phase we take the reflected shock speed ratio $f_{rs} = 0.3$.

Even without numerical computation, the normalization of the axial and radial phases equations has shown us the following regarding the characteristic times and speeds. We note that in the design of plasma focus devices the geometrical ratio c is between 2 and 3 so that the factor $[(c^2-1)/\ln c]^{1/2}$ remains within a small range. Thus basically: $t_a \sim z_o/S$ and $t_p \sim a/S$ and both the axial and radial speeds v_a and v_p are proportional to the drive parameter S .

Over a range of machines, from small to big, it is experimentally observed that the drive parameter is constant for neutron optimized operation in deuterium. This is consistent with observed constant speed over the range of devices. Hence the characteristic times scale with dimension z_o for the axial phase and ‘ a ’ for the radial phase.

Moreover it is implied in the equations 14-16 that at any point in the radial trajectory the values of r_s , r_p and z_f are proportional to ‘ a ’. Hence in the position of maximum compression, which gives the minimum radius $r_{\min}$ of the piston position, the gross parameters $r_{\min}$ and z_p are both proportional to anode radius ‘ a ’.

5. Results of computation and discussion.

Computation for the trajectories and electric current yield the following results. Fig 3 shows the axial phase dynamics in terms of axial position and speed presented in real quantities. It shows that the current sheath reaches the end of the axial phase at $2.87 \mu\text{s}$ with a peak speed of $8.6 \text{ cm}/\mu\text{s}$.

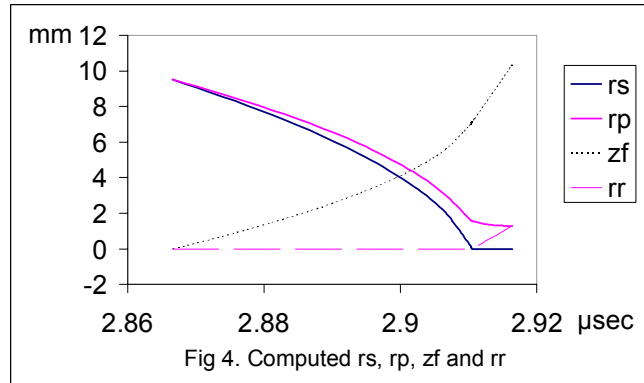
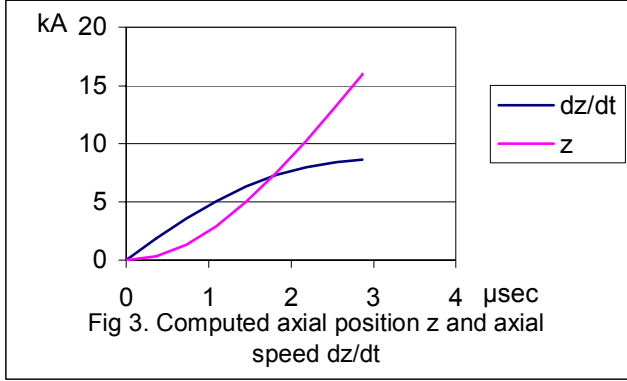


Figure 4 shows the radial phase dynamics. The shock front and the piston start together at $r=a$; the length of the focus pinch is zero at this time. The shock front accelerates onto the axis, hitting the axis 40 ns from the start of the radial phase. At this time the piston position is 1.6mm from the axis and the length of the lengthening focus column is 7.1 mm. According to this computation the speed of the on-axis shock front exceeds $50 \text{ cm}/\mu\text{s}$. The piston speed, which had peaked at $30 \text{ cm}/\mu\text{s}$, reduces sharply as the shock approaches the axis. We note the limitation of the radial model as follows.

Implicit in this radial model is the assumption of instantaneous communication between the piston and the shock front ie an assumption of infinite signal speed. This assumption results in the computed speeds being too high. If we consider the actual communication delay due to the finite small disturbance speed between the piston and shock front (of the order of ns, dependent on plasma slug temperature) the shock front would at any instant feel the considerably smaller pressure of the piston at an earlier time, and likewise the piston would feel the effect of the shock front moving at a slower speed at an earlier

position. This delay effect incorporated into the model, would considerably slow down both the shock front and piston, as they near the axis, to peak values of 25-30 cm/ μ s for the shock front on-axis and to about 20 cm/ μ s a short time before the shock front hits the axis. As the reduction in speeds is only significant near the axis (final 3 mm of shock front travel) the shock transit time is increased only slightly.

When the shock front hits the axis, a reflected shock develops and moves radially outwards. The piston continues to compress inwards until it hits the out-going reflected shock front. We define this point where the piston meets the reflected shock as the point of maximum gross compression, and label this radial position $r_{\min}$. Experimental observations using shadowgraphs complemented by electrical and x-ray measurements, indicate that beyond this point of time the plasma radius remains at about the value of $r_{\min}$ for a short period (some 20ns for the UNU/ICTP PFF) before the pinch disassembles rather violently. For deuterium this model is not extended specifically to compute the dynamics of this phase. For neon we have included this as a radiative phase, adding radiative terms into the computation. Indeed for neon, the radiative terms are energetically significant and even affect the dynamics to give evidence of further compression due to radiative cooling..

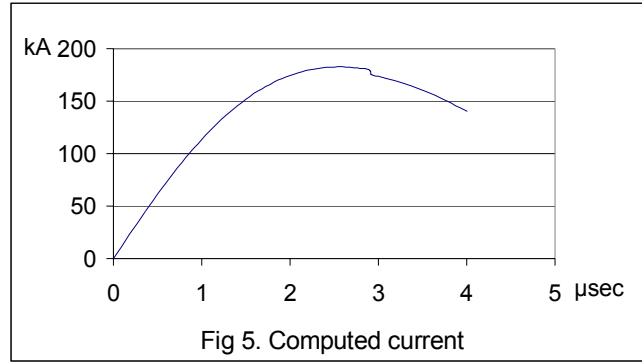


Figure 5 shows the computation of the electric current. This computation was extended to the post-focus phase by using a simple expanded column approximation. The current agrees well with experimentally observed Rogowskii coil measurements.

The key computed pinch parameters for the deuterium focus are as follows:

$$r_{\min} = 0.13 \text{ a} \quad (18)$$

$$z_p = 0.7 \text{ a} \quad (19)$$

$$t_{\text{comp}} = 5 \times 10^{-6} \text{ a} \quad (20)$$

We have not computed t_p but this has been measured experimentally as⁸:

$$t_p = 2 \times 10^{-6} \text{ a} \quad (21)$$

The above results have been verified experimentally^{8,13} with shadowgraphs and measurements of current, voltage, neutron and SXR of the UNU/ICTP PFF operated in deuterium.

6. Scaling of Yield

For a thermalised plasma we can generally assign density n and temperature T . If the particles of density n interact among themselves with a cross section $C(T)$, generally depended on T , we may write down a general radiation yield, Y , relationship as follows:

$$Y \sim n^2 (\text{volume}) (\text{lifetime}) C \quad (22)$$

For the plasma focus, given the dimensional and temporal dependence as shown in equations (18), (19) and (21) we have

$$Y \sim n^2 a^4 C \quad (23)$$

6a. Constant S operation

A survey of plasma focus devices have shown that in deuterium the drive parameter⁸ is $S = 90 \text{ kA per cm per (torr)}^{1/2}$ of deuterium over a range large range of energy from 3kJ – 200kJ. This corresponds to the well-know phenomenon that similar speeds are observed in small devices as well as in large devices. For example the peak axial speeds for neutron-optimised operation is experimentally observed to be 8 – 10 cm/ μs . This has also been expressed as an average axial speed of 5-5.5 cm/ μs . Similarly the radial speeds peak at 25-30 cm/ μs for all devices, big or small. This peak speed occurs as the radial in-going shock goes on-axis. Such constant speeds observed over such a range of device energies indicate that at any point of the plasma focus operation, e.g. end of axial phase or end of inward shock phase as the inward radial shock goes on axis, the temperature in a small focus is the same as the temperature in a big focus. By extension it is inferred that the temperature at the point of maximum compression is also the same for a small focus as for a big focus. This is of course consistent from the viewpoint of energy densities. The quantity S is the magnetic energy density driving the system. Hence since the driving magnet energy is constant over the range of devices, it is consistent that the temperatures generated is also constant over the range of devices.

Given this observed constancy we have:

$$\frac{(I/a)}{\rho^{1/2}} = \text{constant} \quad (24)$$

$$\text{Hence } I \sim a\rho^{1/2} \quad (25)$$

For a deuterium focus, the pinched gas is fully ionised and the number density of deuterium ions in the pinch is proportional to the ambient density ρ .

Hence applying equation (25) to Eq (23) and noting that since $T=\text{constant}$ over the range of devices, the cross section C is also constant and we have⁹:

$$Y \sim I^4 \quad (26)$$

This yield law really applies to the thermonuclear neutron yield. Experimentally for a small neutron-optimised plasma focus it has been shown¹⁰ that the thermonuclear

component of the neutron yield is only 15%, the other 85% being ascribed to beam-target mechanisms. Nevertheless we note that these yield proportions are occurring at a plasma ion temperature of about 1 keV and a beam energy of about 50 keV.

6b. Speed-enhancement of neutron yield

This leads to the concept of speed enhancement^{9,12}. If the driver parameter S is increased, the drive energy density increases, the speed increases and we may expect the focus pinch ion temperature also to increase. This leads to a dramatic increase in neutron yield since in the range of 1 – 10 keV, the neutron fusion cross-section is a rapid function of T with $C \sim T^n$ n being greater than 4. And since $T \sim v^2$, if we keep ‘ a ’ constant as the drive current is increased, we increase the drive parameter S . This will give us a thermonuclear yield component better than

$$Y \sim I^8 \quad (27)$$

Increase in speed will also increase the energy of the beam component. The beam component will however not have a significant increase since above 50 keV the cross section C barely increases with T . This means that as operational speed is increased, the plasma focus neutron yield becomes more thermonuclear. For example an increase in speed by 20% increases the focus pinch temperature by more than 40%, with the thermonuclear yield increasing by more than 5 times, whilst the beam-target component barely increases. This means that even a 20% increase in the drive parameter for the UNU/ICTP PFF could not only significantly increase its neutron but also make this yield predominantly thermonuclear.

However it is not a simple matter to increase the drive speed. Experiments have shown that if the peak axial speed is pushed above 10 cm/ μ s, plasma focus quality simply deteriorates. A force-field flow-field decoupling mechanism³ has been proposed to account for the deterioration of the focus quality. According to shock theory for a gas with specific heat ratio of 5/3, the contact surface or piston should separate from the shock front 1 cm for every 4 cm travelled by the shock front. At lower speeds, although deuterium is already fully ionised at 4 cm/ μ s, there is still considerable diffusion of the magnetic field into the plasma all the way to the shock front so that effectively the centre of drive force field is not significantly behind the centre of the mass field. However at a speed above 9 or 10 cm/ μ s, the electrical conductivity is sufficient to limit the field penetration resulting in a separation of the centre of the force field from the centre of the mass field. This separation grows with distance travelled. If this separation becomes of the order of the anode radius, then when the shock wave sweeps around the anode in the radial compression, the piston is still travelling axially a radius away and when the shock hits the axis the piston may still not have started its radial compression. Such a compression will be weak. It is postulated that this force-mass decoupling mechanism^{3,11} becomes significant above the ‘speed-limit’ of 9-10 cm/ μ s observed for deuterium focus operation.

An experiment^{12,13} was carried out to achieve this speed increase yet overcome the decoupling by keeping the ‘speed-enhanced’ region short. A stepped-anode was used

consisting of a ‘normal’ radius section designed with the normal S value, followed by a short ‘speed-enhanced’ section for which the S value was increased by a reduction of ‘a’. By this means speeds up to 15cm/μs was achieved with focussing. Neutron-yield enhancement was indicated.

However this experiment was severely limited in its scope since ‘speed-enhancement’ was achieved not by an increase of I at fixed ‘a’, rather by effectively fixing I and reducing ‘a’. This was due to the limitations imposed by the UNU/ICTP PFF in its electrical range of operation. It is proposed that speed-enhancement experiments should be carried out in a machine with fixed ‘normal’ ‘a’; and to increase I above ‘normal’ values.

6c Scaling for neon operation

The plasma focus is operated in neon to generate SXR in the wavelength range¹⁷ of 0.8 – 1.4 nm. This wavelength range is found suitable for microelectronics lithography¹⁴ from the point of view of near optimum contrast using existing mask technology. We have performed calculations^{15,16} using the axial-radial model described above but including radiation terms (free-free, free-bound and bound-bound) into the dynamical equations as well as SXR yield equations. These calculations indicate that the focus pinch temperature needs to be adjusted to 300 – 400 eV for optimum yield in the correct wavelength range which arises from He-like and H-like Neon ions. We have selected 350 eV which corresponds to an axial speed of 4.5 cm/μs. For the UNU/ICTP PFF we found that optimum yield is obtained at 14 kV operation when an ambient pressure of 1 torr is used. Because the neon is still ionising (rather than fully ionised) its effective specific heat ratio may be approximated as 1.4. This gives it a thinner slug layer and a smaller compressed radius. Some of the parameters of the neon focus pinch are calculated as:

$$\begin{aligned} r_{\min} &= 0.04a \\ z_{\max} &= 0.8a \\ t_{\text{comp}} &= 4 \times 10^{-6}a \end{aligned}$$

These agree with experimental observations. We have also estimated from experiments:

$$t_p = 1 \times 10^{-6}a$$

Since the pinch temperature needs to be fixed at 350 eV no speed-enhancement is possible for the neon focus operated for microelectronics lithography purposes.

7. Conclusion

We conclude from computation, supplemented by experimental observations that the plasma focus pinch has gross parameters that scale according to anode radius ‘a’ in the following manner:

		Neon (for SXR)	Deuterium
minimum radius	$r_{\min}$	0.13a	0.04a
maximum length	z	0.7a	0.8a
radial shock transit	t_{comp}	$5 \times 10^{-6}a$	$4 \times 10^{-6}a$
pinch lifetime	t_p	$2 \times 10^{-6}a$	$1 \times 10^{-6}a$

where, the times are in sec, when the value of anode radius, a, is in m.

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