The Effect of Specific Heat Ratio on Neutron Yield

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Abstract—It is known that the radius ratio of a plasma pinch depends on the specific heat ratio $\gamma$ of the pinch plasma, where radius pinch ratio is defined as: radius of plasma pinch column/radius of anode. The lower the specific heat ratio the lower would be the pinch radius ratio with corresponding increased compression and pinch density. The deuterium plasma focus pinch is invariably fully ionized and has a specific heat ratio of practically the highest possible value of 5/3. If the deuterium plasma focus could have its specific heat ratio reduced below 5/3 we might expect its radius ratio to be correspondingly reduced, increasing the pinch density, thus improving the D–D fusion neutron yield. To demonstrate this effect we run the Lee model code in deuterium but hypothetically fix the specific heat ratio, reducing it at each run. The results show that indeed the radius ratio is reduced, increasing the compression, and the neutron yield is substantially increased. The effect is used to explain the observed neutron yield enhancement when a deuterium plasma focus doped with a small amount of krypton.

Index Terms—Deuterium, Lee model code, neutron yield, plasma pinching effect, specific heat ratio.

I. INTRODUCTION

THE study of plasma focus (PF) device as a source of fusion neutrons is well established. As a pulsed neutron source, it has applications in the materials research for nuclear fusion reactors and neutron radiography [1], and a portable pulsed source for well-logging applications and for testing neutron shielding to estimate health hazards [2].

It is known that the plasma pinching effect for deuterium occurs at high-temperature range, typically from 100 eV to $>1$ keV. At these temperatures, deuterium is fully ionized and has only 3 DOF. We may write the specific heat ratio $\gamma = 1 + 2/f$ where $f$ is the degree of freedom. We may consider $\gamma$ as an index of compressibility [3] based on nonradiative degree of freedom $f$. The lower the value of $\gamma$ the greater is the compressibility. In a deuterium pinch, with $\gamma$ at the biggest possible value of 5/3, the deuterium pinch may be considered to be the least compressible.

From the point of compressibility one would prefer bigger values of $f$. For example as $f$ increases to large values, the value of $\gamma$ tends toward unity at which limit the gas becomes perfectly compressible since at the hypothetical limit $f = \infty$ the gas cannot heat up and hence has no pressure to resist any compressing force [4]. In planning an experiment one would not be attempting to reach such a limiting situation but one could try to increase the compressibility in searching for a combination of higher density and perhaps better confinement time with sufficient temperature for the purpose of the planned experiment. One example is that of the plasma focus. The results of experiments and numerical experiments have led to a generally agreed conclusion that in the plasma focus the fusion neutrons are produced predominantly by a beam-gas target mechanism [5]–[9]. Thus, an increase in density may be expected to be beneficial to production of fusion neutrons despite a reduction in temperature, within limits. This could be tested by a model which includes both thermonuclear and beam-gas target mechanism. The Lee model code is such a model [10].

We ask the following question. Suppose it is possible to reduce the specific heat ratio $\gamma$ of deuterium to below 5/3, how would this affect the deuterium pinch? Would the resulting increase in compressibility cause a big enough rise in pinch density with a consequent enhanced neutron yield? We run the code in deuterium and hypothetically reduce the specific heat ratio $\gamma$ of the deuterium plasma focus pinch. We use the INTI PF [11], [12] for these numerical experiments. This is the machine operating in the INTI International University, Malaysia. The parameters of the machine are given in Section III below.

The remainder of this paper is organized in the following sections. Section II describes the key feature of Lee model code. Section III describes the procedures for implementing Lee model code to compute the radius ratio of plasma pinch (that is defined as: radius of plasma pinch column/radius of anode) and neutron yield in a range of specific heat ratio. Section IV discusses the computed result for effect of specific heat ratio against the radius ratio of plasma pinch with the corresponding neutron yield. Section V concludes the overall research work and highlights the key implications of the results.

II. LEE MODEL CODE

The Lee model code [10] couples the electrical circuit with PF dynamics, thermodynamics, and radiation. The schematic
of the axial and radial phases used in the modeling is shown in the following Fig. 1.

The details of the radial phases are shown in the radial position versus time schematic of Fig. 2. It is energy-, charge- and mass-consistent. It was initially described in 1983 [13] and used in the design and interpretation of experiments [14]–[17].

An improved five-phase code incorporating finite small disturbance speed [18], radiation, and radiation-coupled dynamics was used [19]–[21]. The Lee model code has been used extensively as a complementary facility in several machines, for example: UNU/ICTP PFF [14], [19]–[21], NX2 [21], [22], NX1 [21], and DENA in a modified version [23]. It has also been used in other machines for design and interpretation including sub-kJ PF machines [24], FNII [25], and the UBA hard X-ray source [26]. Information computed includes axial and radial dynamics [13]–[16], [22], [23], SXR emission characteristics and yield [19]–[22], [27]–[32], design of machines [10], [15], [24], [26], optimization of machines [10], [22], [24], [30], and adaptation to Filippov-type DENA [23]. A speed-enhanced PF [19] was designed based on the code. Recently, PF neutron yield calculations [33], current and neutron yield limitations [34], neutron saturation [9], [35], radiative collapse [36], current-stepped PF [37], and extraction of diagnostic data [32], [38], [39] and anomalous resistance data [40], [41] from current signals have been studied using the code [10]. Most recently, the Lee model code has been used to produce reference numbers for deuteron beam number and energy fluence and flux and scaling trends for these with plasma focus storage energy [42].

Essentially, the five-phase Lee model mainly includes axial phase, radial inward shock phase, radial reflected shock (RS) phase, pinch phase, and expanded column phase. The five phases are summarized as follows.

1) Axial Phase: Described by a snowplow model with an equation of motion coupled to a circuit equation. The equation of motion incorporates empirically the axial phase model parameters: mass and current factors \( f_m \) and \( f_c \), respectively. The mass swept-up factor \( f_m \) accounts for not only the porosity of the current sheet but also for the inclination of the moving current sheet-shock front structure and all other unspecified effects which have effects equivalent to increasing or reducing the amount of mass in the moving structure, during the axial phase. The current factor, \( f_c \), accounts for the fraction of current effectively flowing in the moving structure (due to all effects such as current shedding at or near the back-wall and current sheet inclination). This defines the fraction of current effectively driving the structure, during the axial phase.

2) Radial Inward Shock Phase: Described by four coupled equations using an elongating slug model. The first equation computes the radial inward shock speed from the driving magnetic pressure. The second equation computes the axial elongation speed of the column. The third equation computes the speed of the current sheath, also called the magnetic piston, allowing the current sheath to separate from the shock front by applying an adiabatic approximation. The fourth is the circuit equation. Thermodynamic effects due to ionization and excitation are incorporated into these equations, these effects being important for gases other than hydrogen and deuterium. Temperature and number densities are computed during this phase. A communication delay between shock front and current sheath due to the finite small disturbance speed is crucially implemented in this phase. The model parameters, radial phase mass swept-up and current factors, \( f_m \) and \( f_c \), are empirically incorporated in all three radial phases. The mass swept-up factor \( f_m \) accounts for all mechanisms which have effects equivalent to increasing or reducing the amount of mass in the moving slug, during the radial phase. The current factor, \( f_c \), accounts for the fraction of current effectively flowing in the moving piston forming the back of the slug (due to all effects). This defines the fraction of current effectively driving the radial slug.

3) Radial RS Phase: When the shock front hits the axis, because the focus plasma is collisional, a RS develops which moves radially outward, whilst the radial current-sheath piston continues to move inward. Four coupled equations are also used to describe this phase, these being for the RS moving radially outward, the piston moving radially inward, the elongation of the annular column and the circuit. The same model parameters, \( f_m \) and \( f_c \), are used as in the previous radial phase. The plasma temperature behind the RS undergoes a jump by a factor approximately two.
4) **Pinch Phase:** When the outgoing RS hits the incoming piston the compression enters a radiative phase in which for gases such as neon, radiation emission may actually enhance the compression, where we have included energy loss/gain terms from joule heating and radiation losses into the piston equation of motion. Three coupled equations describe this phase; these being the piston radial motion equation, the pinch column elongation equation and the circuit equation, incorporating the same model parameters as in the previous two phases. Thermodynamic effects are incorporated into this phase. The duration of this slow compression phase is set as the time of transit of small disturbances across the pinched plasma column. The computation of this phase is terminated at the end of this duration. Neutron and soft X-ray production are incorporated into this phase. The fusion neutron yield (when operated in deuterium) incorporates a thermonuclear component and a beam-gas target component.

5) **Expanded Column Phase:** To simulate the current trace beyond this point, we allow the column to suddenly attain the radius of the anode and use the expanded column inductance for further integration. In this final phase the snowplow model is used, and two coupled equations are used; similar to the axial phase aforementioned. This phase is not considered important as it occurs after the focus pinch.

### III. Procedure Used in the Numerical Experiments

In this paper, we configure the code to run as the INTI PF in deuterium.

1) Capacitor bank parameters: \( L_0 = 110 \text{ nH}, C_0 = 30 \mu \text{F}, \) and \( r_0 = 12 \text{ m}\Omega. \)

2) Tube parameters: \( b = 3.2 \text{ cm}, a = 0.95 \text{ cm}, \) and \( z_0 = 16 \text{ cm}. \)

3) Operating parameters: \( V_0 = 12 \text{ kV}; P_0 = 3.5 \text{ torr}. \)

The parameters are: \( L_0 = \) static inductance defined as inductance of discharge circuit without any plasma dynamics (e.g., with the bank short-circuit at the input to the plasma focus tube), \( C_0 = \) bank capacitance, \( r_0 = \) short circuited resistance of the discharge circuit; \( b = \) cathode radius; \( a = \) anode radius; \( z_0 = \) effective anode length; \( V_0 = \) bank charging voltage, and \( P_0 = \) operating pressure. In addition, the typical Lee model code parameters are set to be: 1) for axial phase \( f_{an} = 0.08 \) and \( f_c = 0.7 \) and 2) for radial phase \( f_{mr} = 0.16 \) and \( f_{ct} = 0.7 \).

For each shot we fix the specific heat ratio at a value less than 5/3, forcing the code to run with this hypothetical value of \( \gamma \). After running a shot, we record the radius ratio to note if the reduced \( \gamma \) has any effect on radius ratio. We also record the neutron yield to note whether there is any enhancement.

### IV. Results

The radius ratio is plotted as a function of \( \gamma \) in Fig. 3. Our results (Fig. 3) show that as \( \gamma \) is reduced the radius ratio becomes reduced. This reduction in radius ratio means an increase in plasma density. When the specific heat ratio is at \( \gamma = 1.667 \), the pinch radius ratio 0.13. This is the typical situation of a deuterium plasma focus pinch [19]. When \( \gamma = 1.1 \), the respective pinch radius ratio is reduced to a much smaller value of 0.02. This means that the pinch density has been increased by a factor of 42.3. We expect such a large increase in pinch density to affect the neutron yield whilst the change in temperature has little effect because the thermonuclear component is insignificant compared with the beam-gas target component in the range of operation of all present plasma focus devices.

The neutron yield versus \( \gamma \) is plotted in Fig. 4(a). Our numerical experiments show that as \( \gamma \) is reduced, there is enhancement of neutron yield. For example at \( \gamma = 1.667 \) the neutron yield is \( 1.2 \times 10^7 \). When \( \gamma \) is reduced to 1.5, the yield is increased to \( 3.2 \times 10^7 \). When \( \gamma = 1.1 \), the neutron yield is at the large value of \( 180 \times 10^7 \).

In practical situations, we expect that we are able to reduce the specific heat ratio by a small amount. For example by doping deuterium with krypton the mixture will have fully ionized deuterium and krypton ions which are still in the process of ionizing. This ionizing krypton will provide additional degrees of freedom beyond the translational degrees. The mixture will on average (per particle) have a degree of freedom slightly >3. This will result in a value of \( \gamma \) which is slightly less than 5/3. In this situation, we can expect a small enhancement in neutron yield by about 40%, taking the example of \( \gamma = 1.6 \) as shown in Fig. 4(b). Another contributing factor is that the induced voltage also increases with reduction in \( \gamma \) due to higher current sheath compression resulting in significantly greater inward radial speeds. This higher induced voltage increases the effective deuteron beam speed resulting in higher beam-gas target fusion cross section.

For example at 15 kV 5 torr D the code configured for the INTI PF with standard \( \gamma \) of 1.667 produces D beam ion energy of 63.3 keV. Hypothetically reducing the \( \gamma \) to 1.6 increases pinch speeds and compressibility resulting in D beam ion energy of 70.8 keV. There is an increase of \( Y_n \) from 1.27 \( \times 10^7 \) to 1.88 \( \times 10^7 \) (48%) due to this change of \( \gamma \). Checking the D–D fusion cross section [43], there is an increase from 0.009794 b to 0.00988 b (24%) due to the increased beam ion energy. This contributes to the increase in neutron yield. These numerical experiments indicate that such levels of neutron enhancement.
can be expected when compared with the practical experiment of krypton doping on D–D reaction [44].

V. CONCLUSION

In this paper, the study of the hypothetical effect of specific heat ratio against neutron yield has been carried out. The results show that there is a large enhancement of density and neutron yield if the value of could be reduced to small values like 1.1.

However, practically we can conceive of only a reduction of γ by a small amount below 5/3. This is possible by doping deuterium with krypton or xenon. For doping with krypton, preliminary consideration shows that the γ of a lightly doped deuterium is around 1.6; which will give neutron yield enhancement of 40% for the doped deuterium. Detailed work on specific heat ratio of mixed gases [44] needs to be carried out to more closely correlate with actual experiments such as those recently reported [45].
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