From Beam-target to Thermonuclear Fusion in the Dense Plasma Focus Pinch: Energy throughput scaling

[1,2,3,4]S Lee, [2,3]S H Saw
[1]Physics Department, University of Malaya, 50603 Kuala Lumpur, Malaysia
[2]Institute for Plasma Focus Studies, 32 Oakpark Drive, Chadstone VIC3148, Australia
[3]Nilai University, 71800 Nilai, Malaysia
[4]INTI International University, 71800 Nilai, Malaysia

Outline of talk:
I: Background concepts:

- Electromagnetic drive, MRN and typical speeds, Speed factor S.
- Mach >> 1 driven plasmas, temperature vs speed
- Cross-sections for nuclear fusion: beam-target and thermonuclear

Outline of talk:

II: DPF Fusion: Beam-target predominance: why? - Throughput scaling

- Inductive voltages generate tens of keV, driving beam ions to 100 keV energy – optimum fusion x-section
- Shock speed generates < 0.5 keV kinetic temperature; fusion x-section 10^{-13} times below optimum – negligibly small
- Hence Beam-target fusion mode predominates
- Throughput (Output/Input) Scaling to break-even Q =1
- Breakeven point found through numerical experiments

Note: Discuss Gross PF pinch (scalable), NOT hot spots

Outline of talk:
III: Transitioning to thermonuclear mode

- How?
- Optimum conditions for neutron yield in thermonuclear mode
- Throughput (Output/Input) Scaling to break-even Q =1
- Breakeven point found through numerical experiments

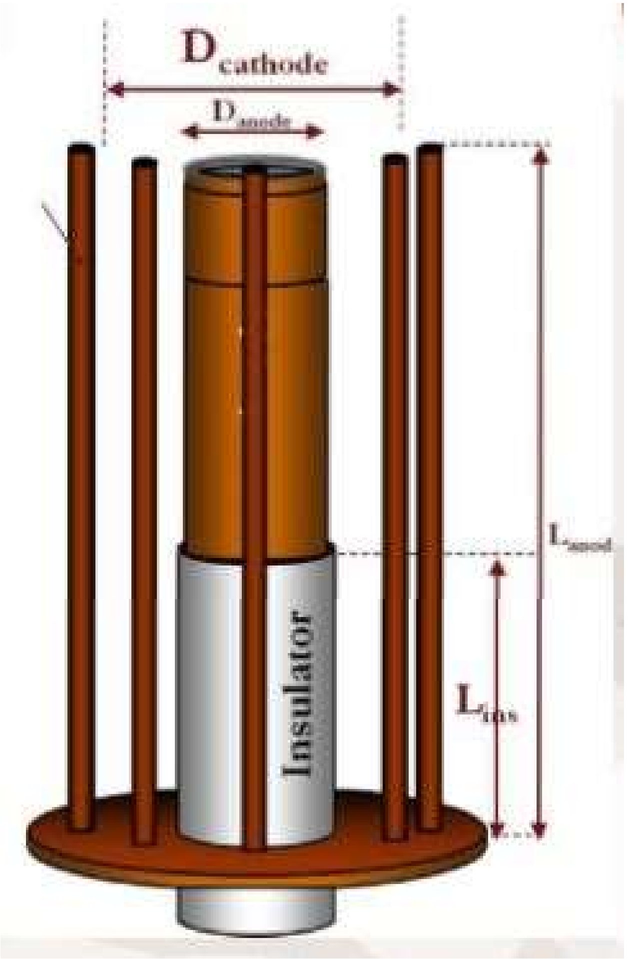
Outline of talk:

IV: Comparison:

Beam-target (DPFQ1) breakeven point versus Thermonuclear breakeven point

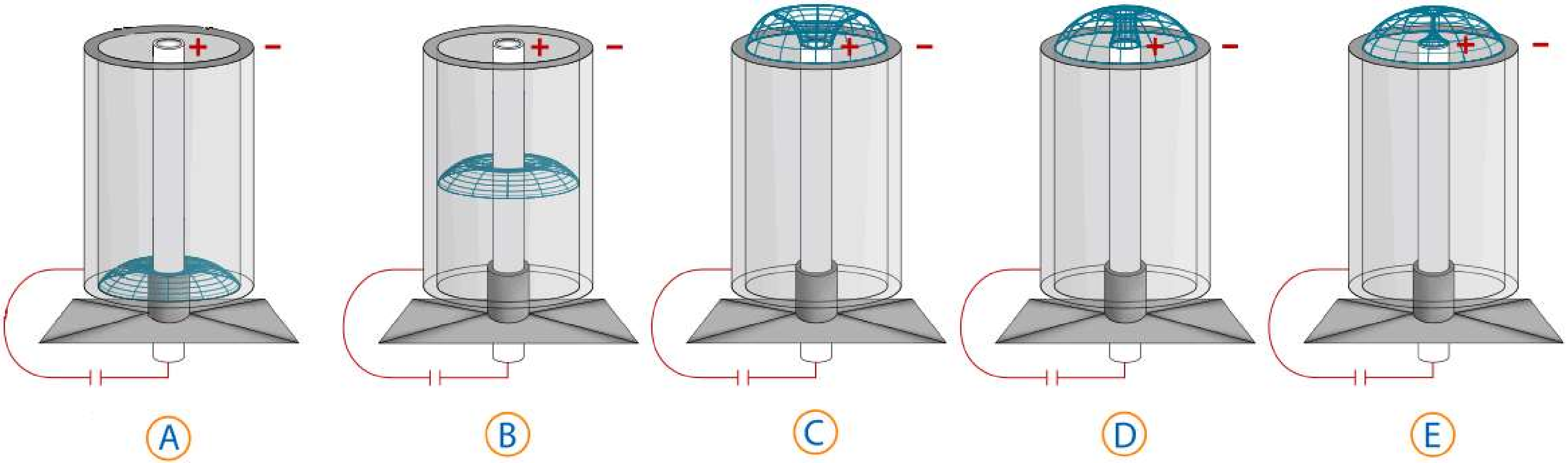
- **Proposing a feasible test point DPF0.01**
- **Conclusions**

Some introductory slides: DPF electrodes – (from Ali Abdou)



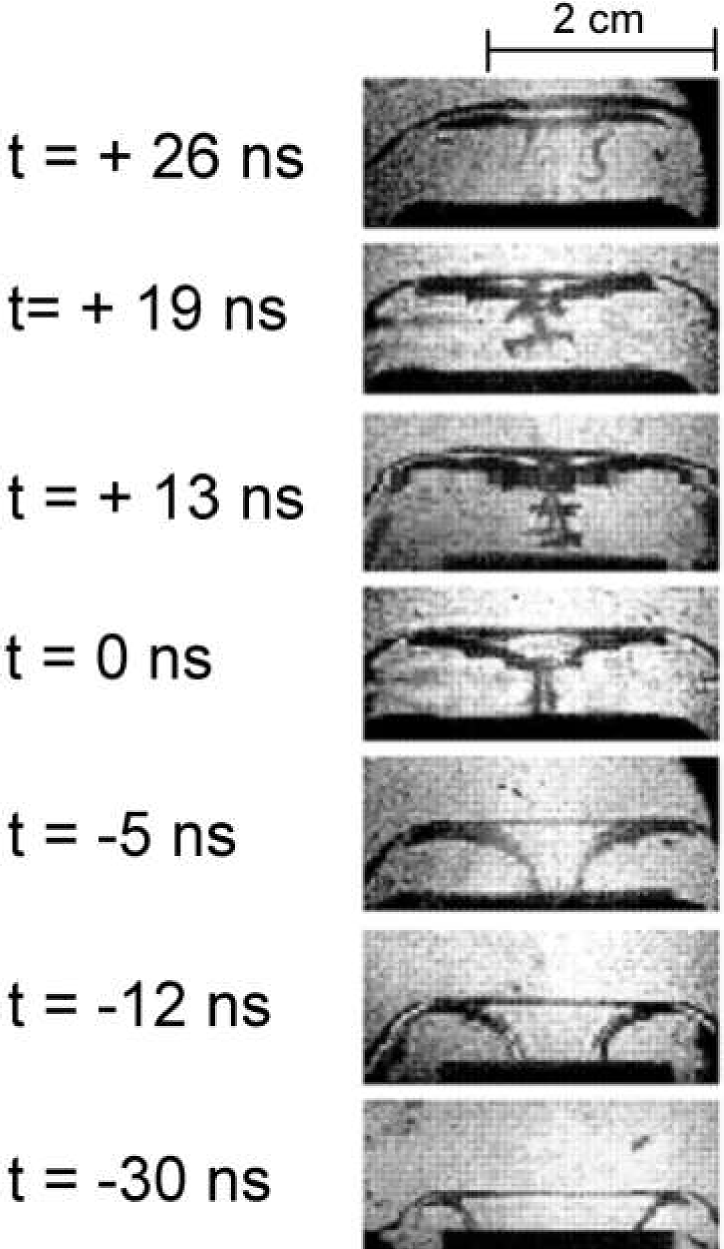
DPF dynamics – axial phase, radial phase –

from internet

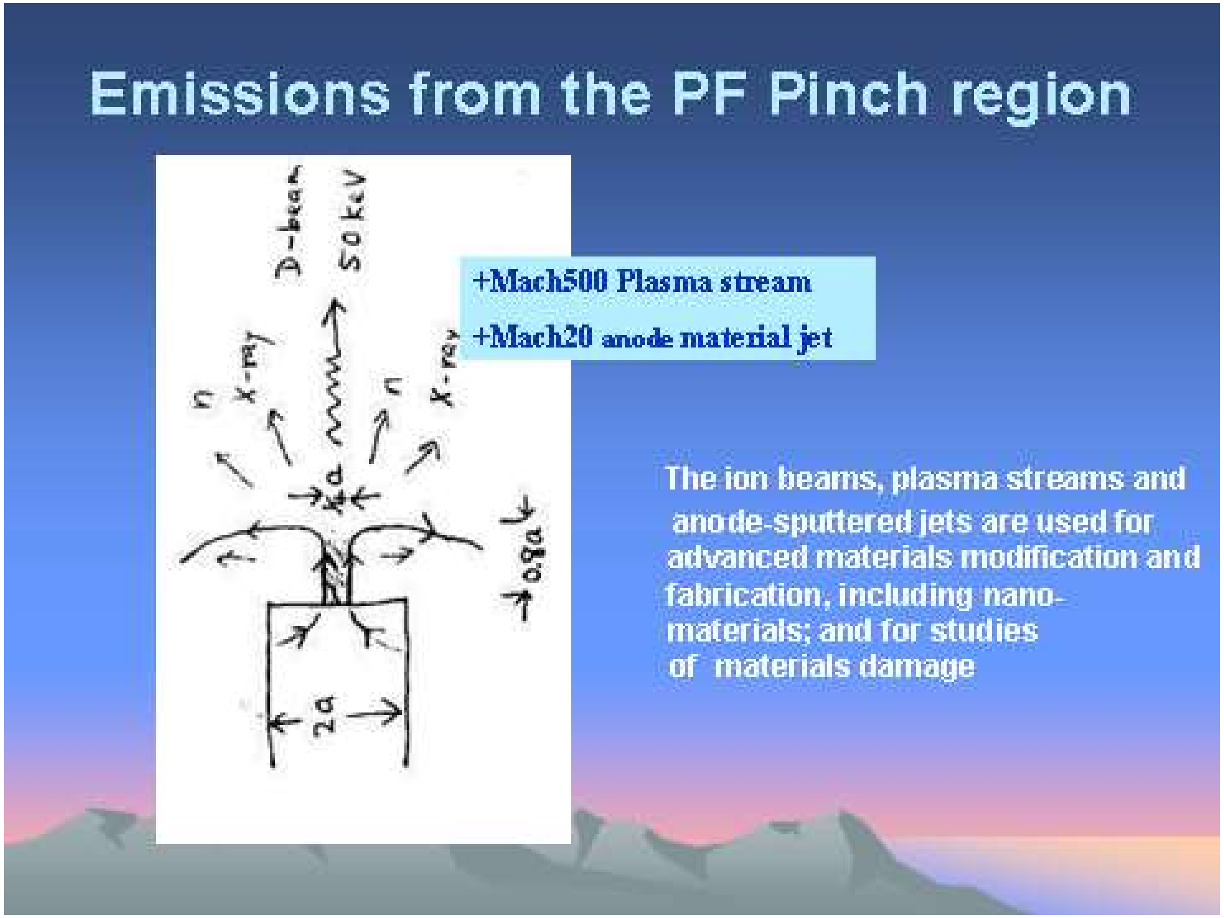


Radial dynamics – confirmed by our ns Shadowgraphs –

(from M Shahid Rafique, PhD thesis, NTU, Singapore [1])



Among radiations from DPF pinch are the fusion neutrons- 50%-50% D-T has the best cross-section for nuclear fusion; producing 14.1 MeV neutrons



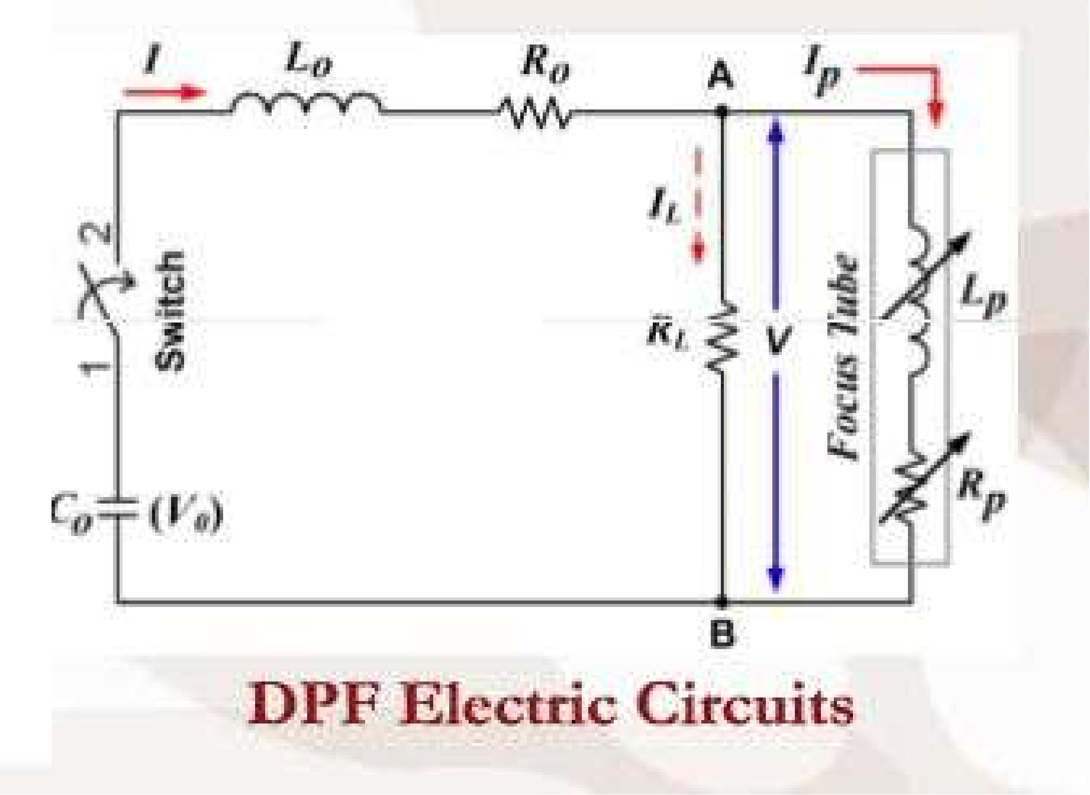
Emissions from the PF Pinch region

+Mach500 Plasma stream

+Mach20 anode material jet

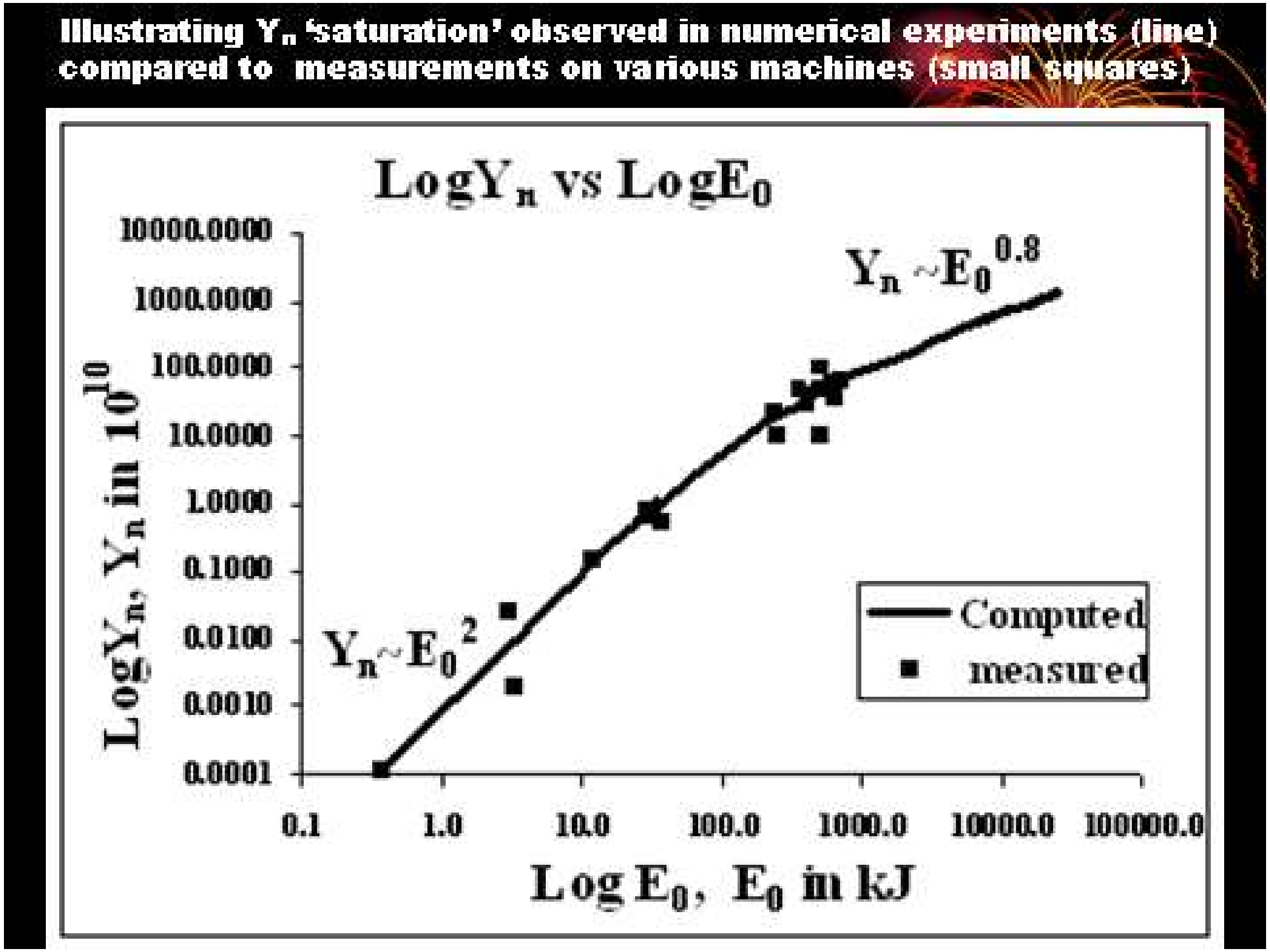
The ion beams, plasma streams and anode-sputtered jets are used for advanced materials modification and fabrication, including nano-materials; and for studies of materials damage

DPF circuit representation



DPF Electric Circuits

Initial hope: $Y_n \sim E_0^2$ [2],suggesting breakeven at hundreds MJ . However dominance of dynamic resistance for very large capacitor banks causes current- scaling deterioration [3], leading to yield- scaling deterioration to $Y_n \sim E_0^{0.8}$– suggests no break-even **unless** capacitor voltages are greatly increased [4] aided by increase in pressure [4].



Illustrating Y_n 'saturation' observed in numerical experiments (line) compared to measurements on various machines (small squares)

I: Some background concepts for DPF:

1. Electromagnetic drive:

- Interaction of electric current and magnetic field (JXB) produces high plasma speeds and temperature. Electromagnetic drive is efficient when the magnetic Reynolds number M_{RN} is high.

2. Magnetic Reynold's number M_{RN} and speed factor S:

- For electromagnetic em drive to be efficient, the condition is that $M_{RN} >>1$.
- For high Mach shock waves we have shown [5]

  $M_{RN} \sim v^4$; with transition point around 5 cm/μs for D-T plasma.
- $M_{RN} >>1$ is fulfilled in the DPF because plasma speeds are highly supersonic.
- For electromagnetic drive, the speed is governed by a fundamental factor $S= (I/a) / \rho^{0.5}$

Over whole **range of existing DPF's (sub kJ to MJ)** S ~ 70 – 200, practically constant [6]

- S= (I/a) / $\rho^{0.5}$

Value of S: typically 100 (kA/cm) Torr$^{0.5}$

(I/a) : Typically **200 kA/cm**

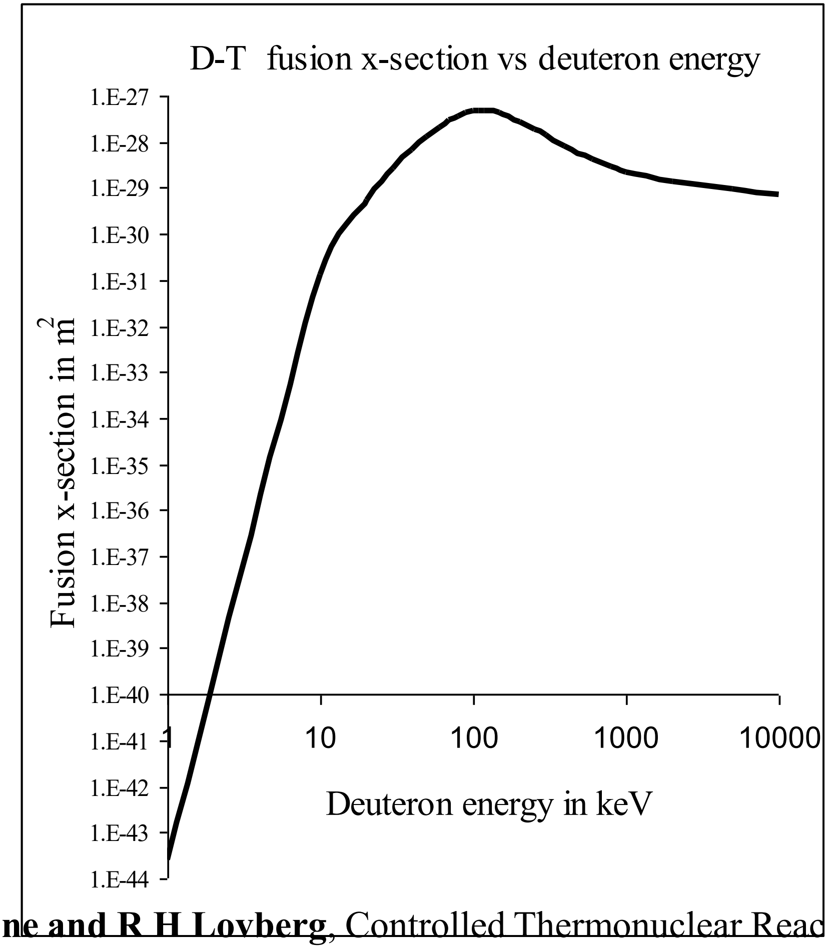
Operational pressure: Typically 4 Torr **(~ 0.01 atm)**

3. High Mach (Mach >> 1) shock waves:

- A convenient unit of speed for electromagnetically driven systems is cm/μs. (1 cm/μs = 10^4 m/μs ~ Mach 10 in D-T).
- In DPF's, speeds are typically 10 cm/μs and higher. Thus DPF plasma drives are characterized by Mach > 100 supersonic shock waves.
- Shock wave systems are equi-partitioned with approximately equal amounts of energy in the thermal modes and the kinetic components.
- The shock conservation equations enable the plasma temperature T to be computed [6a] from the shock speed v.
- For a 50%-50% D-T shock system:

$$\mathbf{T = 2.8x10^{-5}v^2}$$

4. Cross-sections for D-T nuclear fusion : applicable to **Beam- (plasma) target:**



- 10 keV σ is **10^{13}** higher than σ at 1 keV.
- 10 keV to peak σ at ~ 100 keV, another increase of **10^3**.
- Optimum σ occurs at ~100 keV.
- Higher beam energy σ drops.

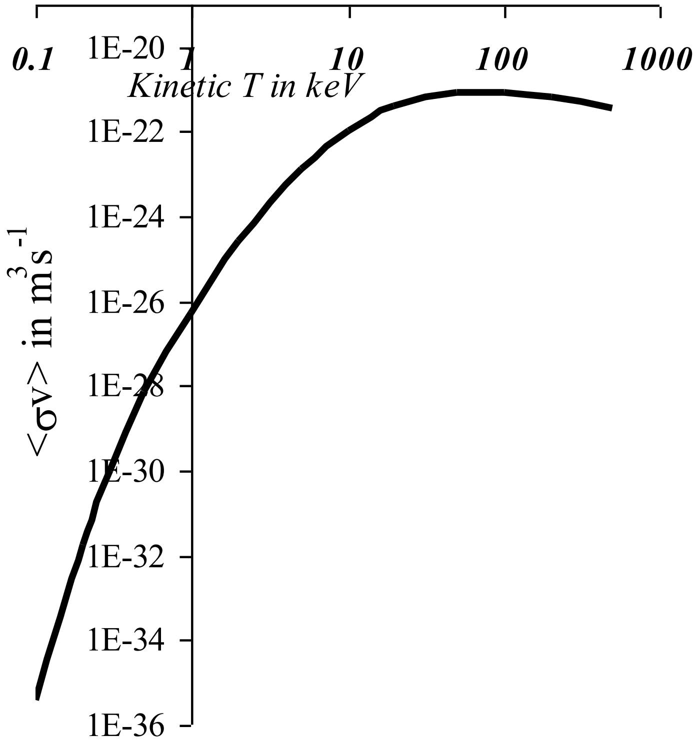
1 MeV σ **has dropped almost 100x**

[7] **S Glasstone and R H Lovberg**, Controlled Thermonuclear Reactions. D Van Nostrand Company, New Jersey **(1960)**
[8] **J.D.Huba**. 2006 Plasma Formulary pg44 Http://wwwppd.nrl.navy.mil/nrlformulary/NRL_FORMULARY_07.pdf]

5. Cross-sections for nuclear fusion: Thermonuclear:

Relevant cross-section is the <σv> ie product of cross-section and particle speed v averaged over Maxwellian distribution

<σ v> vs kinetic temperature kT



[7] [8]

- From 0.1 to 1 keV <σv> **increases 10^9**.
- From 1 to 10 keV <σv> **increases 10^4**.
- From 10 keV, the <σv> increases <10 to its peak value at 70 keV.
- From <σv> point of view, good operational point is around **70 keV**

II: Present-generation DPF (sub kJ to MJ) operate predominantly in beam-target mode: why?

- All present DPF's (sub kJ to MJ) operate with same speed: axial around 10 cm/μs and radial around 20 cm/μs [6].
- This gives a temperature in D-T of ~ 0.3 x 10^6 K for the axial phase plasma and ~ 1.2 x 10^6 for the radial phase on axis shock; to about 2.4 million K in the stagnated plasma column on shock reflection on-axis.
- The gross pinch **temperature** is **typically < 0.5 keV** (1 keV = 1.14 x 10^7 K) – very low temperature from fusion point of view.
- Resulting from highly supersonic piston action, inductive voltages (back EMF motor effect, if we recall) typically 20 – 40 kV are generated, producing 60 – 120 keV ions. These energies are near optimum from fusion point of view.
- **Thermonuclear** yield: **low**; **Beam – target** yield: **high**
- Advantage: **low investment in plasma energy vs optimum beam energy**

B-t fusion scaling: output fusion energy to input energy.

- We ask the question: How much D-T fusion energy (from beam-target) do we get per unit energy needed to drive the DPF?
- We first ask the question: How many D-T neutrons (from beam-target) do we get per unit pinch energy.

Modelling by the Lee code provides the number of beam-target neutrons [9-11] as follows:

- $Y_{b\text{-}t} = C_n n_i I_{pinch}{}^2 z_p{}^2 (\ln(b/r_p))[\sigma/V_{max}{}^{1/2}]$

pinch radius r_p, pinch length z_p, pinch ion density n_i driven by pinch current I_{pinch} ; with beam ion energy V_{max} and corresponding beam-target fusion cross-section σ.

Here all quantities are in SI units with V_{max} in keV and the constant $C_n = 1.4 \times 10^7$ (calibrated)

The pinch energy [12] at temperature T is

- $\boldsymbol{E_{pinch} = [kT/ (\gamma\text{-}1)]\, n_i(1+Z_{eff})\, \pi r_p^{\,2} z_p}$

k = Boltzmann constant, γ = specific heat ratio = 5/3 and Z_{eff} =1 (for fully ionized D-T plasma).

- Assume an equilibrium pinch, equate the confining magnetic pressure to the hydrostatic plasma pressure. Thus:
- $I_{pinch}^{\;2} = 2\pi \text{x} 10^7\, n_i kT(1+Z_{eff}) r_p^{\;2}$

Divide $Y_{b\text{-}t}$ by E_{pinch}, replacing I_{pinch}^2 in $Y_{b\text{-}t}$ by the RHS of Eq (4) we get the required number of beam-target neutrons per unit pinch energy. Note: $((\ln(b/r_p)) \sim 2$ and $z_p \sim 1.4$ a ([6] for fully ionized hollow anode DPF)

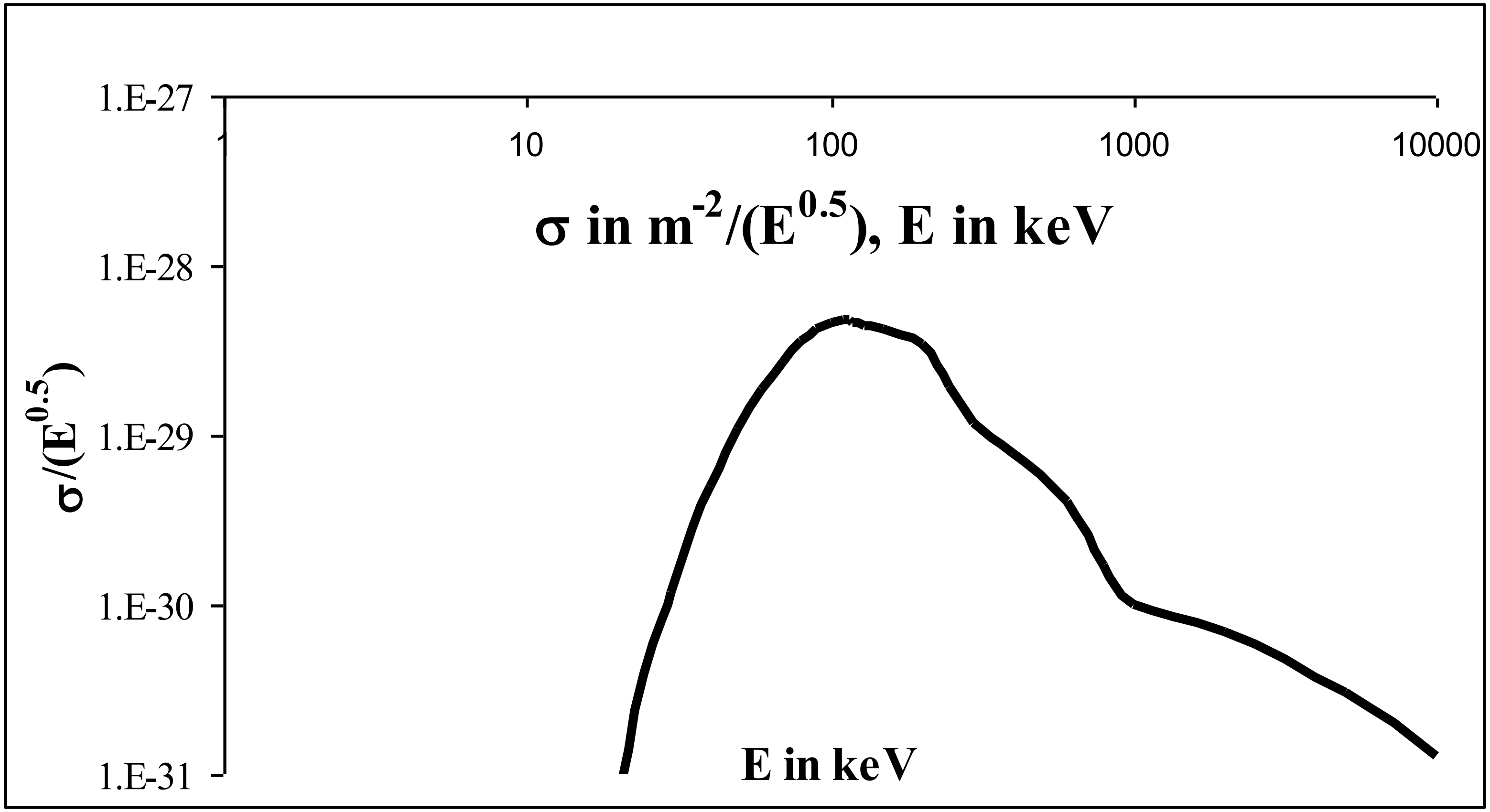
General scaling for number of D-T DPF beam-target neutrons

- $\mathbf{Y_{b\text{-}t} / E_{pinch} = 5 \times 10^{14}\, n_i\, a\, [\sigma/V_{max}^{1/2}]}$

This general scaling stipulates that the number of beam-target neutrons depends on the pinch ion density, the anode radius ‘a’ and the energy of the D-T beam ions through the fraction $[\sigma/V_{max}^{1/2}]$.

Optimising beam – target scaling by choosing optimum beam energy through factor $[\sigma/V_{max}^{1/2}]$

Optimum value of $[\sigma/V_{max}^{1/2}]$ occurs at V_{max} = 115 keV with value **5 x 10^{-29} m^2.**
Note: **broad flat maximum**, in range of beam ion energy 90 – 120 keV



Operate the DPF at **optimum deuteron beam energy around 100 keV** then the scaling for $Y_{b\text{-}t} / E_{pinch}$ is optimized:

- **$(Y_{b\text{-}t} / E_{pinch})_{100keV} = 2.5 \times 10^{-14}\, n_i\, a$**

A D-T neutron has energy of 14.1 MeV ie 2.26×10^{-12} J ; also estimating that E_{pinch} ~10% of the stored energy E_0, then we have the ratio

Q_{100keV} = **$(E_{b\text{-}t} / E_0)_{100keV} = \underline{6 \times 10^{-27}\, n_i\, a}$**

Better than break- even

for $Q > 1$

$6 \times 10^{-27} n_i a > 1$

Example: **‘a’= 1 m** then for $Q>1$

$n_i > 1.7 \times 10^{26}\ m^{-3}$ (100 keV)

(ie **6 atm** of fill pressure is sufficient for break-even).

Numerical experiments - disagree

- In fact thousands of runs have been made using the Lee code [9,11] in DPF’s of meter-size anode radius at beam-target conditions (following present generation DPF’s with axial speed around 10 cm/μs) – over years!
- Nothing approaching anywhere near break-even has been observed at such DPF sizes and densities; and even bigger sizes and densities!

Discrepancy- is due to **condition of 100 keV beam ion** energy, which is **not met**

$$V = \frac{\mu}{2\pi}\left[(\ln c)z_0 + \left(\ln\frac{b}{r_p}\right)z_f\right]f_c\frac{dI}{dt} + \frac{\mu}{2\pi}\left[\left(\ln\frac{b}{r_p}\right)\frac{dz_f}{dt} - \frac{z_f}{r_p}\frac{dr_p}{dt}\right]f_c I$$

V = **V1** + **V2**

V = **V1** + **V21** V22

	Tube	**dI/dt*position**	**speed*I**			**Effective**
	Voltage kV	term	term	dz/dt term	drp/dt term	Beam ion
	V	**V1**	**V2**	V21	V22	Energy keV
NX2 15kV	**25.1**	**-10.6**	**35.7**	17.1	18.6	**75**
PF1000 27kV	**25.7**	**-29.8**	**55.5**	29.1	26.4	**77**
PF1000 27kV RESF=0.1	**57.4**	**-65.6**	**123**	62.9	59.7	**172**
25MJ 35kV	**62**	**-285**	**347**	185	162	**186**
162MJ 90kV	**106**	**-484**	**590**	308	282	**318**
16200MJ 900kV	**1460**	**-6121**	**7580**	3970	3610	**4380**

Scaled up DPF requires 500 MA. The dynamic impedance of 2mΩ requires **bank voltage of 1 MV**; Thus induced beam ion energy ~ 5 MeV

- For b-t DPF's the values of speeds are kept at low levels of **10 cm/μs** for axial and 20 cm/μs for the radial phases. The speed factor

  $S = (I/a)/\rho_0^{1/2}$ needs to be kept at around **100**. Thus as 'a' increases, I increases proportionally; so with (I/a) = 200 kA per cm anode radius, we need **20 MA for 1 m** anode radius. The increase in **operational pressure to atm** will require a further increase of current to around 500 MA.
- **Thus the scaled-up m-sized DPF will have induced voltages increased to multiple MV; way past optimum** $[\sigma/V_{max}^{1/2}]$. Indeed at **10 MeV** this cross-section parameter has **dropped by 1000 times** from optimum value.
- These estimates provide a guide for numerical experiments for a scaled-up DPF, to obtain an operational point for a B-T device; although we know the beam ions will be far too energetic.

Beam-target numerical experiment (1.2 MV, 5 atm) generating Q = 0.002

Lo	Co	b	a	zo	ro mOhm
20	10000	120	100	60	0.15

massf	currf	massfr	currfr	Model Parameters
0.08	0.7	0.2	0.7	

Vo	Po	MW	A	At-1 mol-2	Operational Parameters
1200	4000	5	1	2	

E_0 kJ	RES F	c=b/a	I_{peak} kA	T_p 10^6 K	v_a cm/μs	v_s cm/μs	v_p cm/μs	r_{min} cm	z_{max} cm	pinch τ ns	V_{max} kV	n_i pinch $10^{23}/m^3$
7.20E+06	0.11	1.20	3.5E+05	0.11	8.3	8.5	6.8	30.0	161.7	6527	**1,337**	1051

Y_n 10^{10}	EINP %	SF	ID kA/cm	E_N kJ	**E_N/E_0**	Y_{th}	Y_{b-t}	Y_n
6.1E+08	43.4	55	3476	1.4E+04	**0.002**	5.0E-12	6.1E+18	**6.1E+18**

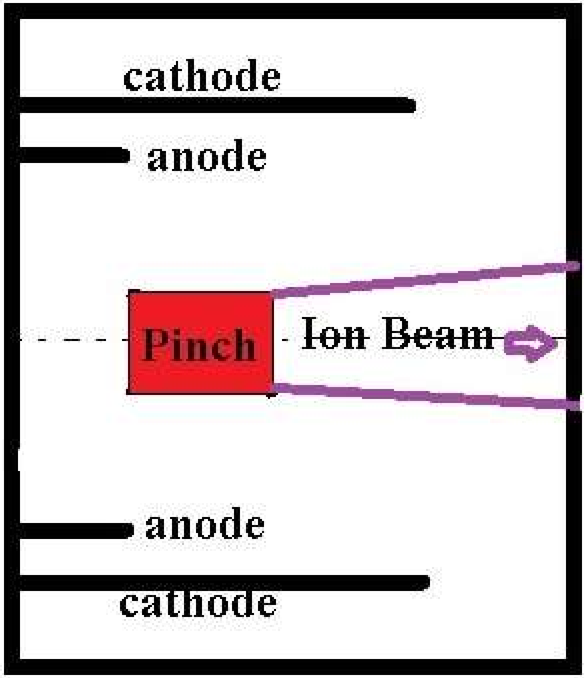
Beam ion energy 4 MeV; number of ions in beam = 1.3 x 10^{22}

[$\sigma/V_{max}^{0.5}$] is down from optimum value by almost 1000 times

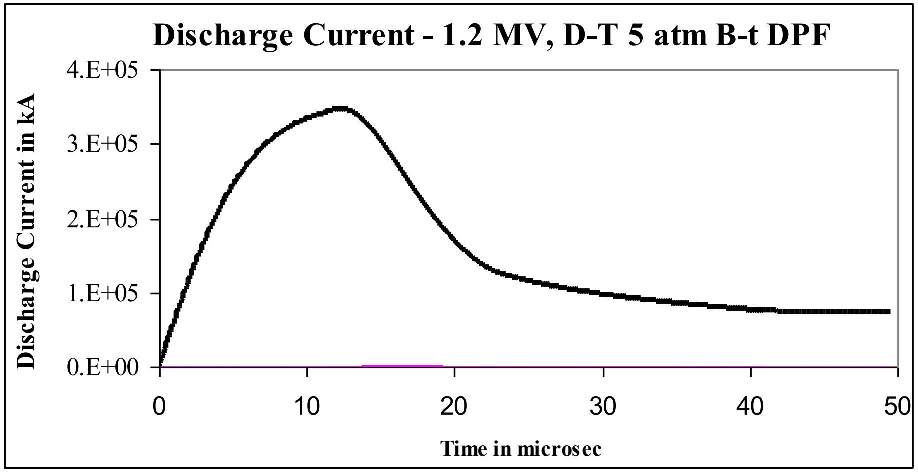
SIRIM estimates by M Akel [13] indicate **1 m path in 5 atm D-T sufficient to slow Deuteron- Triton Beam to 100 keV**.

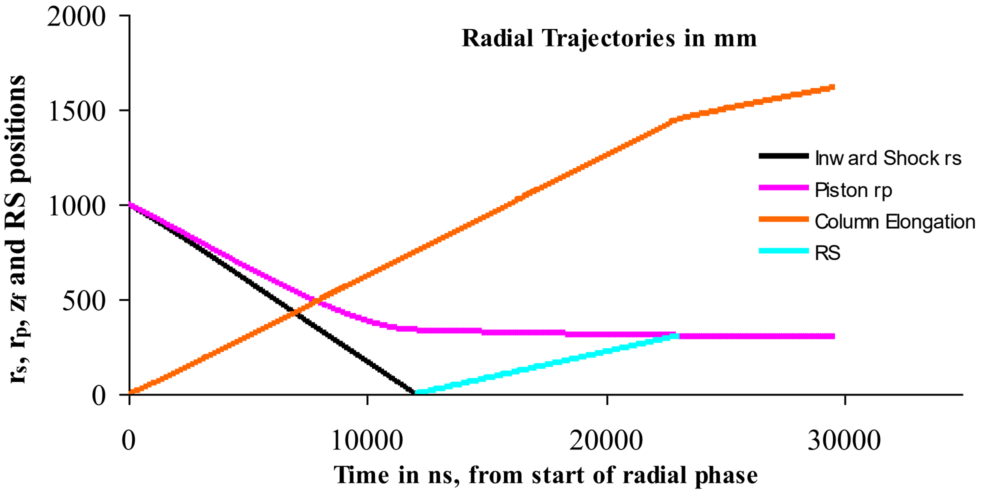
In schematic shown below the ion beam will be moderated to 100 keV in its path (> 1m) before leaving the chamber.

In this schematic Q>1 enhancing the Lee Code calculation of Q = 0.002.



Discharge current waveform and radial trajectories of 1.2 MV, 5 atm, a = 1m; D-T DPF





III: Transitioning to thermonuclear mode

- To operate the DPF pinch at thermonuclear conditions for D-T the pinch temperature needs to be increased from the sub-keV of present-day DPF’s to ~70 keV
- at which the optimum thermalized cross-section $<\sigma v>$ for D-T fusion occurs.
- This requires faster plasma speeds than presently used.
- How fast?

III: Transitioning to thermonuclear mode - how?

- How to get to such high temperature?
- The primary mechanism in the DPF is the electromagnetic drive generating high plasma speeds. Thermonuclear regime is 50 times hotter; requires shock speed 7 times higher.
- Radial shock speed needs to increase to **150 cm/μs.**
- Speed factor S needs to increase to **1000** (from present day 100)
- Density is ~1000x higher. (as will be seen)
- Hence current per unit anode radius need to increase to 60 MA/cm (from present-day 200 kA/cm) !!

Thermonuclear scaling: derive the ratio: Y_{th} / E_{pinch} at thermonuclear pinch conditions.

- $Y_{th} = 0.5\, n_i^2\, \pi r_p^2 z_f <\sigma v> \tau$

where $<\sigma v>$ is the thermalised fusion cross section-velocity product corresponding to the plasma temperature [149], for the lifetime of the pinch τ.

Dividing this number by E_{pinch}:

- $Y_{th} / E_{pinch} = 0.17 <\sigma v> /(kT)]\, n_i\, \tau$ number of D-T neutrons per J of pinch energy.

General scaling of Q as function of $n_i\tau$ at pinch temperature T

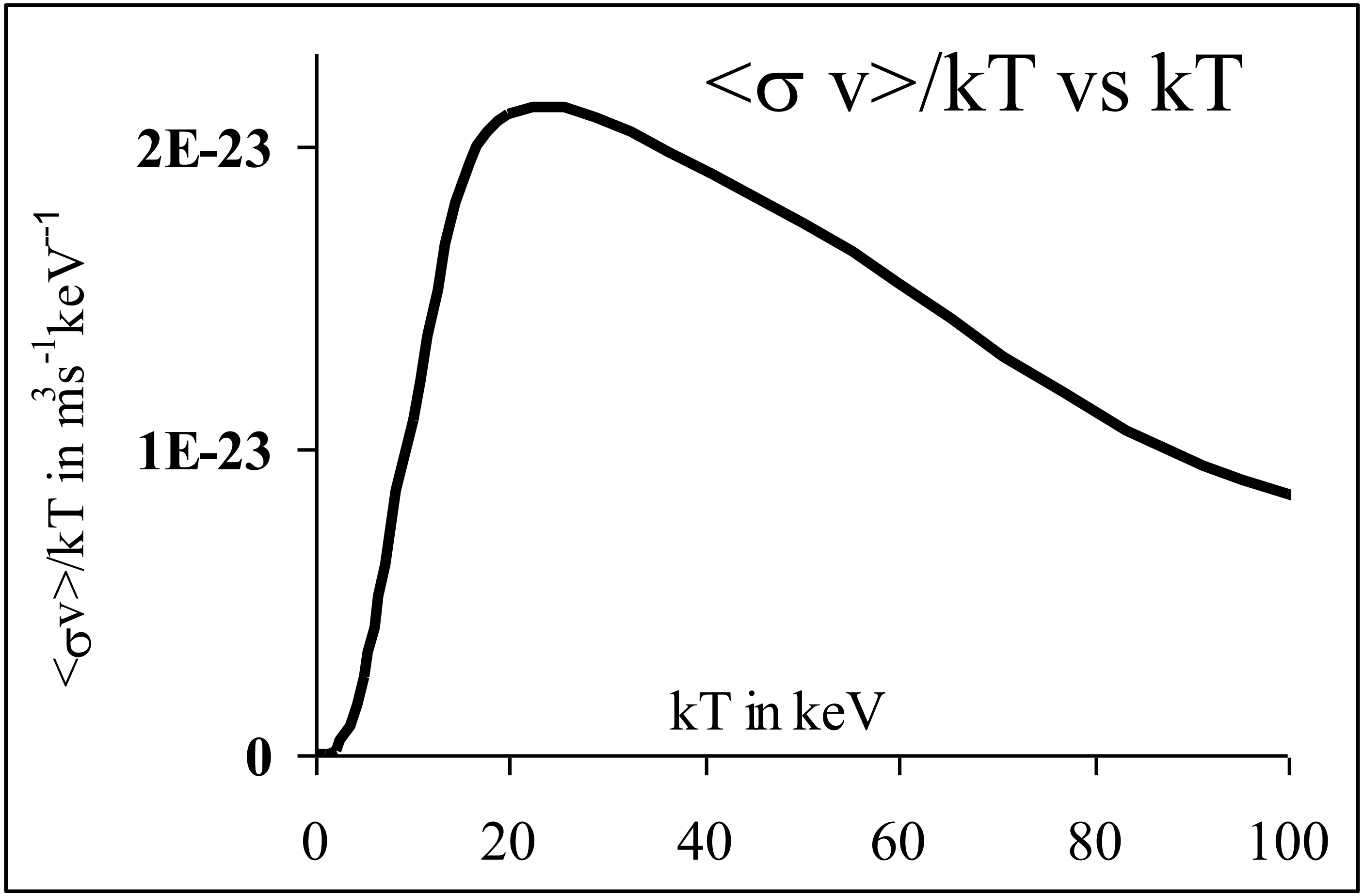
- $E_{th}/E_0 = \quad 240[<\sigma v>/kT]\, n_i\, \tau$

where kT is in keV. Here estimate $E_{pinch} = 0.1\, E_0$ and energy of 1 D-T neutron as 14.1×10^3 keV.

Note the Q value is a function of $(n_i\, \tau)$

and the fusion x-section parameter $[<\sigma v>/kT]$

Optimum [<σv>/kT]



Selecting optimum operational temperature

- From [<σv>/kT] curve, o**ptimum** occurs at T = 20 keV with value = **2.1×10^{-23}** $m^3s^{-1}(keV)^{-1}$.
- Such a concept was expressed in seminal form by Lawson who selected 25 keV

[14. https://en.wikipedia.org/wiki/Lawson_criterion]

[15.Lawson, J. D. (December 1955). *Some Criteria for a Power producing thermonuclear reactor* (PDF) (Technical report). Atomic Energy Research Establishment, Harwell, Berkshire, U. K.]

Scaling of Q as function of $n_i\ \tau$ at optimum temperature of 20 keV

- $E_{th}/E_0 = 5\text{x}\ 10^{-21}\ n_i\ \tau$
- Fixing operation at this optimum temperature, the requirement for better than break-even ie $E_{th}/E_0 > 1$ is
- **$n_i\ \tau > 2 \text{ x } 10^{20}\ \text{m}^{-3}\text{s}$ 20 keV**
- This criterion applied specifically to the DPF is comparable to the well-known Lawson's $n_i\ \tau$ criterion ($n_i\ \tau > 1.5 \text{ x } 10^{20}$). [14]

Example

- For present-day DPF operating at T ~ 0.5 keV pinch duration is ~10-20 ns per cm ‘a’, governed by transit time of small disturbance speed across the pinch diameter.
- At **20 keV the lifetime is around 2 ns per cm ‘a’.**
- For **a = 1 m the lifetime τ is 200 ns**.
- For Q > 1, the requirement is **$n_i > 10^{27}$; close to 50 atm**
- These estimates provide guidance for numerical experiments to find an operational point for Q>1.

Numerical experiments found an operational thermonuclear point for Q>1

L_0	**C_0**	**b**	**a**	**z_0**	**r_0 mΩ**
18	**7,000**	**160**	**155**	**13,500**	**0.1**
massf	**currf**	**massfr**	**currfr**	Model Parameters	
0.1	**0.7**	**0.4**	**0.8**		
V_0	**P_0**	**MW**	**A**	**At-1 mol-2**	Operational
800,000	**55,000**	**5**	**1**	**2**	Parameters

E_0	RESF	c=b/a	I_{peak}	T_p	v_a	v_s	v_p	r_{min}	z_{max}	pinch τ	V_{max}	n_i pinch
kJ			kA	10^6 K	cm/µs	cm/µs	cm/µs	cm	cm	ns	kV	$10^{23}/m^3$
2.24E+12	0.062	1.032	6.2E+07	267	242	257	205	30.2	258.3	250.3	1.2E+07	6.8E+04

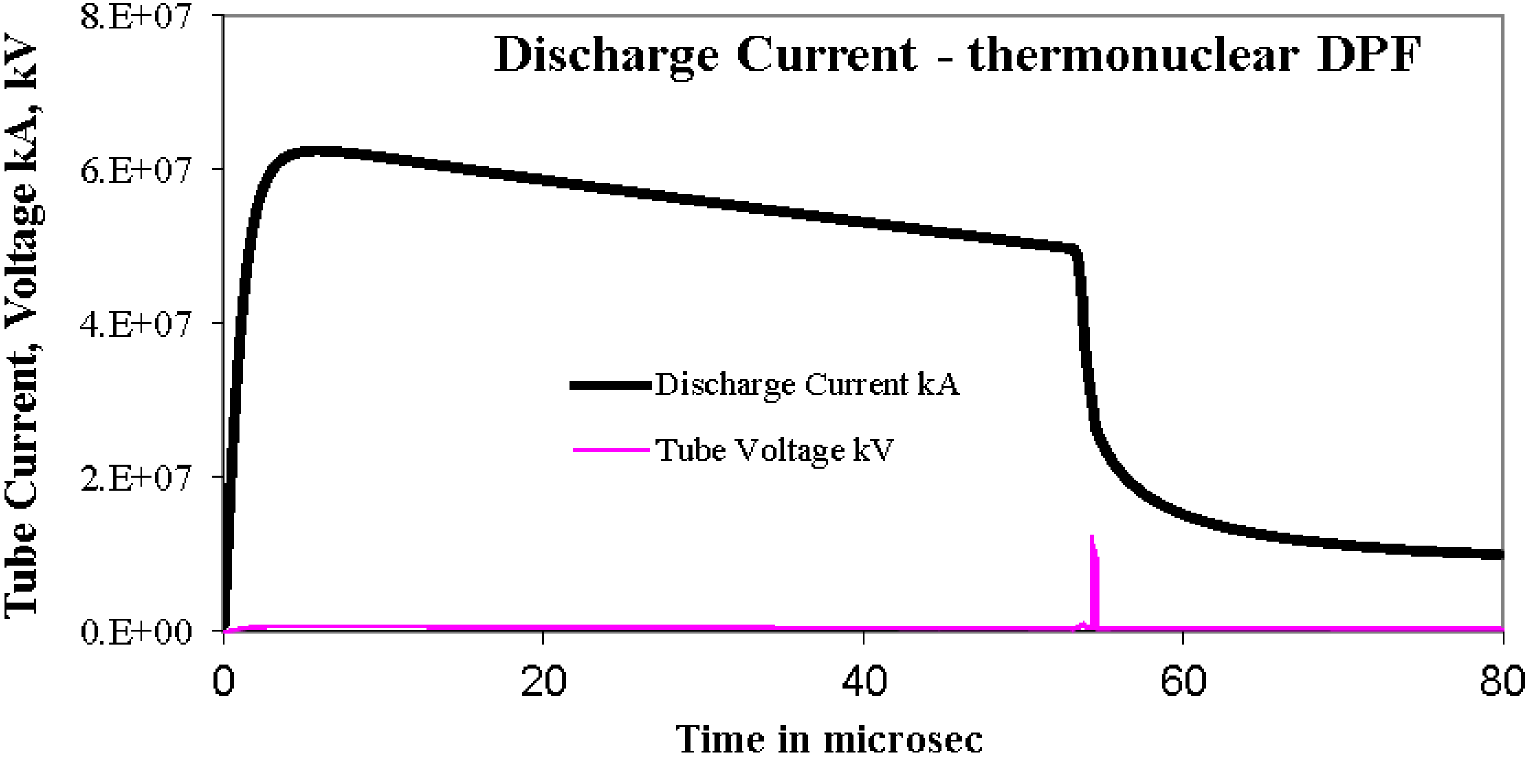
Y_n	EINP	SF	ID	E_N kJ	E_N/E_0	Y_{th}	Y_{b-t}	Y_n
	%		kA/cm					
1.1E+27	45.7	1,719	4.0E+05	2.4E+12	1.09	1.1E+27	2.9E+22	**1.1E+27**

Number beam ions 1.7 x 10^{24} number of ions in radial phase original space = 1.5 x 10^{28}

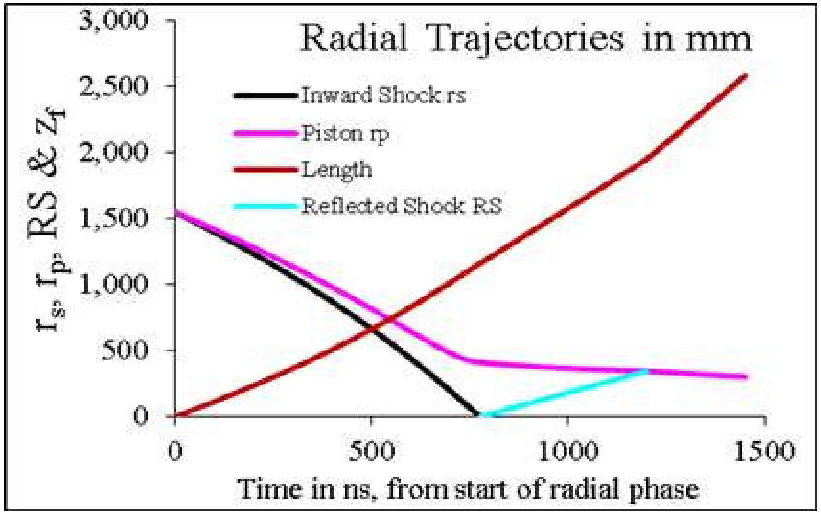
Numerical experiment with our standard code

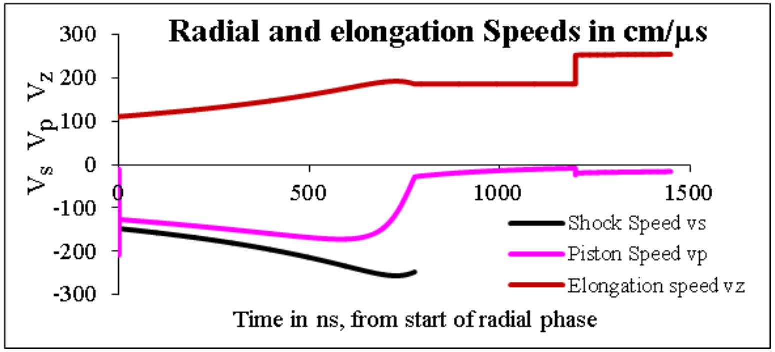
- The code found the following **possible point of operation**:
- Capacitor bank, **7,000 μF, 800 MV**;
- DPF operating pressure: **70 atm. Anode** is 13.5 m long, a= 1.55 m.
- The annular channel for axial phase is 5 cm with c = b/a = 1.03. Such a small channel and radius ratio is necessary to reduce the dynamic impedance of the high shock speed; otherwise the necessary high value of current will require even higher operating voltage.
- The pinch temperature is **23 keV**; pinch ion density is 7 x 10^{27} m^{-3}, estimated duration of 250 ns.
- Thus the **nτ is 18 x 10^{20}**. The D-T neutrons are thermonuclear (99.99%)
- Yield of 1.1 x 10^{27} carrying energy of 2.4x10^{15} J compared to a capacitor bank stored energy of 2.2 x10^{15} J,
- Q almost 1.1.
- The current waveform and radial dynamics and temperature profiles are shown in the following figures.

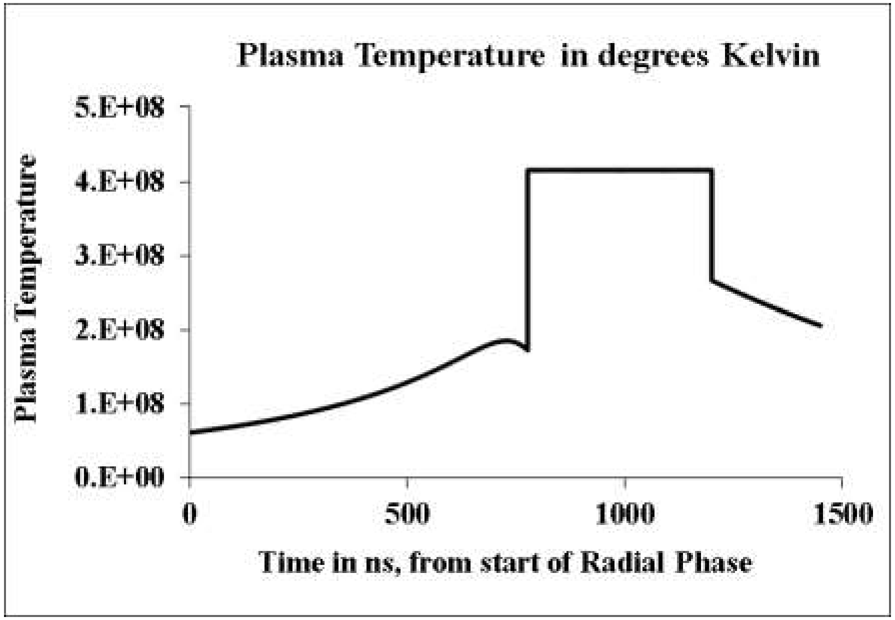
Thermonuclear DPF- current waveform



Thermonuclear DPF - Radial Dynamics and Temperature

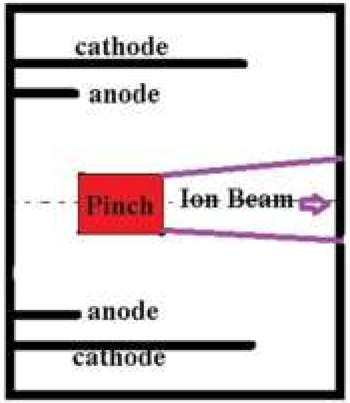

Radial Trajectories in mm


Radial and elongation Speeds in cm/µs


Plasma Temperature in degrees Kelvin

IV: Summary: B-T vs Thermonuclear

D-T	E_0	V_0	P_0	a	T	U	Y_n	Q	Effective Q
	GJ	MV	Atm	m	keV	keV	At pinch		Ion path beyond pinch
B-T (**DPFQ1**)	**7.2**	**1.2**	**5.3**	**1**	**0.1**	**4.0E+03**	**6.0E+18**	**0.002**	**~1**
Thermo-nuclear	2.2E+06	800	72.3	1.6	23	1.2E+07	1.1E+27	1.1	



The B-T shot has 1.3 x 10^{22} ions in the beam (unreacted as computed by the code). These 5 MeV D-T ions will undergo fusion reactions within the high pressure D-T gas to raise the value of Q to greater than 1.

Comparison: **B-T vs Thermonuclear**

- The B-T point (DPFQ1) much less extreme; though still technically far away

DPFQ1 – technically too difficult

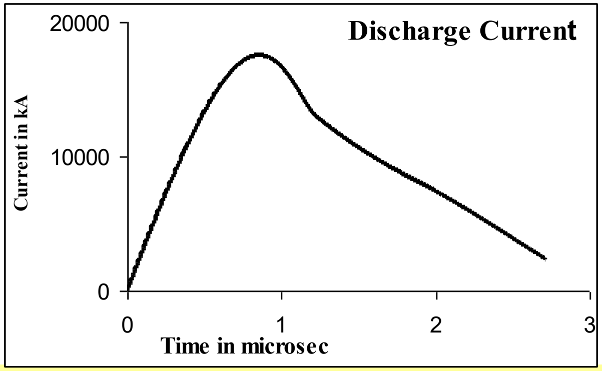
- A **technically feasible device** could be attempted first – DPF0.01

Proposed B-t trial point – DPF0.01- technically feasible – to reach Q~0.01

Lo nH	Co µF	b cm	a cm	zo cmro	mΩ
30	20	18	15	3	3
massf	currf	massfr	currfr		
0.08	0.7	0.2	0.7		
Vo kV	Po Torr	MW	A	At-1 mol-2	
900	**100**	5	1	2	

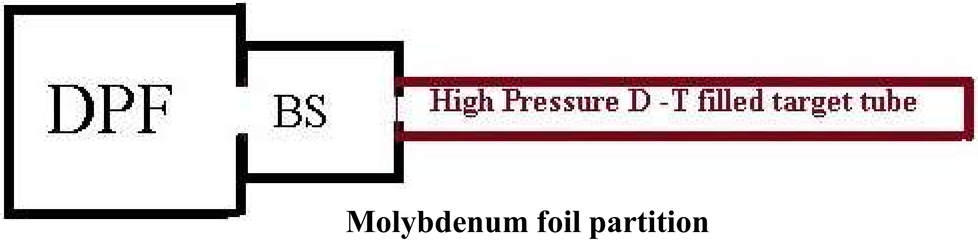
Note on High pressure DPF operation: ICDMP in its recent (September 2017) meeting has discussed plans to look at experimental feasibility of the concept of high pressure optimized operation of DPF

Numerical experiments- DPF0.01



Eo kJ	RESF	c=b/a	I_{peak} kA	T_p 10^6	v_a cm/µs	v_s cm/µs	v_p cm/µs	r_{min} cm	z_{max} cm	pinch τ ns	V_{max} kV	n_i pinch $10^{23}/m^3$
8.10E+03	0.08	1.20	1.8E+04	2.3	11.9	27.3	18.1	4.1	23.9	282	746	31.5

Y_n	EINP %	SF	ID kA/cm	E_N kJ	E_N/E_0	Y_{th}	$Y_{b\text{-}t}$	Yn	N_{ion} in beam
6.6E+13	36.1	0.5	117	0.15	2E-05	4.5E+06	6.6E+13	6.6E+13	8.4E+18



Beam ion energy is 2.5 MeV, beyond optimum 100 keV
BS = reduce ion beam divergence, using pulsed magnetic field.
1 m long Target tube is filled with 10 atm D-T gas to interact with remnant ion beam, increase yield ~ 500x to reach Q~0.01 .

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Conclusion

- Beam-target scaling at optimum beam ion energy of 100 keV suggests 1.2 MV 5 atm D-T DPF would suffice for breakeven at stored capacitor energy of 7 GJ. (**DPFQ1**)
- For comparison scaling of thermonuclear DPF is obtained, guiding numerical experiments to operational point of 800 MV 70 atm, 23 keV [achieving Q = 1.2 at stored energy of 2 million GJ.]
- Comparison of the two possible operational points shows that the Beam-target Q~1 point is 700 times lower in operational voltage, 300,000 times lower in capacitor energy, 14 times lower in operational pressure than the thermonuclear Q ~ 1 operational point.
- The Beam-target operational point (DPFQ1) is much closer to present-day DPF in every operational parameter than the thermonuclear breakeven point.
- A present-day technologically feasible point: **DPF0.01**: 900 kV (8 MJ) 100 Torr with Q ~ 0.01 is proposed for initial test- (incorporates a fusion booster 10 atm D-T target tube).
- This presentation deals with **gross DPF pinch** (scalable); have not discussed structures within the pinch (such as **hot spots**) which could modify gross scaling.

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